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Revisiting the Economic Value of Groundwater

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Key Points:

- Revisited the theoretical framework of computing the economic value of groundwater in a dynamic context.
- Proved the existence of the dynamic reallocation value, which is generated by an intertemporal intake reallocation of groundwater users.
- Disregarding this new value can underestimate the value of groundwater as an essential instrument for climate adaptation.

19 **Abstract**

20 This paper revisits the theoretical framework of computing the economic value of groundwater
21 in a dynamic context. Specifically, we prove that an additional type of economic value exists,
22 that is, the dynamic reallocation value (DRV), which has been overlooked in existing studies,
23 and we propose a new construction of the total economic value of groundwater with social
24 implications for the role of groundwater in climate adaptation. We examine the existence of this
25 new value and its underlying behavioural mechanism using a simple two-stage model, and then
26 generalise the specification to a dynamic model with an arbitrary number of stages. We find that
27 behind the positive values of DRV, users intentionally destabilize total water use by amplifying
28 their reactions against surface water fluctuations and still realize a higher total expected benefit
29 than in the case without uncertainty. We show that this behaviour is an intertemporal reallocation
30 of groundwater intake against changes in intertemporal cost allocations caused by the users'
31 stabilizing behaviours. Disregarding the DRV underestimates the economic value of
32 groundwater as an essential instrument for climate adaptation.

33

34 **1 Introduction**

35 Over the past few decades, a considerable number of studies have attempted to quantify
36 the economic value of groundwater in various locations worldwide and have explored improved
37 groundwater management systems (e.g., Burt, 1964; Kim et al., 1989; Tsur, 1990; Tsur &
38 Graham-Tomasi, 1991; Ramasamy, 1996; Amigues et al., 1997; National Research Council,
39 1997; Hernández-Mora et al., 2003; Pulido-Velázquez et al., 2004; Ranganathan & Palanisami,
40 2004; Syaukat & Fox, 2004; Kakumanu & Bauer, 2008; Diao et al., 2008; Palanisami et al.,
41 2008; Marques et al., 2010; Ananthini & Palanisami 2010; Reichard et al., 2010;
42 Nanthakumaran & Palanisami, 2011; Gomez & Rola, 2011; Palanisami et al., 2012; Kovacs et
43 al., 2015; Rouhi Rad et al., 2017; Foster et al., 2017; MacEwan et al., 2017; Ashwell et al., 2018;
44 Quintana-Ashwell & Gholson, 2022; Msangi & Hejazi, 2022). Most of these attempts are
45 grounded in theoretical frameworks traced back to Tsur's seminal papers on the buffering role of
46 groundwater (Tsur et al., 1989; Tsur, 1990; Tsur & Graham-Tomas, 1991; Gemma & Tsur,
47 2007).

48 The basic construction of such frameworks is as follows: the total economic value (TEV)
49 of groundwater can be divided into the augmentation value (AV) and the stabilization value
50 (SV). The AV is the value of being augmented by an increase in the average water intake
51 through the exploitation of groundwater resources in addition to surface water. The SV is the
52 value of mitigating the impact of surface water fluctuations by adjusting groundwater intake.
53 Typically, groundwater extraction increases during periods of surface water shortage and
54 decreases during periods of surface water abundance. Tsur presented a methodological
55 framework for computing the values of these components.

56 The present paper revisits this framework in a dynamic context. Specifically, it proves the
57 existence of an additional type of economic value, that is, the dynamic reallocation value (DRV),
58 which has been overlooked in previous studies, including those conducted in a dynamic context.
59 Furthermore, we propose a new construction of the total economic value of groundwater, with
60 social implications for the role of groundwater in climate adaptation.

61 Similar to the SV, the DRV is derived from the adaptive behaviours of economic agents
 62 against surface-water variations under uncertain environments. However, they are conducted
 63 with different economic intentions and movements in opposite directions. They are optimizations
 64 against the changes in intertemporal cost allocations that occur as a reflection of stabilizing
 65 behaviours. Therefore, disregarding the DRV underestimates the economic value of groundwater
 66 as an essential instrument for climate adaptation.

67 Similar to most relevant studies (e.g., Peter et al., 2020; Monobina & Kurt, 2014; Abell et
 68 al., 2017; Cécile & Marine, 2019; Msangi & Hejazi, 2022), the present paper limit its attention to
 69 industrial and agricultural use of groundwater. We therefore do not deal with the economic
 70 benefits of nonconsumptive water use, such as landscapes, amenities, and tourism. In addition,
 71 we do not consider the environmental impacts of groundwater extraction, such as salt damage,
 72 land subsidence, and other externalities on human society and ecosystems. However, we discuss
 73 some policy implications of our findings regarding these issues in the discussion section.

74 The remainder of this paper is organised as follows: section 2 reviews the theoretical
 75 background of the economic value of groundwater. Section 3 describes our model formulation.
 76 Section 4 proves the existence of DRV, and discusses its underlying mechanism using a simple
 77 two-stage model. Section 5 generalises the findings to a model with an arbitrary number of
 78 stages, and presents some numerical illustrations of the DRV. Finally, Section 6 concludes the
 79 paper.

80 2. Theoretical Background

81 The basic idea of the Tsur's framework is the following. To compute the economic value
 82 of groundwater, we first use the difference between the expected net economic benefit of using
 83 both surface water and groundwater conjunctively and that of using only surface water, taking
 84 the latter as a baseline (Tsur, 1990; Reichard & Raucher, 2003; Sato, 2015). Specifically, we can
 85 calculate the economic value of groundwater V^u as follows:

$$86 \quad V^u \triangleq E[F(w_u) - C(G_u) \cdot (w_u - S)] - E[F(S)], \# \quad (1)$$

87
 88 where $F(\cdot)$ is a concave benefit function, w_u the benefit-maximizing total water use, $C(\cdot)$ a unit
 89 extraction cost that depends on the groundwater stock G_u , and S uncertain surface water whose
 90 known mean value is \bar{S} . In most groundwater literature, a unit cost function depends on the
 91 distance between the water table and ground surface. Although we implicitly incorporate the
 92 mathematical transformation from the stock amount to the above distance in the form of the
 93 function $C(\cdot)$ to simplify calculations, this doesn't have any effect on the essence of the solutions
 94 and conclusions below. We assume $C(\cdot)$ is strictly decreasing, that is, the smaller the
 95 groundwater stock, the higher the unit cost is. For simplicity, we assume that the user can utilize
 96 the surface water for free; therefore, the remaining $w_u - S$ represents the amount of groundwater
 97 used.

98 The difference obtained in (1) however contains both the AV and SV. To eliminate the
 99 AV and extract a pure SV, Tsur uses the difference in benefits when there is no uncertainty in S
 100 as another baseline. That is,

$$V^c \triangleq F(w_c) - C(G_c) \cdot (w_c - \bar{S}) - F(\bar{S}), \# \quad (2)$$

102

103 where w_c is the benefit-maximizing total water use in the case without uncertainty and G_c is the
104 groundwater stock. The SV is then given by

105

$$SV \triangleq V^u - V^c. \# \quad (3)$$

106

107 In this simplified static problem, if the groundwater stocks are equal, that is, $G_u = G_c$, and
108 so are the unit costs, the benefit-maximizing amount of water use are also the same, thereby
109 indicating that $w_u = w_c$, and so are the expected pumping costs. This is because the benefit-
110 maximizing amount of water used is determined at the level at which the marginal net benefit
111 $F'(w)$ is equal to the marginal cost (unit cost) $C(G)$. Therefore, the user pumps an amount that
112 can completely offset surface water fluctuations and stabilize the net benefit. Accordingly, the
113 SV can eventually be computed as the difference in benefits with and without uncertainty when
114 the user can only use surface water.

115

$$SV = V^u - V^c = F(\bar{S}) - E[F(S)]. \# \quad (4)$$

116

117 Thus, the SV can be expressed as a risk premium that the user is willing to pay to stabilize the
118 surface water flow at the mean (Gemma & Tsur, 2007).

119 Using (4), the augmentation value can be computed as the remainder, $V^u - SV$:

120

$$AV \triangleq F(w_c) - C(G_c) \cdot (w_c - \bar{S}) - F(\bar{S}). \# \quad (5)$$

121

122 The total economic value of the groundwater is the sum $TEV \triangleq SV + AV$:

123

$$TEV = F(w_c) - C(G_c) \cdot (w_c - \bar{S}) - E[F(S)]. \# \quad (6)$$

124

125 Various studies have applied this approach to evaluate the economic value of
126 groundwater in actual water environments (e.g., for cases in India, Ramasamy (1996),
127 Ranganathan & Palanisami (2004), Gemma & Tsur (2007), Kakumanu & Bauer (2008),
128 Palanisami et al. (2008), Ananthini & Palanisami (2010), Nanthakumaran and Palanisami
129 (2011), and Palanisami et al. (2012); for cases in the United States, Tsur (1997), Kovacs et al.
130 (2015), Kovacs & West, 2016; MacEwan et al. (2017), and Msangi & Hejazi (2022); for cases in
131 Israeli, Tsur (1990)).

132 However, the transformation from (3) to (4) is not applicable to dynamic cases in general,
133 even if the initial groundwater stocks were the same. Gemma and Tsur (2007) seem to be aware

134 of this point. Hence, in an attempt to extend the Tsur (1990)'s framework to a dynamic
 135 environment, they avoided using a simple analogy of the risk premium in equation (4) but did
 136 not explore what exists in the gap between (3) and (4). The most recent attempt to apply Tsur's
 137 framework to a dynamic environment is Msangi and Hejazi (2022), which analyzes the impact of
 138 suboptimal behaviours and the physical constraints of extraction abilities on the economic value
 139 of groundwater. Through an empirical application to California, they showed that suboptimal
 140 behaviours diminish the AV while keeping the SV unaffected in the unconstrained case;
 141 however, the SV could be diminished in the constrained case. We will come back to this point in
 142 later sections.

143 On the other hand, the present paper argues that an additional type of economic value is
 144 hidden in the difference between V^u and V^c , that is,

$$V^u - V^c = SV + DRV. \# \quad (7)$$

146

147 Thus, the total economic value of the groundwater is composed of three components:

148

$$TEV = AV + SV + DRV. \# \quad (8)$$

149

150 **3 Model Formulation**

151 In each of the following analyses, we consider models with N users for the sake of
 152 generality, and denote the user set $\{1, \dots, N\}$ as \mathcal{N} . This enables us to examine the economic
 153 value of groundwater in both optimal and suboptimal environments. The former type of solution
 154 is described by a single decision-maker model, where the social planner distributes groundwater
 155 intake to each user during each time period to maximize the intertemporal sum of the aggregate
 156 net economic benefits of all users (henceforth, *single decision-maker regime*). The other type of
 157 solution is described by a multiple-user model in which each user plays a noncooperative
 158 dynamic game in choosing the amount of groundwater intake with the aim of maximizing its
 159 own intertemporal sum of net economic benefits (henceforth, *multiple-user regime*). Replacing
 160 $N = 1$ provides simpler scenarios for a single user.

161 The water environment in both regimes is governed by a stochastic dynamic process
 162 determined by two state variables: $G_{t-1} \in \mathcal{G}$, the groundwater stock, and $S_t \in \mathcal{S}$, the surface
 163 water flow, both available to users at the beginning of period t , where \mathcal{G} and \mathcal{S} represent sets of
 164 possible amount of the groundwater stock and surface water flow, respectively. The transition
 165 equation for the groundwater stock is as follows:

166

$$G_t = f(G_{t-1}, R_t, g_{1t}, \dots, g_{Nt}) \triangleq G_{t-1} + R_t - \sum_{\mathcal{N}} g_{it}, \quad \# \quad (9)$$

167

168 where $g_{it} (\geq 0)$ is the groundwater intake by user i in period t and $R_t (\geq 0)$ denotes the
 169 groundwater recharge in period t . Groundwater dynamics can be governed by a variety of

170 interconnected hydrological processes driven by various climatic, topographic, and
 171 hydrogeological factors (Cuthbert et al., 2019). Therefore, more complex mechanisms, such as
 172 stochastic and spatially heterogeneous groundwater recharge, which are affected by local
 173 precipitation and surface water intake, can be introduced. However, for analytical simplicity, we
 174 don't touch on such complexities and use a fixed value, R , throughout all periods. However, such
 175 simplifications do not invalidate the essence of our argument on the existence of a new value,
 176 because the behavioural mechanism that generates it is the users' natural reactions to the
 177 underlying nature of the groundwater stock transition as argued below.

178 The surface flow S_t is given by:

$$179 \quad S_t = \bar{S}_t + \xi_t, \quad \# \quad (10)$$

180
 181 where \bar{S}_t is the average flow amount that is expected in period t in normal years and ξ_t denotes
 182 the fluctuation from the average in period t , where $\xi_t > 0$ means a period of abundant water
 183 supply and $\xi_t < 0$ a period of water scarcity. For the analytic approach in the following section,
 184 we assume, like most groundwater literature (e.g., Burt, 1964; Tsur & Graham-Tomasi, 1991;
 185 Provencher & Burt, 1994; Knapp & Olson, 1995; Joodavi et al., 2015), that ξ_t is a stationary,
 186 temporally independent random variable of a known distribution with a zero mean and variance
 187 of σ^2 .

188 Users make decisions on groundwater intake after observing the realization of surface
 189 water flows during the current period. Let $s_{it} = \varepsilon_i S_t$ denote the amount of surface water utilized
 190 by user i in period t , where ε_i is the share of user i and $\sum_{\mathcal{N}} \varepsilon_i = 1$. For simplicity, we assume
 191 that users can use surface water within this range at no additional cost. Let w_{it} be the total
 192 amount of water used by user i in period t ; thus, $w_{it} = g_{it} + s_{it}$.

193 $F_i(w_{it})$ represents the instantaneous benefit accruing to user i in period t , which is
 194 assumed to be quadratic for acquiring analytical solutions:

$$195 \quad F_i(w_{it}) \triangleq a_i w_{it} - b w_{it}^2, \quad \#$$

196
 197 where a_i and b are positive constants. This represents diminishing returns to production, which
 198 accords with most production practices as reported in many groundwater literature (e.g., Gisser
 199 & Sánchez, 1980; Provencher & Burt, 1994; Gardner et al., 1997; Msangi & Hejazi, 2022;
 200 Quintana-Ashwell & Gholson, 2022). Based on this, we introduce user heterogeneity by
 201 differentiating parameter a_i s. Although we do not differentiate parameter b to obtain analytical
 202 solutions for the dynamic game, this differentiation allows us to cover a broad range of
 203 heterogeneity in terms of production scale and technology.

204 Let $C_i(G_t)$ denote the unit cost of user i for pumping groundwater to the surface, which
 205 depends on the groundwater stock.

$$206 \quad C_i(G_t) \triangleq c_i - d G_t, \quad \#$$

207

208 where c_i and d are positive constants. Therefore, the cost is inversely proportional to the total
 209 inventory. This is consistent with the assumptions of most groundwater studies such as those of
 210 Gisser and Sánchez (1980) and Gardner et al. (1997). Moreover, although we do not differentiate
 211 the parameter d to obtain analytical solutions, the differentiation of c_i enables us to represent a
 212 considerable amount of heterogeneity in pumping facilities and the spatial diversity of an aquifer.
 213 Again, the specifications of these parameters do not invalidate our arguments on the new value.

214

215 The instantaneous net benefit, including the pumping cost, for user i in period t is given
 216 by:

217

$$\pi_i(g_{it}, G_{t-1}, S_t) \triangleq F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1})g_{it}. \#$$

218

219 The period set $\{1, \dots, T\}$ is denoted by \mathcal{T} , and let $\Pi_i: (\mathcal{G} \times \mathcal{S} \times U_{i1} \times U_{-i1}) \times \dots \times (\mathcal{G} \times \mathcal{S} \times$
 220 $U_{iT} \times U_{-iT}) \rightarrow \mathbb{R}_{\geq 0}$ denote the discounted intertemporal sum of user i 's expected net benefits:

221

$$\Pi_i(G_0, S_1, g_{i1}, g_{-i1}, \dots, G_{T-1}, S_T, g_{iT}, g_{-iT}) \triangleq E \left[\sum_{t \in \mathcal{T}} \beta^{t-1} [F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1})g_{it}] \right], \quad (11)$$

222

223 where U_{it} is the set of admissible actions of user i in period t , and $\beta \in [0, 1]$ is a discount factor.
 224 Symbols with the subscript $-i$ indicate that they are a variable or set for the users excluding user
 225 i . The social planner maximizes the discounted intertemporal sum of the aggregate expected net
 226 benefits $\Pi: (\mathcal{G} \times \mathcal{S} \times U_{11} \times \dots \times U_{N1}) \times \dots \times (\mathcal{G} \times \mathcal{S} \times U_{1T} \times \dots \times U_{NT}) \rightarrow \mathbb{R}_{\geq 0}$:

227

$$\begin{aligned} \Pi(G_0, S_1, g_{11}, \dots, g_{N1}, \dots, G_{T-1}, S_T, g_{1T}, \dots, g_{NT}) &\triangleq \sum_{i \in \mathcal{N}} \Pi_i(G_0, S_1, g_{i1}, g_{-i1}, \dots, G_{T-1}, S_T, g_{iT}, g_{-iT}) \\ &= E \left[\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} \beta^{t-1} [F_i(g_{it} + \varepsilon_i S_t) - C_i(G_{t-1})g_{it}] \right], \end{aligned}$$

228

229 subject to equations (9) and (10), and the initial stock level G_0 . One of the possible requirements
 230 for admissible actions is, $U_{it} := [0, G_{t-1}]$, that is, users can exploit the aquifer to its whole stock
 231 level. In the following, we assume that the total groundwater intake does not exceed the current
 232 groundwater stock within a single period. We come back to a drawback of this simplification in
 233 the discussion section.

234 In the multiple-user regime, user i maximizes the discounted intertemporal sum of the
 235 expected net benefits (11) subject to (10), the initial stock level G_0 , and the transition equations
 236 of the groundwater stock:

237

$$G_t = f_i(G_{t-1}, g_{it}, g_{-it}) \triangleq G_{t-1} + R - g_{it} - \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} g_{jt}, \quad t \in \mathcal{T}.$$

238

239 Let γ_{it} denote an admissible strategy of user i for $S_t \in \mathcal{S}, G_{t-1} \in \mathcal{G}, t \in \mathcal{T}$, and let Γ_{it} denote the
 240 set of admissible strategies. We can then describe the dynamic process as an N -user T -stage
 241 discrete-time stochastic dynamic noncooperative game defined by
 242 $\{\mathcal{N}, \mathcal{T}, \mathcal{G}, \mathcal{S}, \{U_{it}\}_{i \in \mathcal{N}, t \in \mathcal{T}}, \{f_{it}\}_{i \in \mathcal{N}, t \in \mathcal{T}}, \{\Gamma_{it}\}_{i \in \mathcal{N}, t \in \mathcal{T}}, \{\Pi_i\}_{i \in \mathcal{T}}\}$.

243

244 4 Two-stage Model

245 We start by demonstrating the existence of a new value using a simple two-stage model
 246 and examine the underlying economic mechanisms.

247 4.1 Existence of the DRV

248 For the two-stage model, by solving backwards from the second stage, we obtain unique
 249 solutions for each regime and for cases with and without uncertainty (See S11 in the Supporting
 250 Information for solutions and derivation). In the following discussion, we use the notations in
 251 Table 1 for the variables derived from these solutions:

252

253 **Table 1.** Notations for the variables derived from the solutions.

254

255 (a) Single decision-maker regime

Notation	Description
$\pi_u^{\text{single}} = \pi_{u1}^{\text{single}} + \pi_{u2}^{\text{single}}$	aggregate expected net benefit (and its temporal decomposition) in the uncertain case
$\pi_c^{\text{single}} = \pi_{c1}^{\text{single}} + \pi_{c2}^{\text{single}}$	aggregate expected net benefit (and its temporal decomposition) in the certain case
$w_{u1}^{\text{single}}(S_1)$	aggregate water use at the first stage after observing S_1 in the uncertain case
$w_{c1}^{\text{single}}(\bar{S})$	aggregate water use at the first stage after observing \bar{S} in the certain case
$g_{u1}^{\text{single}}(S_1)$	aggregate groundwater intake at the first stage after observing S_1 in the uncertain case
$g_{c1}^{\text{single}}(\bar{S})$	aggregate groundwater intake at the first stage after observing \bar{S} in the certain case

256

257 (b) Multiple-user regime

Notation	Description
$\pi_u^{\text{multi}} = \pi_{u1}^{\text{multi}} + \pi_{u2}^{\text{multi}}$	aggregate expected net benefit (and its temporal decomposition) in the uncertain case
$\pi_c^{\text{multi}} = \pi_{c1}^{\text{multi}} + \pi_{c2}^{\text{multi}}$	aggregate expected net benefit (and its temporal decomposition) in the certain case
$w_{u1}^{\text{multi}}(S_1)$	aggregate water use at the first stage after observing S_1 in

	the uncertain case
$w_{c1}^{\text{multi}}(\bar{S})$	aggregate water use at the first stage after observing \bar{S} in the certain case
$g_{u1}^{\text{multi}}(S_1)$	aggregate groundwater intake at the first stage after observing S_1 in the uncertain case
$g_{c1}^{\text{multi}}(\bar{S})$	aggregate groundwater intake at the first stage after observing \bar{S} in the certain case

258

259 Note that the expected net benefits in the table are the expected values evaluated before
 260 the realization of surface water in the first period, whereas water use and groundwater intake are
 261 the values that users determine after observing it. In addition, we don't use the discount factor to
 262 evaluate the expected net benefits, although the solutions used here, that is, $g_{u1}^{\text{single}}(S_1)$, $g_{c1}^{\text{single}}(\bar{S})$,
 263 $g_{u1}^{\text{multi}}(S_1)$ and $g_{c1}^{\text{multi}}(\bar{S})$, are the results of users' decisions with discounting. Therefore, we
 264 evaluate the economic value of each period equally. Summing up the discounted net benefits is
 265 another option for evaluating the economic value of groundwater in a dynamic context and may
 266 sometimes be more appropriate for resource management practices. However, as researchers, we
 267 take a different approach for our analytical purpose to evaluate users' behaviours equally
 268 throughout the period.

269 Although we explain the reason behind the name later, we define the dynamic
 270 reallocation value (DRV) as follows:

271

272 **Definition 1.** The dynamic reallocation value (DRV) is the difference in the intertemporal sum
 273 of the aggregate expected net benefit in cases with and without uncertainty in surface water:

274

$$\begin{aligned} DRV_{\text{single}} &\triangleq \pi_u^{\text{single}} - \pi_c^{\text{single}}, \\ DRV_{\text{multi}} &\triangleq \pi_u^{\text{multi}} - \pi_c^{\text{multi}}. \# \end{aligned} \tag{12}$$

275

276 We can easily derive the following from the solutions of the two-stage model:

277

278 **Proposition 1.** The dynamic reallocation value (DRV) is positive in both the single decision-
 279 maker and multiple-user regimes. That is,

280

$$\begin{aligned} DRV_{\text{single}} &= \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2} \sigma^2 > 0, \\ DRV_{\text{multi}} &= \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2} \sigma^2 > 0. \# \end{aligned} \tag{13}$$

281

282 For the full proof, see SI2 in the Supporting Information.

283 This requires significant reconsideration of the specifications of the economic value of
 284 groundwater used in the literature, which indicates that the above differences are zero. First, the
 285 transformation from (3) into (4) is incorrect in dynamic environments. Second, we argue that the
 286 specification of the SV in (3) is not appropriate, because the difference $V^u - V^c$ contains a
 287 different type of economic value. That is,

$$\begin{aligned} V_{\text{single}}^u - V_{\text{single}}^c &= SV_{\text{single}} + DRV_{\text{single}}, \\ V_{\text{multi}}^u - V_{\text{multi}}^c &= SV_{\text{multi}} + DRV_{\text{multi}}, \end{aligned} \quad (3')$$

289 where for the computation of SV_{single} and SV_{multi} , we use the specification in (4). In the two-
 290 stage model:
 291

$$SV_{\text{single}} = SV_{\text{multi}} = \sum_{t=1}^2 \sum_{i \in \mathcal{N}} \{F_i(\varepsilon_i \bar{S}) - E[F_i(\varepsilon_i S_t)]\} = 2b \left(\sum_{i \in \mathcal{N}} \varepsilon_i^2 \right) \sigma^2. \# \quad (4')$$

293 In the latter half of this section, we explain why the dynamic reallocation value should not be
 294 considered part of the SV.
 295

296 Third, the above considerations redefine the composition of the total economic value of
 297 groundwater. Based on (5), the augmentation values can be derived as follows:

$$\begin{aligned} AV_{\text{single}} &= \pi_c^{\text{single}} - \sum_{t=1}^2 \sum_{i \in \mathcal{N}} F_i(\varepsilon_i \bar{S}), \\ AV_{\text{multi}} &= \pi_c^{\text{multi}} - \sum_{t=1}^2 \sum_{i \in \mathcal{N}} F_i(\varepsilon_i \bar{S}). \end{aligned} \quad (5')$$

299 We can therefore derive a new composition:
 300

$$\begin{aligned} TEV_{\text{single}} &\triangleq \pi_u^{\text{single}} - \sum_{t=1}^2 \sum_{i \in \mathcal{N}} E[F_i(\varepsilon_i S_t)] = AV_{\text{single}} + SV_{\text{single}} + DRV_{\text{single}}, \\ TEV_{\text{multi}} &\triangleq \pi_u^{\text{multi}} - \sum_{t=1}^2 \sum_{i \in \mathcal{N}} E[F_i(\varepsilon_i S_t)] = AV_{\text{multi}} + SV_{\text{multi}} + DRV_{\text{multi}}. \end{aligned} \quad (14)$$

302 Studies that measure the economic value of groundwater using the specification of SV in
 303 (3) most likely overestimate the magnitude of SV, and those that use the specification of (4)
 304 overestimate the magnitude of AV.
 305

306

307 4.2 Behavioural mechanisms of the DRV

308 However, what is DRV and why should it be distinguished from SV and AV? To answer
 309 this, the behavioural mechanisms of the users that generate this value need to be
 310 comprehensively understood. From the solutions shown in SI1 in the Supporting Information, we
 311 can easily demonstrate how the users' groundwater intake reacts to surface water fluctuations.

312

313 **Proposition 2.** When the surface water in the first period, S_1 , deviates from its mean value by
 314 $S_1 - \bar{S}$, the aggregate groundwater intake responds to it by more than $S_1 - \bar{S}$ in both the single
 315 decision-maker and multiple-user regimes. That is,

316

$$g_{u1}^{\text{single}}(S_1) - g_{u1}^{\text{single}}(\bar{S}) = g_{u1}^{\text{single}}(S_1) - g_{c1}^{\text{single}}(\bar{S}) = -\frac{4b^2}{4b^2 - N^2d^2\beta}(S_1 - \bar{S}), \quad (15)$$

$$g_{u1}^{\text{multi}}(S_1) - g_{u1}^{\text{multi}}(\bar{S}) = g_{u1}^{\text{multi}}(S_1) - g_{c1}^{\text{multi}}(\bar{S}) = -\frac{4b^2}{4b^2 - Nd^2\beta}(S_1 - \bar{S}). \#$$

317

318 This is significantly different from the stabilizing behaviour implied by previous studies
 319 in the specification of Equation (4), where the groundwater intake responds to the surface water
 320 fluctuation on a one-to-one basis to ensure that the former movement perfectly offsets the latter
 321 change. If the surface-water content increases by $S_1 - \bar{S}$, the groundwater intake declines by
 322 $S_1 - \bar{S}$. If the surface water decreases by $S_1 - \bar{S}$, the groundwater intake increases by $S_1 - \bar{S}$.
 323 However, Proposition 2 suggests that groundwater intake not only stabilizes the fluctuation but
 324 also destabilizes the total water use. From Equation (15), we can easily derive the following:

325

$$w_{u1}^{\text{single}}(S_1) - w_{u1}^{\text{single}}(\bar{S}) = w_{u1}^{\text{single}}(S_1) - w_{c1}^{\text{single}}(\bar{S}) = -\frac{N^2d^2\beta}{4b^2 - N^2d^2\beta}(S_1 - \bar{S}), \quad (16)$$

$$w_{u1}^{\text{multi}}(S_1) - w_{u1}^{\text{multi}}(\bar{S}) = w_{u1}^{\text{multi}}(S_1) - w_{c1}^{\text{multi}}(\bar{S}) = -\frac{Nd^2\beta}{4b^2 - Nd^2\beta}(S_1 - \bar{S}). \#$$

326

327 In specification (4), the surface water fluctuation has no effect on the total water use because it is
 328 perfectly absorbed by the offsetting movement of the groundwater intake; however, Equation
 329 (16) reveals that it has an effect. When the amount of surface water increases, the total water
 330 declines and as the surface-water decreases, the total water increases.

331 This intended destabilization decreases the expected benefit of the first period, but it is
 332 more than covered in the second period, as shown in the next proposition, which can easily be
 333 calculated from the results shown in SI1 in the Supporting Information. This leads to the
 334 intertemporal sum of the expected benefit being greater than that in a certain case, as shown in
 335 Proposition 1.

336

337 **Proposition 3.** The aggregate expected net benefit in the first period in the case with uncertainty
 338 is less than that in the case without uncertainty, whereas the aggregate expected net benefit in the
 339 second period in the case with uncertainty is greater than that in the case without uncertainty.
 340 That is,

$$\begin{aligned}
 341 \quad \pi_{u1}^{\text{single}} - \pi_{c1}^{\text{single}} &= -\frac{N^3 b d^4 \beta^2}{(4b^2 - N^2 d^2 \beta)^2} \sigma^2 < 0, \\
 \pi_{u2}^{\text{single}} - \pi_{c2}^{\text{single}} &= \frac{4N b^3 d^2}{(4b^2 - N^2 d^2 \beta)^2} \sigma^2 > 0, \\
 \pi_{u1}^{\text{multi}} - \pi_{c1}^{\text{multi}} &= -\frac{N b d^4 \beta^2}{(4b^2 - N d^2 \beta)^2} \sigma^2 < 0, \\
 \pi_{u2}^{\text{multi}} - \pi_{c2}^{\text{multi}} &= \frac{4N b^3 d^2}{(4b^2 - N d^2 \beta)^2} \sigma^2 > 0. \#
 \end{aligned} \tag{17}$$

342
 343 From these results, we can expect that there is another consideration in users' intake
 344 decisions that differs from the stabilizing behaviour. Therefore, we aim to elucidate the reason
 345 behind users' intentionally destabilizing water use and why such behaviours generate higher total
 346 benefit than that in cases without uncertainty.

347 To examine these points graphically, we further simplify the model in four respects: first,
 348 we consider a single user model with the instantaneous benefit function $F(w_t) = aw_t - bw_t^2$;
 349 second, we consider that the surface water takes between two values S_L ($= 0$ for simplicity) and
 350 S_H with a probability of $1/2$ for each and with the mean value \bar{S} ($= S_H/2$); third, there is no
 351 groundwater recharge ($R = 0$); and fourth, the discount factor $\beta = 1$. These simplifications are
 352 only for graphical illustration, and the argument below holds for the more general specifications
 353 discussed thus far.

354 In the first stage, after observing surface water S_1 , the user faces the following problem:

$$355 \quad \max_{g_1} F(S_1 + g_1) - C(G_0)g_1 + E_1[F(S_2 + g_2(S_2, g_1)) - C(G_0 - g_1)g_2(S_2, g_1)], \#$$

356
 357 where $g_2(S_2, g_1)$ is the solution in the second period with stock level $G_0 - g_1$ and the
 358 observation of S_2 :

$$359 \quad g_2(S_2, g_1) = \frac{1}{2b}(a - c + d(G_0 - g_1)) - S_2. \# \tag{18}$$

360
 361 As discussed in the previous section, we excluded cases in which the user exploits the entire
 362 stock in a single period. The first-order condition then provides the benefit-maximizing intake
 363 g_1^* :

364

$$F'(S_1 + g_1^*) = C(G_0) + E_1[-C'(G_0 - g_1^*)g_2(S_2, g_1^*)]. \# \quad (19)$$

365

366 The benefit-maximizing groundwater intake is therefore ensured when the marginal benefit is
 367 equal to the sum of the unit cost of the first period (the first term on the right side) and the
 368 marginal user cost (the second term). The latter is the future pumping cost that would have been
 369 saved by decreasing a marginal unit of groundwater intake in the first period. In other words, this
 370 is the opportunity cost of the current extraction.

371 We examine this mechanism in two steps. First, we introduce a policy in which the user
 372 absorbs the surface water fluctuation perfectly in the first period and keeps the total water use for
 373 that period constant (at the mean value). This is not the optimal behaviour but provides a very
 374 good case for understanding the behavioural mechanism of dynamic reallocation. We call this
 375 *Policy E* (where *E* represents *exact stabilization*) and denote it by g_{Et} . Next, we introduce the
 376 optimal policy described in Proposition 2. In this policy, the user amplifies its reaction against
 377 surface water fluctuation to generate an artificial destabilization but can achieve a full dynamic
 378 reallocation value. We call this *Policy R* (where *R* represented *reallocation*) and denote it by g_{Rt} .
 379 In addition, we call a reference policy that the user would take when there is no uncertainty
 380 *Policy C* (where *C* represents *certainty*) and denote it by g_{Ct} . In the following figures, we
 381 describe the user's intake decisions and the corresponding benefits and costs after observing (a)
 382 S_H and (b) S_L during the first period.

383

384 Policy E

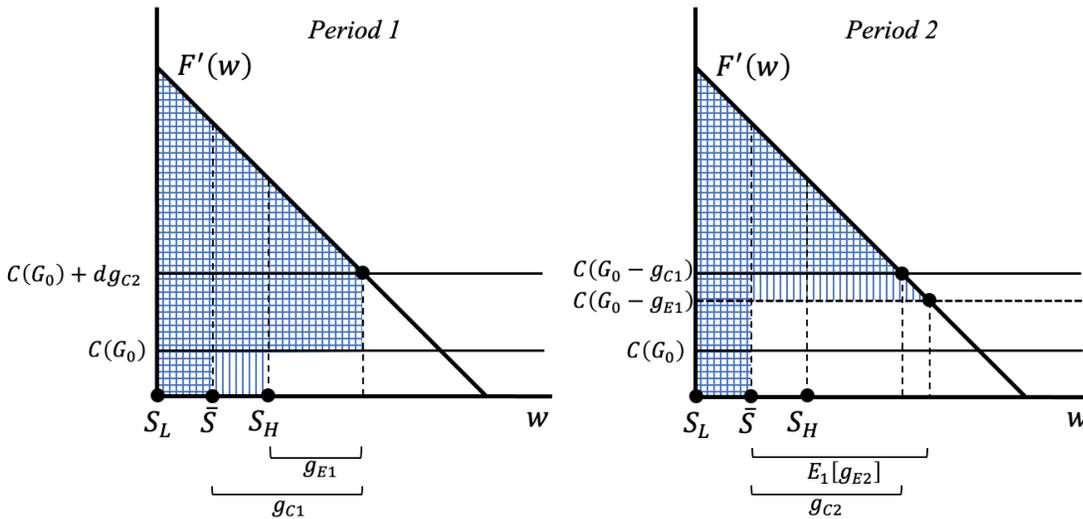
385 Figure 1 shows a comparison of Policies E and C. In Policy C, the total water use in the
 386 first period is determined at the intersection of the marginal benefit curve $F'(w)$ and the sum of
 387 the unit cost and marginal user cost $C(G_0) + dg_{C2}$. Policy E also maintains this amount by
 388 changing the groundwater intake g_{E1} to offset the surface water fluctuation in an exact manner.
 389 The expected net benefits evaluated in period 0 are the same for both policies. This is exactly the
 390 same situation as that captured by the simplification of Equation (4). Therefore, the SV in period
 391 1 is evaluated purely by the risk premium in (4).

392 But the truth is that the impact of the fluctuation does not disappear at all. It is transferred
 393 to period 2 through the corresponding change in the groundwater stock and unit cost, which is
 394 represented by the differences between the solid and dotted horizontal lines on the right side of
 395 Figure 1(a) and (b). Note that the intake of Policy E in period 2 (g_{E2}) is shown as the expected
 396 amount evaluated before the realization of surface water in this period.

397 Surprisingly, even in Policy E, which replicates the standard stabilizing behaviour, if we
 398 stand at period 0 (the moment before observing S in period 1), the expected net benefit is larger
 399 than that of Policy C. Why does the case with uncertainty achieve a higher expected net benefit
 400 than that of the case without uncertainty, even with a concave benefit function (i.e. a risk-averse
 401 agent)? Figure 2 shows the increments and decrements in benefits and costs over the values of
 402 Policy C. When considering the benefit side only, policy E obtains a lower expected value by the
 403 amount corresponding to the area of the triangle in the grey shaded area on the left. This is
 404 normal for risk-averse agents. However, on the cost side, it achieves a higher expected reduction

405 by the amount corresponding to the shaded square in the middle. Consequently, the expected net
 406 benefit of Policy E is higher than that of Policy C, as indicated by the area of the shaded triangle
 407 on the right. Therefore, the source of the higher net benefit is the cost side. Why, however, does
 408 Policy E achieve a larger cost reduction? In period 1, the user increases the intake when it
 409 observes S_H and decreases it when S_L to stabilize the benefit in the period. These behaviours can
 410 simultaneously be seen as an intertemporal reallocation of the groundwater intake, which in turn
 411 affects the intertemporal allocation of groundwater stock and thereby that of unit pumping cost.
 412 In the case of our two-stage model, the increase (decrease) in intake in period 1 increases
 413 (reduces) the unit pumping cost in period 2. This makes the relative price of groundwater in
 414 period 2 to period 1 higher (lower) than that of Policy C. Thus, transferring the intake from
 415 period 2 to period 1 or from period 1 to period 2 reduces the pumping cost in period 2. In other
 416 words, the intertemporal reallocation of groundwater intake, which occurs as a result of the
 417 stabilizing behaviour in period 1, generates a higher expected net benefit in Policy E than in
 418 Policy C through a cost reduction realized by the corresponding intertemporal reallocation of the
 419 unit pumping cost.

420

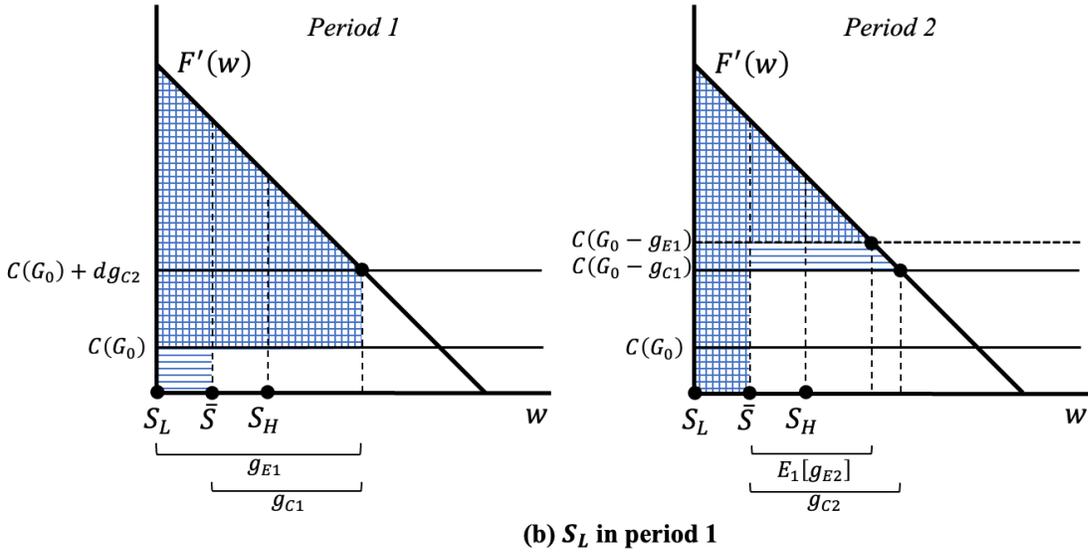


(a) S_H in period 1

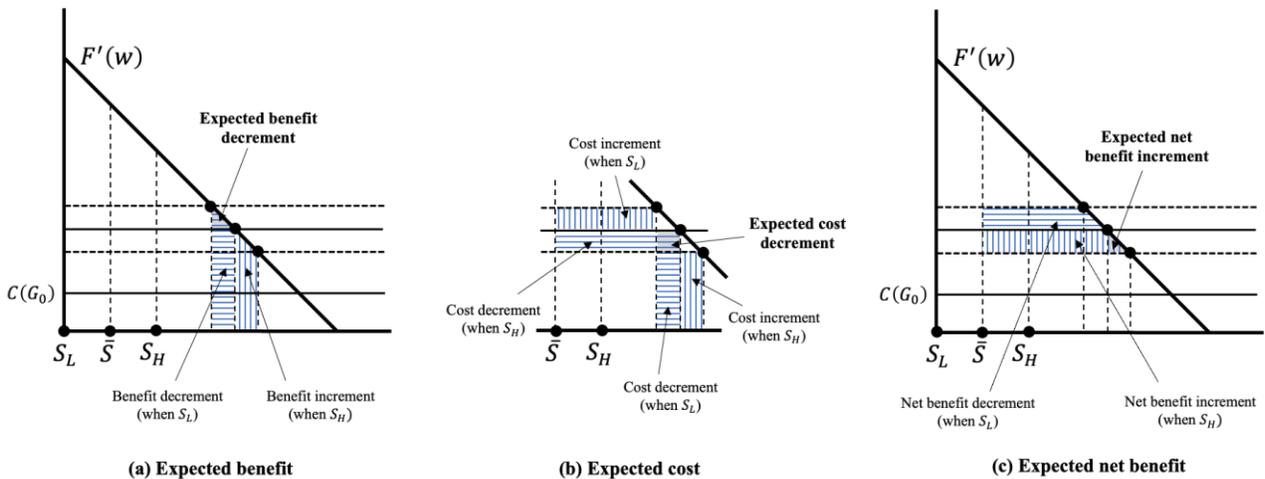
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422

423



424
 425 **Figure 1.** User's intake decisions and corresponding net benefits for Policies E and C. The line
 426 segment that is declining to the right is the marginal benefit curve $F'(w)$. The horizontal lines
 427 represent the unit cost or the sum of the unit cost and marginal user cost. The areas in the vertical
 428 stripes represent the net benefits achieved by Policy E and the horizontal stripes represent those
 429 achieved by Policy C.
 430
 431



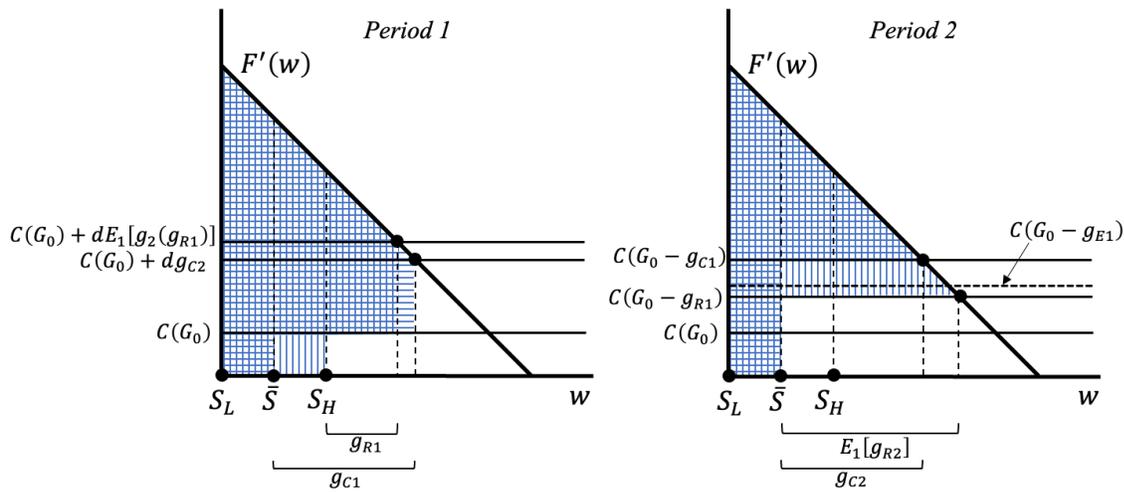
432
 433 **Figure 2.** Increments and decrements in expected benefit and cost of Policy E over Policy C. The
 434 areas in the vertical stripes represent the increments and the horizontal stripes represent the
 435 decrements in (a) benefit, (b) cost, and (c) net benefit. The areas of the shaded triangles or
 436 squares represent the increments or decrements in the expected amount evaluated in period 0.
 437
 438

439 Policy R

440 Policy E is not optimal because the intake in period 1 is a simple reaction to the surface
 441 water fluctuation of the period and not the benefit-maximizing intake derived from equation (19).
 442 In Policy R, the user determines the intake to equate the marginal benefit with the sum of the unit

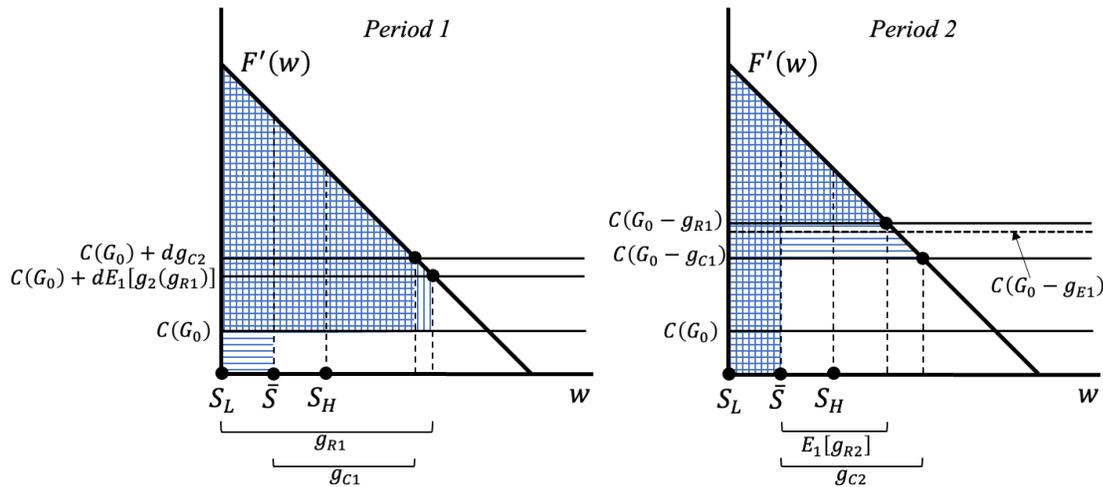
443 cost and marginal user cost, which reflects the relative price of groundwater in period 2 over
 444 period 1. Figure 3 illustrates these behaviours. In period 1, the user increases the intake to more
 445 than that of Policy E when it observes S_H and decreases the intake to more than that of Policy E
 446 when it observes S_L . This destabilizes the benefit in period 1 and lowers the expected net benefit
 447 of the period. However, it achieves a much larger cost reduction in period 2 than that of Policy E
 448 and generates a higher total expected net benefit. This is why the artificial destabilization
 449 described in Proposition 2 decreases the expected net benefit in the first period but increases it in
 450 the second period, as stated in Proposition 3, and finally results in an increased total expected net
 451 benefit, as stated in Proposition 1.

452
 453
 454
 455



(a) S_H in period 1

456
 457



(b) S_L in period 1

458
 459
 460
 461

Figure 3. User's intake decisions and corresponding net benefits for Policies R and C. The areas in the vertical stripes represent the net benefits achieved by policy R and the horizontal stripes represent those achieved by policy C. The line segment that is declining to the right is the

462 marginal benefit curve $F'(w)$. The horizontal lines represent the unit cost or the sum of the unit
 463 cost and marginal user cost.

464

465 In summary, the DRV is derived from users' optimization to the changes in intertemporal
 466 cost allocations that occur as a reflection of their stabilizing behaviours. Users actively reallocate
 467 their groundwater intake intertemporally to save their pumping costs throughout the periods,
 468 thereby achieving a higher total benefit even in the case with uncertainty than in the case without
 469 uncertainty. We, therefore, call this value the "dynamic reallocation value."

470

471 5 Dynamic Model with An Arbitrary Number of Stages

472 5.1 Generalization of the DRV

473 We first generalise the formulation of the DRV in Equation (13) to models with an
 474 arbitrary number of stages T . Subsequently, we examine how the generalized DRV reacts to
 475 changes in major parameters using some numerical illustrations.

476

477 **Proposition 4.** In the single decision-maker regime, the dynamic reallocation value (DRV) in a
 478 dynamic problem of maximizing $\Pi: (\mathcal{G} \times \mathcal{S} \times U_{11} \times \dots \times U_{N1}) \times \dots \times (\mathcal{G} \times \mathcal{S} \times U_{1T} \times \dots \times$
 479 $U_{NT}) \rightarrow \mathbb{R}_{\geq 0}$ subject to (9), (10), and the initial stock level G_0 is given by $DRV_{\text{single}} = \sum_{\mathcal{T}} \Xi_t$,
 480 where

481

$$\begin{aligned} \Xi_t \triangleq & -\frac{b}{N}(1 + \Psi(t))^2 \sigma^2 \\ & + \frac{\Phi(t)(Nd - b\Phi(t))}{N} \left\{ [\Psi(t-1)]^2 + [\Psi(t-2)(1 - \Phi(t-1))]^2 + \dots \right. \\ & \left. + \left[\Psi(1) \prod_{\tau=2}^{t-1} (1 - \Phi(\tau)) \right]^2 \right\} \sigma^2, \quad 4 \leq t \leq T, \end{aligned}$$

$$\begin{aligned} \Xi_3 \triangleq & -\frac{b}{N}(1 + \Psi(t))^2 \sigma^2 \\ & + \frac{\Phi(t)(Nd - b\Phi(t))}{N} \left\{ [\Psi(t-1)]^2 + [\Psi(t-2)(1 - \Phi(t-1))]^2 \right\} \sigma^2, \\ & t = 3, \end{aligned}$$

$$\Xi_2 \triangleq -\frac{b}{N}(1 + \Psi(t))^2 \sigma^2 + \frac{\Phi(t)(Nd - b\Phi(t))}{N} [\Psi(t-1)]^2 \sigma^2, \quad t = 2,$$

$$\Xi_1 \triangleq -\frac{b}{N}(1 + \Psi(t))^2 \sigma^2, \quad t = 1.$$

482

483 For the definition of the functions Ψ and Φ and the proof, see SI3 in the Supporting Information.

484

485 **Proposition 5.** In the multiple-user regime, the dynamic reallocation value (DRV) in a N -user
 486 discrete-time stochastic infinite dynamic noncooperative game of a finite horizon,
 487 $\{\mathcal{N}, \mathcal{T}, \mathcal{G}, \mathcal{S}, \{U_{it}\}_{i \in \mathcal{N}, t \in \mathcal{T}}, \{f_{it}\}_{i \in \mathcal{N}, t \in \mathcal{T}}, \{\Gamma_{it}\}_{i \in \mathcal{N}, t \in \mathcal{T}}, \{\Pi_i\}_{i \in \mathcal{T}}\}$, is given by $DRV_{\text{multi}} = \sum_{\mathcal{T}} \tilde{\Xi}_t$,
 488 where

489

$$\begin{aligned} \tilde{\Xi}_t \triangleq & -\frac{b}{N} \left(1 + \tilde{\Psi}(t)\right)^2 \sigma^2 \\ & + \frac{\tilde{\Phi}(t) (Nd - b\tilde{\Phi}(t))}{N} \left\{ [\tilde{\Psi}(t-1)]^2 + [\tilde{\Psi}(t-2) (1 - \tilde{\Phi}(t-1))]^2 + \dots \right. \\ & \left. + \left[\tilde{\Psi}(1) \prod_{\tau=2}^{t-1} (1 - \tilde{\Phi}(\tau)) \right]^2 \right\} \sigma^2, \quad 4 \leq t \leq T, \end{aligned}$$

$$\begin{aligned} \tilde{\Xi}_3 \triangleq & -\frac{b}{N} \left(1 + \tilde{\Psi}(t)\right)^2 \sigma^2 \\ & + \frac{\tilde{\Phi}(t) (Nd - b\tilde{\Phi}(t))}{N} \left\{ [\tilde{\Psi}(t-1)]^2 + [\tilde{\Psi}(t-2) (1 - \tilde{\Phi}(t-1))]^2 \right\} \sigma^2, \\ & t = 3, \end{aligned}$$

$$\tilde{\Xi}_2 \triangleq -\frac{b}{N} \left(1 + \tilde{\Psi}(t)\right)^2 \sigma^2 + \frac{\tilde{\Phi}(t) (Nd - b\tilde{\Phi}(t))}{N} [\tilde{\Psi}(t-1)]^2 \sigma^2, \quad t = 2,$$

$$\tilde{\Xi}_1 \triangleq -\frac{b}{N} \left(1 + \tilde{\Psi}(t)\right)^2 \sigma^2, \quad t = 1.$$

490

491 For the definition of the functions $\tilde{\Psi}$ and $\tilde{\Phi}$ and the proof, see SI4 in the Supporting Information.

492

493 5.2 Numerical illustrations

494 To analyze how the dynamic reallocation value reacts to changes in major parameters,
 495 such as the number of stages or the variance of surface water fluctuation, and how such reactions
 496 differ between the single-decision-maker regime and the multiple-user regime, this subsection
 497 provides some numerical illustrations of each type of economic value by applying a set of
 498 sample parameter values to the analytical results of the previous section and subsection
 499 (especially, Propositions 4 and 5). The values used are listed in Table 2. Note that the purpose of
 500 this subsection is not to simulate the concrete values of the DRV using actual water data. Rather,
 501 we aim to examine the basic responses of the DRV to changes in major parameters in a

502 theoretical setting. So the values in the table are arbitrarily chosen to allow clearer graphical
 503 demonstrations in the figures below, and they do not have concrete physical and monetary units.
 504

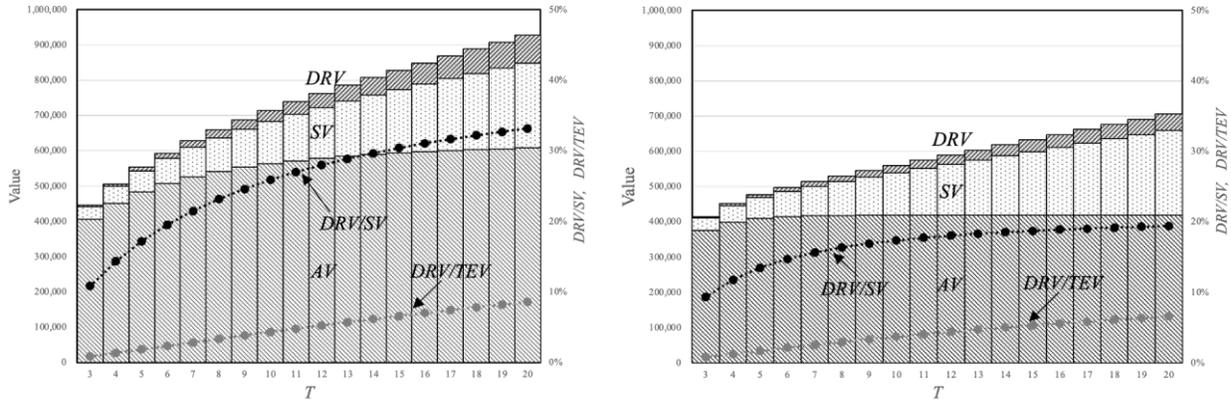
Table 2. Parameter Values Used in Numerical Illustration

Parameter	Description	Value
a_i	First-order coefficient of instantaneous benefit function	12,200
b	Second-order coefficient of instantaneous benefit function	300
c_i	Pumping cost intercept	21,000
d	Pumping cost slope	[15, 20]
G_0	Initial groundwater stock	1,000
\bar{S}	Average surface water supply	100
σ^2	Variance of surface water supply	[0, 600]
R	Natural groundwater recharge	0.1
N	Number of users	10
ε_i	Share of water right	$1/N$
β	Discount factor	0.98
T	Number of stages	{3, 4, ..., 20}

505 Figure 4 shows the composition of the three value types at a different number of stages
 506 ($T = 3, 4, \dots, 20$, $d = 20$, $\sigma^2 = 400$). First, we note that all values, including the DRV, increase
 507 as the number of stages T increases, but in different manners. The increment in the AV over T s
 508 decreases as T increases. This is because of the users' intertemporal levelling behaviour of
 509 groundwater use within a given stock amount. The SV increases linearly; the increment in the
 510 SV over T s is constant. This is natural if we consider the SV specification in Equation (4).
 511 However, the increment in the DRV increases as T increases. This is because, as was revealed in
 512 the previous section, the source of the DRV is the intertemporal reallocation of groundwater
 513 intake, and it is transferred to the following stages through the corresponding change in stock and
 514 cost. Every intake at each stage impacts the following stages; hence, the DRV increases with
 515 increasing increments as the time horizon is prolonged. As a result, the share of the DRV in the
 516 total economic value of groundwater increases as T increases, and the ratio of the DRV to the SV
 517 also increases as T increases.

518 Second, the multiple-user regime exhibits lower values than the single-decision-maker
 519 regime exhibits, except for the SV, which is the same between the two regimes. In addition, the
 520 share of the DRV in the total economic value or to the SV is lower in the multiple-user regime
 521 than in the single-decision-maker regime. The results for the AV and SV are consistent with the
 522 findings of previous studies (e.g., Gemma & Tsur, 2007). A new finding is about the DRV. If we
 523 compare the equation of (15) between the two regimes, the users respond to the surface water
 524 fluctuations by more than the amount of fluctuation, but the extent is weaker in the multiple-user
 525 regime. Overexploitation of groundwater in a suboptimal environment hinders users from fully
 526 utilizing reallocation opportunities.

527



(a) Single decision-maker regime

(b) Multiple-user regime

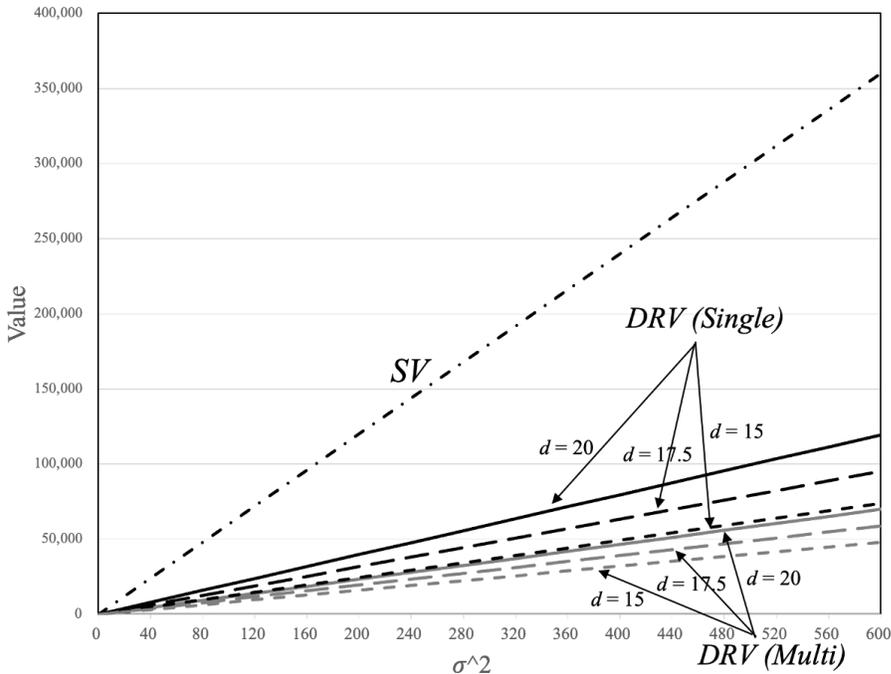
528
 529 **Figure 4.** Composition of economic value in various numbers of stages. The bar charts represent
 530 the values of AV, SV, and DRV, and the line graphs represent the DRV/SV and DRV/TEV
 531 ratios.

532

533 Figure 5 shows how the SV and DRV change as the variance of the surface water
 534 fluctuation increases. As can be predicted from the formulas, both SV and DRV respond linearly
 535 to the variance increase; however, the figure indicates that the slope of the DRV is smaller than
 536 that of the SV. It is not easy to show the reason for the smaller slope analytically, but an intuitive
 537 explanation may be, as we discussed in the previous section, the DRV can be seen as a by-
 538 product of the users' stabilizing behaviour. Therefore, the DRV utilizes surface water
 539 fluctuations to a lesser extent than the SV does. Again, the slope of the DRV is smaller in the
 540 multiple-user regime than in the single-decision-maker regime.

541 Figure 5 also shows how DRV responds to different levels of the pumping slope
 542 parameter d , which is the marginal unit cost with respect to stock level G . The larger the
 543 parameter value, the more the unit cost responds to a marginal change in stock level. As shown
 544 in the figure, the slope of the DRV curve increases as d increases.

545



546
547
548

Figure 5. SV and DRV for different levels of variance in surface water fluctuation.

549 6 Discussion and Conclusions

550 In this study, we revisited the total economic value of groundwater. Specifically, we
551 proved the existence of a dynamic reallocation value and proposed a new construction of the
552 total economic value of groundwater comprising three components: augmentation value (AV),
553 stabilization value (SV), and dynamic reallocation value (DRV).

554 Furthermore, we showed the economic mechanisms underlying the DRVs using simple
555 analytical models. Similar to the SV, the DRV is derived from the adaptive behaviours of
556 economic agents against surface-water variations under uncertain environments. However, they
557 are conducted with different economic intentions and movement in opposite directions. Our
558 model results showed that users intentionally destabilize their water use by increasing or
559 decreasing their groundwater intake by more than the amount of surface water fluctuations. Such
560 seemingly irrational behaviours arise from their optimization against changes in intertemporal
561 cost allocations that occur as a reflection of stabilizing behaviours. That is, the stabilization
562 behaviour of one period can simultaneously be seen as an intertemporal reallocation of
563 groundwater intake from or to the following periods. Such reallocations change the unit pumping
564 cost, and thereby, the relative price of groundwater in the future. Users actively take advantage
565 of this to save their pumping costs throughout the period and achieve a higher total benefit, even
566 in cases with uncertainty than in cases without uncertainty.

567 In addition, we analyzed how the DRV reacts to changes in parameters such as the
568 number of stages or the variance of surface water fluctuation using numerical illustrations. First,
569 we found that the share of DRV in the total economic value of groundwater increases as the time
570 horizon increases. Second, DRV diminishes in a suboptimal environment with multiple users

571 because the overexploitation of groundwater hinders users from fully utilizing reallocation
572 opportunities.

573 Unfortunately, the DRV has been overlooked in all existing studies, including those
574 conducted in dynamic contexts. Typically, studies using the simplified specification of the SV
575 are likely to include the DRV in the AV unconsciously, and thereby overestimate the AV.
576 Therefore, they estimate the value of groundwater to adapt to climate instability only in terms of
577 its stabilization function. However, as shown in this study, users can derive additional value from
578 groundwater than simply offsetting surface water fluctuations. In other words, even if the TEV
579 itself is not affected, disregarding DRV can underestimate the value of groundwater as an
580 essential instrument for climate adaptation. Although the present paper did not apply our results
581 to actual water data, it is preferable that the economic valuations of existing empirical studies be
582 re-examined using our new framework incorporating DRV.

583 The major methodological limitations of this paper are as follows. First, similar to almost
584 all existing groundwater studies (e.g., Gisser & Sánchez, 1980; Provencher & Burt, 1994;
585 Gardner et al., 1997; Msangi & Hejazi, 2022; Quintana-Ashwell & Gholson, 2022), we used a
586 quadratic form for the benefit function (production function), which enabled us to derive simple
587 analytical and even reduced-form solutions. Although we believe that our conclusions are not
588 affected by function types, as long as they allow for diminishing marginal benefits, an
589 assumption that accords with most production practices, we can numerically examine other types
590 of benefit functions in future studies. Second, we used a stationary, temporally independent
591 random variable for surface water fluctuations. This is because the typical situations that the
592 current study addresses are those in which industrial or agricultural users tackle fluctuations in a
593 relatively short period of time, for example, monthly. However, we can examine our findings in
594 broadened environments, such as Markovian disturbances (e.g., Srikanthan & McMahon, 1985,
595 2001) or even in cases in which distributions are completely unknown, through numerical
596 simulations using reinforcement learning. Third, the present study used a relatively simple
597 setting for hydrological processes, such as deterministic recharge; however, we can examine our
598 framework under more complex interactions between precipitation, surface water flow, and
599 groundwater recharge both natural and artificial (e.g., Barlow et al., 2003; Vedula et al., 2005;
600 Hantush, 2005; Fleckenstein et al., 2006; Pulido-Velázquez et al., 2006; Pulido-Velázquez et al.,
601 2007; Marques et al., 2010; Reznik et al., 2022). Finally, we excluded cases in which the entire
602 stock is exploited or should be kept above a threshold level, or cases in which groundwater
603 supply is physically limited or reduced by its depletion. These cases have been extensively
604 studied in some literature (e.g., Gisser & Sánchez, 1980; Gisser & Allen, 1984; Zeitouni, 2004;
605 Msangi & Hejazi, 2022; Rouhi Rad et al., 2017; Foster et al., 2017). Although excluding these
606 allows us to focus on a simple analytical demonstration of the DRV, it can take away the
607 possibilities of considering different types of responses to intertemporal reallocation of intake
608 that can generate the DRV. For example, it is known that, when the stock is binding, the user
609 cost comprises not only the depth cost but also the stock cost (Provencher and Burt, 1993). It is
610 therefore very likely that the DRV increases when user consider the latter type of user cost. We
611 leave the evaluation of DRV in such cases for future study.

612 Finally, let us discuss some policy implications that we can derive from the study
613 findings. First, the existence of the DRV augments the importance of sustainable groundwater
614 management, particularly in areas threatened by surface-water fluctuations under climate change.
615 Groundwater can provide those areas with larger economic benefits beyond its stabilizing

616 effects. Second, overexploitation can reduce these benefits under insufficient regulation. Proper
617 regulations are essential not only for avoiding the exhaustion of resources but also for fully
618 utilizing the SV and DRV of groundwater. Third, although this paper did not directly address
619 issues related to non-consumptive water use and externalities of groundwater extraction, the
620 discovery of the new value indirectly contributes to addressing such issues because, as discussed
621 above, the DRV provides users of groundwater with stronger incentives for its sustainable
622 management. Finally, a growing body of literature have simulated optimized conjunctive
623 management of surface water and groundwater using machine learning models including genetic
624 algorithm (e.g., Safavi et al., 2010; Safavi & Esmikhani, 2013 & 2016; Safavi & Falsafioun,
625 2016; Rezaei et al., 2017; Sepahvand et al., 2019). Although, most of these literatures have not
626 captured dynamic reallocation behaviours presented in this paper explicitly, it is valuable to
627 separate them from other types of optimization using these models and quantify the economic
628 benefit of such behaviours.

629

630 **Acknowledgments**

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632

633 **Conflicts of interest**

634 The authors declare no conflicts of interest relevant to this study.

635

636

637 **Data availability statement**

638 The present paper is supplemented by the Supporting Information. The data used for the
639 numerical illustrations in Figure 4 and 5 are available at Zenodo via
640 <https://doi.org/10.5281/zenodo.10887433>.

641

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