Water Resources Research

Supporting Information for

Revisiting the Economic Value of Groundwater

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SI1, SI2, SI3, and SI4.

Introduction

We supplement the solution process of the two-stage model and the proof of Propositions 1, 4, and 5 in this supporting material.

SI1.

1. Single decision-maker regime

Since the intertemporal net benefit function is additively separable with respect to the instantaneous sum of the users' net benefits, we can solve the problem in two steps: the determination of the total water intake g_t for each period, where $g_t = \sum_{i\in\mathcal{N}} g_{it}$, and the allocation of water pumping to each user within period t taking the total water intake g_t as given.

Consider the problem of the second step first:

$$\max_{g_{1,2},\dots,g_{N,2}} \sum_{i\in\mathcal{N}} [F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1) g_{i2}],$$

subject to $\sum_{i\in\mathcal{N}} g_{it} = g_t$. By solving the problem, we get

$$\begin{aligned} g_{it} &= \frac{\tilde{a}_i}{2b} + \frac{S_t}{N} + \frac{g_t}{N} - \varepsilon_i S_t \\ w_{it} &= \frac{\tilde{a}_i}{2b} + \frac{S_t}{N} + \frac{g_t}{N}, \end{aligned} \tag{A.1}$$

for all $i \in \mathcal{N}$ and for all $t \in \{1, 2\}$, where $\tilde{a}_i \triangleq a_i - a/N$ and $a \triangleq \sum_{i\in\mathcal{N}} a_i$. Therefore, the maximized instantaneous aggregate net benefit for given S_t, G_{t-1}, and g_t is given by

$$\pi_t(g_t, S_t, G_{t-1}) \triangleq W(S_t) + (H(S_t) + dG_{t-1}) g_t - \frac{b}{N} g_t^2, \tag{A.2}$$

where

$$W(S_t) \triangleq \sum_{i=1}^{N}\left[a_i\left(\frac{\tilde{a}_i}{2b}+\frac{S_t}{N}\right)\right]-b\sum_{i=1}^{N}\left(\frac{\tilde{a}_i}{2b}+\frac{S_t}{N}\right)^2,$$
$$H(S_t) \triangleq \frac{a-Nc-2bS_t}{N}.$$

Next, we consider the problem of determining the total water intake g_t for each period. By solving backword from period 2, we obtain the following solution:

$$g_2(G_1,S_2)=\frac{a-Nc+NdG_1}{2b}-S_2. \tag{A.3}$$

The problem of the first period in the uncertain case is then given by:

$$\max_{g_1} W(S_1)+(H(S_1)+dG_0)g_1-\frac{b}{N}g_1^2+\beta E_1[\pi_2(g_2(G_1,S_2),S_2,G_0)|S_1].$$

Subsequently, we get

$$g_{u1}^{\text{single}}(S_1)=\frac{1}{4b^2-N^2d^2\beta}[(2b-Nd\beta)X-2b(2bS_1-Nd\beta\bar{S})-N^2d^2\beta R], \tag{A.4}$$

where $X \triangleq a-Nc+NdG_0$ and $w_{u1}^{\text{single}}(S_1)$ is given by $w_{u1}^{\text{single}}(S_1)=g_{u1}^{\text{single}}(S_1)+S_1$.

For the above solution to satisfy the necessary and sufficient conditions, we further require the following from the second-order condition:

$$4b^2-N^2d^2\beta>0. \tag{A.5}$$

Using (A.1), (A.3), and (A.4), we get:

$$\begin{aligned}\pi_{u1}^{\text{single}} &= \left(\sum\nolimits_{i\in\mathcal{N}}a_i\varepsilon_i\right)\bar{S}-b\left(\sum\nolimits_{i\in\mathcal{N}}\varepsilon_i^2\right)\bar{S}^2+\frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i-2b\varepsilon_i\bar{S})^2-\frac{Nd^2\beta^2(2b-Nd)^2}{4b(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})^2\\ &\quad-\frac{N^3bd^4\beta^2}{(4b^2-N^2d^2\beta)^2}R^2-\frac{N^2d^3\beta^2(2b-Nd)}{(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})R-\frac{N^3bd^4\beta^2}{(4b^2-N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A.6}$$

$$\begin{aligned}\pi_{u2}^{\text{single}} &= (c-dG_0)\bar{S}+\frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}X_i^2-\frac{d(2b-Nd\beta)(8b^2-N^2d^2\beta-2Nbd)}{4b(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})^2\\ &\quad+\frac{4Nb^3d^2}{(4b^2-N^2d^2\beta)^2}R^2+\frac{4b^2d(2b-Nd)}{(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})R+\frac{4Nb^3d^2}{(4b^2-N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A.7}$$

$$\begin{aligned}\pi_{u}^{\text{single}} &= \left(\sum\nolimits_{i\in\mathcal{N}}a_i\varepsilon_i+c-dG_0\right)\bar{S}-b\left(\sum\nolimits_{i\in\mathcal{N}}\varepsilon_i^2\right)\bar{S}^2+\frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i-2b\varepsilon_i\bar{S})^2+\frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}X_i^2\\ &\quad-\frac{d[Nd\beta^2(2b^2+N^2d^2-2Nbd)+2b^2(4b-Nd-2Nd\beta)]}{2b(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})^2\\ &\quad+\frac{Nbd^2(4b^2-N^2d^2\beta^2)}{(4b^2-N^2d^2\beta)^2}R^2+\frac{d(2b-Nd)(4b^2-N^2d^2\beta^2)}{(4b^2-N^2d^2\beta)^2}(X-2b\bar{S})R\\ &\quad+\frac{Nbd^2(4b^2-N^2d^2\beta^2)}{(4b^2-N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A.8}$$

where $X_i \triangleq a_i - c + dG_0$.

Similarly, the problem of the first period in the certain case is given by:

$$\max_{g_1} W(\bar{S}) + (H(\bar{S}) + dG_0)g_1 - \frac{b}{N}g_1^2 + \beta\pi_2(g_2(G_1,\bar{S}),\bar{S},G_0).$$

Subsequently, we get:

$$g_{c1}^{\text{single}}(\bar{S}) = \frac{1}{4b^2 - N^2d^2\beta}[(2b - Nd\beta)(X - 2b\bar{S}) - N^2d^2\beta R]. \tag{A.9}$$

In addition, $w_{c1}^{\text{single}}(\bar{S})$ is given by $w_{c1}^{\text{single}}(\bar{S}) = g_{c1}^{\text{single}}(\bar{S}) + \bar{S}$. Using solutions (A.1), (A.3), and (A.9), we obtain:

$$\begin{aligned}\pi_{c1}^{\text{single}} &= \left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &- \frac{Nd^2\beta^2(2b - Nd)^2}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 - \frac{N^3bd^4\beta^2}{(4b^2 - N^2d^2\beta)^2}R^2 - \frac{N^2d^3\beta^2(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.10}$$

$$\begin{aligned}\pi_{c2}^{\text{single}} &= (c - dG_0)\bar{S} + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 - \frac{d(2b - Nd\beta)(8b^2 - N^2d^2\beta - 2Nbd)}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 \\ &+ \frac{4Nb^3d^2}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.11}$$

$$\begin{aligned}\pi_{c}^{\text{single}} &= \left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 \\ &- \frac{d[Nd\beta^2(2b^2 + N^2d^2 - 2Nbd) + 2b^2(4b - Nd - 2Nd\beta)]}{2b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 \\ &+ \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{d(2b - Nd)(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R.\end{aligned} \tag{A.12}$$

2. Multiple-user regime

User i's problem of the second period for given S_2 and G_1 is:

$$\max_{g_{i2}} F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1)g_{i2}.$$

Hence, the solution for this is:

$$g_{i2}(G_1, S_2) = \frac{a_i - c + dG_1}{2b} - \varepsilon_i S_2. \tag{A.13}$$

User i's problem of the first period in the uncertain case is given by

$$\max_{g_{i1}} F_i(g_{i1} + \varepsilon_i S_1) - C_i(G_0)g_{i1} + \beta E_1[\pi_i(g_{i2}(G_1,S_2),G_1,S_2)|S_1].$$

Subsequently, we obtain:

$$g_{u1}^{\text{multi}}(S_1) = \frac{1}{4b^2 - Nd^2\beta}[2b(X - 2bS_1) - d\beta(X - 2b\bar{S}) - Nd^2\beta R], \tag{A.14}$$

and $w_{u1}^{\text{multi}}(S_1)$ is given by $w_{u1}^{\text{multi}}(S_1) = g_{u1}^{\text{multi}}(S_1) + S_1$.

For the above solution to satisfy the necessary and sufficient conditions, we require the following from the second-order condition:

$$4b^2 - d^2\beta > 0 \tag{A.15}$$

Using the solutions (A.13) and (A.14), we get:

$$\begin{aligned}\pi_{u1}^{\text{multi}} = &\left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &+ \frac{d^3\beta^2(2b - d\beta)(8\beta^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 - \frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}R^2 \\ &- \frac{d^3\beta^2(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R - \frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}\sigma^2,\end{aligned} \tag{A.16}$$

$$\begin{aligned}\pi_{u2}^{\text{multi}} = &(c - dG_0)\bar{S} - \frac{d(2b - d\beta)(8b^2 - Nd^2\beta - 2Nbd)}{4b(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 \\ &+ \frac{4Nb^3d^2}{(4b^2 - Nd^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R + \frac{4Nb^3d^2}{(4b^2 - Nd^2\beta)^2}\sigma^2,\end{aligned} \tag{A.17}$$

$$\begin{aligned}\pi_{u}^{\text{multi}} = &\left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 \\ &- \frac{d(2b - d\beta)(4b^2 - d^2\beta^2)(8b^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}R^2 \\ &+ \frac{d(4b^2 - d^2\beta^2)(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R + \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}\sigma^2.\end{aligned} \tag{A.18}$$

Similarly, the problem of the first period in the certain case is given by:

$$\max_{g_{i1}} F_i(g_{i1} + \varepsilon_i\bar{S}) - C_i(G_0)g_{i1} + \beta\pi_i(g_{i2}(G_1, \bar{S}), G_1, \bar{S}).$$

Subsequently, we get:

$$g_{c1}^{\text{multi}}(\bar{S}) = \frac{1}{4b^2 - Nd^2\beta}[(2b - d\beta)(X - 2b\bar{S}) - Nd^2\beta R^g]. \tag{A.19}$$

In addition, $w_{c1}^{\text{multi}}(\bar{S})$ is given by $w_{c1}^{\text{multi}}(\bar{S}) = g_{c1}^{\text{multi}}(\bar{S}) + \bar{S}$. Using solutions (A.13) and (A.19), we obtain:

$$\begin{aligned}\pi_{c1}^{\text{multi}} = &\left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum\nolimits_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &+ \frac{d^3\beta^2(2b - d\beta)(8\beta^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 - \frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}R^2 \\ &- \frac{d^3\square\square^2(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.20}$$

$$\begin{aligned}\pi_{c2}^{\text{multi}} = &(c - dG_0)\bar{S} - \frac{d(2b - d\beta)(8b^2 - Nd^2\beta - 2Nbd)}{4b(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 \\ &+ \frac{4Nb^3d^2}{(4b^2 - Nd^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.21}$$

$$
\begin{aligned}
\pi_c^{\text{multi}} = &\left(\sum\nolimits_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum\nolimits_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum\nolimits_{i\in\mathcal{N}} (X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum\nolimits_{i\in\mathcal{N}} X_i^2 \\
&- \frac{d(2b - d\beta)(4b^2 - d^2\beta^2)(8b^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}R^2 \\
&+ \frac{d(4b^2 - d^2\beta^2)(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R.
\end{aligned} \tag{A. 22}
$$

SI2.

Proof of Proposition 1

From (A.8) and (A.12), we obtain:

$$DRV_{\text{single}} = \pi_u^{\text{single}} - \pi_c^{\text{single}} = \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}\sigma^2.$$

From (A.5), we can demonstrate $DRV_{\text{single}} > 0$.

Similarly, from (A.18) and (A.22), we obtain:

$$DRV_{\text{multi}} = \pi_u^{\text{multi}} - \pi_c^{\text{multi}} = \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}\sigma^2.$$

From (A.15), we can demonstrate $DRV_{\text{multi}} > 0$.

SI3.

Proof of Proposition 4

First we find a general solution for groundwater intake in the case of an arbitrary number of stages. Let $\gamma_t = (\gamma_{1t}, \ldots, \gamma_{Nt}) \in \Gamma_t = \Gamma_{1t} \times \ldots \times \Gamma_{Nt}$ denote an admissible action rule of the social planner, where Γ_{it} is the set of admissible action rules concerning user i in period t. Let $V(t, G_{t-1}, S_t)$ denote the optimal value function in period $t \in \mathrm{T}$ given the current groundwater stock G_{t-1} and the realization of surface flow S_t,

$$V(t, G_{t-1}, S_t) \triangleq \max_{\gamma_t \in \Gamma_t, \ldots, \gamma_T \in \Gamma_T} E_t\left[\sum\nolimits_{t\in\mathrm{T}}\sum\nolimits_{i\in\mathrm{N}} \beta^{t-1}\left[F_i(\gamma_{i\tau} + \varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}\right]\right]. \tag{C. 1}$$

The recursive structure of the returns leads to the following Bellman optimality equation (Bellman 1952; Basar, 2012):

$$
\begin{aligned}
V(t, G_{t-1}, S_t) = \max_{\gamma_t \in \Gamma_t} \sum\nolimits_{i\in\mathrm{N}} &\left[F_i(\gamma_{i\tau} + \varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}\right] + \beta E_{t+1}\left[V(t+1, G_t, S_{t+1})\right], \\
&V(T+1, G_T, S_{T+1}) = 0.
\end{aligned} \tag{C. 2}
$$

Now we prove the following action rules constitute a unique solution for groundwater intake.

$$
\begin{aligned}
\gamma_{iT}^*(S_T, G_{T-1}) &= \frac{1}{2b}\left[\Theta_i(S_T) - Nd^2\beta G_{T-1}\right], \\
\gamma_{it}^*(S_t, G_{t-1}) &= \frac{1}{v_t}\Big[\frac{v_t}{2b}\Theta_i(S_t) + \frac{Nd^2\beta\rho_{t+1}}{2b}\Theta(S_t) - d\beta\rho_{t+1}\Theta(\bar{S}) - Nd^2\beta\eta_t R \\
&\quad + d(v_{t+1} - Nd\beta\rho_{t+1})G_{t-1}\Big], \qquad t \le T-1,
\end{aligned} \tag{C. 3}
$$

where

$$\Theta_i(S_t) \triangleq a_i - c_i - 2b\varepsilon_i S_t, \qquad \Theta(S_t) \triangleq \sum\nolimits_{i\in\mathcal{N}} (a_i - c_i) - 2bS_t,$$

$$\rho_t \triangleq \begin{cases} 1, & t = T \\ \upsilon_{t+1} - 2\beta\rho_{t+1}(Nd - b), & t \le T-1 \end{cases}$$

$$\upsilon_t \triangleq \begin{cases} 2b, & t = T \\ 2b\upsilon_{t+1} - N^2d^2\beta\rho_{t+1}, & t \le T-1 \end{cases}$$

$$\eta_t \triangleq \begin{cases} 0, & t = T \\ \beta\eta_{t+1}(2b - Nd) + \rho_{t+1}, & t \le T-1. \end{cases}$$

Also consider

$$E_{t-1}\left[\frac{\partial V(t, G_{t-1}, S_t)}{\partial G_{t-1}}\right] = \frac{d}{\upsilon_t}[\rho_t\Theta(\bar{S}) + Nd\beta\eta_t(2b - Nd)R + Nd\rho_t G_{t-1}]. \tag{C.4}$$

For $t = T$ and $T - 1$, solving backward from T, we can easily show (C.3) and (C.4) are true. Assume that they also hold for some $t = k + 1$ $(1 \le k \le T - 2)$:

$$\begin{aligned} \gamma_{ik+1}^*(S_{k+1}, G_k) = \frac{1}{\upsilon_{k+1}}\Big[&\frac{\upsilon_{k+1}}{2b}\Theta_i(S_{k+1}) + \frac{Nd^2\beta\rho_{k+2}}{2b}\Theta(S_{k+1}) - d\beta\rho_{k+2}\Theta(\bar{S}) \\ &-Nd^2\beta\eta_{k+1}R + d(\upsilon_{k+2} - Nd\beta\rho_{k+2})G_k\Big], \\ E_k\left[\frac{\partial V(k+1, G_k, S_{k+1})}{\partial G_k}\right] = \frac{d}{\upsilon_{k+1}}&[\rho_{k+1}\Theta(\bar{S}) + Nd\beta\eta_{k+1}(2b - Nd)R + Nd\rho_{k+1}G_k]. \end{aligned} \tag{C.5}$$

Consider the problem for $t = k$:

$$\max_{g_{1,k},\cdots,g_{N,k}} \Omega(S_k) + \sum\nolimits_{i\in\mathcal{N}} [\Theta_i(S_k) + dG_{k-1}]g_{ik} - b\sum\nolimits_{i\in\mathcal{N}} g_{ik}^2 + \beta E_k[V(k+1, G_k, S_{k+1})|S_k],$$

where $\Omega(S_t) \triangleq (\sum_{\mathcal{N}} a_i\varepsilon_i)S_t - b(\sum_{\mathcal{N}} \varepsilon_i^2)S_t^2$. By using (C.5), we obtain the following solution:

$$\begin{aligned} g_{ik} = \gamma_{ik}^*(S_t, G_{t-1}) = \frac{1}{\upsilon_k}\Big[&\frac{\upsilon_k}{2b}\Theta_i(S_k) + \frac{Nd^2\beta\rho_{k+1}}{2b}\Theta(S_k) - d\beta\rho_{k+1}\Theta(\bar{S}) \\ &-Nd^2\beta\eta_k R + d(\upsilon_{k+1} - Nd\beta\rho_{k+1})G_{k-1}\Big]. \end{aligned} \tag{C.6}$$

By using (C.6), we can demonstrate the following:

$$E_{k-1}\left[\frac{\partial V(k, G_{k-1}, S_k)}{\partial G_{k-1}}\right] = \frac{d}{\upsilon_k}[\rho_k\Theta(\bar{S}) + Nd\beta\eta_k(2b - Nd)R + Nd\rho_k G_{k-1}]. \tag{C.7}$$

From equation (C.6) and (C.7), equation (C.3) and (C.5) also holds for $t = k$. By mathematical induction, they are true for all $t \le T - 1$.

Subsequently, we find the DRV. The aggregate groundwater intake is given by:

$$\begin{aligned} g_T &= \frac{1}{\upsilon_T}[\Theta(S_T) - N^2d^2\beta\eta_T R + NdG_{T-1}], \\ g_t &= \frac{1}{\upsilon_t}[\upsilon_{t+1}\Theta(S_t) - Nd\beta\rho_{t+1}\Theta(\bar{S}) - N^2d^2\beta\eta_t R + Nd(\upsilon_{t+1} - Nd\beta\rho_{t+1})G_{t-1}], \qquad t \le T-1. \end{aligned} \tag{C.8}$$

We rewrite (C.8) as:

$$g_t = \Lambda(t) + \Phi(t)G_{t-1} + \Psi(t)S_t, \tag{C.9}$$

where

$$\Lambda(t) \triangleq \begin{cases} \dfrac{1}{\upsilon_T}\left[\sum_{i\in\mathcal{N}}(a_i - c_i) - N^2 d^2 \beta \eta_T R\right], & t = T \\ \dfrac{1}{\upsilon_t}\left[\upsilon_{t+1}\sum_{i\in\mathcal{N}}(a_i - c_i) - Nd\beta\rho_{t+1}\Theta(\bar{S}) - N^2 d^2 \beta \eta_t R\right], & t \le T-1 \end{cases}$$

$$\Phi(t) \triangleq \begin{cases} \dfrac{Nd}{\upsilon_T}, & t = T \\ \dfrac{Nd(\upsilon_{t+1} - Nd\beta\rho_{t+1})}{\upsilon_t}, & t \le T-1 \end{cases}$$

$$\Psi(t) \triangleq \begin{cases} -\dfrac{2b}{\upsilon_T}, & t = T \\ -\dfrac{2b\upsilon_{t+1}}{\upsilon_t}, & t \le T-1. \end{cases}$$

Using this, the groundwater stock G_{t-1} can be transformed into:

$$\begin{aligned} G_{t-1} &= \left[\prod_{\tau=1}^{t-1}\big(1-\Phi(\tau)\big)\right] G_0 \\ &- \left\{\Psi(t-1)S_{t-1} + \Psi(t-2)\big(1-\Phi(t-1)\big)S_{t-2} + \cdots + \left[\Psi(1)\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]S_1\right\} \\ &+ \left\{1 + \big(1-\Phi(t-1)\big) + \cdots + \left[\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]\right\}R \\ &- \left\{\Lambda(t-1) + \big(1-\Phi(t-1)\big)\Lambda(t-2) + \cdots + \left[\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]\Lambda(1)\right\}. \end{aligned} \tag{C.10}$$

In addition, the solutions (C.3) can be transformed into:

$$g_{it} = \gamma_{it}^*(S_t, G_{t-1}) = \frac{\hat{a}_i}{2b} + \frac{\Lambda(t)}{N} + \frac{\Phi(t)}{N}G_{t-1} + \frac{1}{N}\big(1 - N\varepsilon_i + \Psi(t)\big)S_t, \tag{C.11}$$

where $\hat{a}_i \triangleq a_i - c_i - \frac{1}{N}\sum_{i=1}^{N}(a_i - c_i)$. Substitute (C.11) into the aggregate instantaneous net benefit

$$\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t) \triangleq \sum_{i\in\mathcal{N}}[F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1)g_{i2}]. \tag{C.12}$$

Extracting only the terms with $S_1^2, \dots, S_T^2$ from $\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)$ by using (C.10), we obtain

$$\begin{aligned} -\frac{b}{N}\big(1+\Psi(t)\big)^2 S_t^2 + \frac{\Phi(t)\big(Nd - b\Phi(t)\big)}{N}\Big[\Psi(t-1)^2 S_{t-1}^2 + \Psi(t-2)^2\big(1-\Phi(t-1)\big)^2 S_{t-2}^2 \\ + \cdots + \Psi(1)^2\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)^2 S_1^2\Big]. \end{aligned} \tag{C.13}$$

If we take the expected value of $E_0[\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)]$, the terms with σ^2 are generated by replacing $S_1^2, \dots, S_T^2$ in (C.13) with σ^2. They give $\Xi_1, \dots, \Xi_T$ in Proposition 4.

SI4.

Proof of Proposition 5

The procedure is the same as in the proof of Proposition 4 (SI3). We first prove the following strategy constitutes a unique feedback Nash equilibrium solution for $\{\mathcal{N},\mathcal{T},\mathcal{G},\mathcal{S},\{U_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}},\{f_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}},\{\Gamma_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}},\{\Pi_i\}_{i\in\mathcal{T}}\}$.

$$\begin{gathered}
\gamma_{iT}^{**}(G_T,S_T)=\frac{1}{2b}[\Theta_i(S_T)+dG_{T-1}],\\
\gamma_{it}^{**}(G_{t-1},S_t)=\frac{1}{\tilde{\upsilon}_t}\left[\frac{\tilde{\upsilon}_t}{2b}\Theta_i(S_t)-\frac{d\beta\tilde{\upsilon}_t(\tilde{\rho}_{t+1}+N\tilde{\varphi}_{t+1})}{2b\tilde{\upsilon}_{t+1}}\Theta_i(\bar{S})+\frac{d^2\beta\tilde{\rho}_{t+1}}{2b}\Theta(S_t)\right.\\
\left.-\frac{d\beta(d^2\beta\tilde{\rho}_{t+1}^2-\tilde{\upsilon}_t\tilde{\varphi}_{t+1})}{2b\tilde{\upsilon}_{t+1}}\Theta(\bar{S})-d^2\beta\tilde{\eta}_tR+d(\tilde{\upsilon}_{t+1}-d\beta\tilde{\rho}_{t+1})G_{t-1}\right],\\
t\le T-1,
\end{gathered}\tag{D.1}$$

where

$$\tilde{\rho}_t\triangleq\begin{cases}1, & t=T\\ \tilde{\upsilon}_{t+1}-\beta\tilde{\rho}_{t+1}(Nd+d-2b)-\dfrac{(N-1)d\beta\tilde{\rho}_{t+1}(2b-Nd)(\tilde{\upsilon}_{t+1}-d\gamma\tilde{\rho}_{t+1})}{\tilde{\upsilon}_t}, & t\le T-1\end{cases}$$

$$\tilde{\upsilon}_t\triangleq\begin{cases}2b, & t=T\\ 2b\tilde{\upsilon}_{t+1}-Nd^2\beta\tilde{\rho}_{t+1}, & t\le T-1\end{cases}$$

$$\tilde{\eta}_t\triangleq\begin{cases}0, & t=T\\ \beta\tilde{\eta}_{t+1}\tilde{\mu}_{t+1}(2b-Nd)+\tilde{\rho}_{t+1}, & t\le T-1\end{cases}$$

$$\tilde{\mu}_t\triangleq\begin{cases}1, & t=T\\ \dfrac{2b\tilde{\upsilon}_{t+1}-d^2\beta\tilde{\rho}_{t+1}}{\tilde{\upsilon}_t}, & t\le T-1\end{cases}$$

$$\tilde{\varphi}_t\triangleq\begin{cases}0, & t=T\\ \dfrac{\beta\tilde{\mu}_t(2b-Nd)[d\tilde{\rho}_{t+1}(\tilde{\upsilon}_{t+1}-d\beta\tilde{\rho}_{t+1})+\tilde{\upsilon}_t\tilde{\varphi}_{t+1}]}{2b\tilde{\upsilon}_{t+1}}, & t\le T-1.\end{cases}$$

Moreover, consider

$$\begin{gathered}
E_{t-1}\left[\frac{\partial V^i(t,G_{t-1},S_t)}{\partial G_{t-1}}\right]=\frac{d}{\tilde{\upsilon}_t}[(\tilde{\rho}_t+N\tilde{\varphi}_t)\Theta_i(\bar{S})-\tilde{\varphi}_t\Theta(\bar{S})\\
+d\beta\tilde{\eta}_t\tilde{\mu}_t(2b-Nd)R+d\tilde{\rho}_tG_{t-1}].
\end{gathered}\tag{D.2}$$

For $t=T$ and $T-1$, solving backward from T, we can show that (D.1) and (D.2) are true. Assume they hold for $t=k+1$ $(1\le k\le T-2)$, and we can prove they are also true for all $t\le T-1$ in the same way as SI3.

The aggregate groundwater intake is given by:

$$\begin{gathered}
g_T=\frac{1}{\tilde{\upsilon}_T}[\Theta(S_T)-Nd^2\beta\tilde{\eta}_TR+NdG_{T-1}],\\
g_t=\frac{1}{\tilde{\upsilon}_t}[\tilde{\upsilon}_{t+1}\Theta(S_t)-d\beta\tilde{\rho}_{t+1}\Theta(\bar{S})-Nd^2\beta\tilde{\eta}_tR+Nd(\tilde{\upsilon}_{t+1}-d\beta\tilde{\rho}_{t+1})G_{t-1}],\qquad t\le T-1.
\end{gathered}\tag{D.3}$$

Hence, we rewrite (D.3) as:

$$g_t=\widetilde{\Lambda}(t)+\widetilde{\Phi}(t)G_{t-1}+\widetilde{\Psi}(t)S_t,\tag{D.4}$$

where

$$
\tilde{\Lambda}(t) \triangleq \begin{cases} \dfrac{1}{\tilde{v}_T}\left[\displaystyle\sum_{i=1}^{N}(a_i - c_i) - Nd^2\beta\tilde{\eta}_T R\right], & t = T \\ \dfrac{1}{\tilde{v}_t}\left[\tilde{v}_{t+1}\displaystyle\sum_{i=1}^{N}(a_i - c_i) - d\beta\tilde{\rho}_{t+1}\Theta(\bar{S}) - Nd^2\beta\tilde{\eta}_t R\right], & t \le T-1 \end{cases}
$$

$$
\tilde{\Phi}(t) \triangleq \begin{cases} \dfrac{Nd}{\tilde{v}_T}, & t = T \\ \dfrac{Nd(\tilde{v}_{t+1} - d\beta\tilde{\rho}_{t+1})}{\tilde{v}_t}, & t \le T-1 \end{cases}
$$

$$
\tilde{\Psi}(t) \triangleq \begin{cases} -\dfrac{2b}{\tilde{v}_T}, & t = T \\ -\dfrac{2b\tilde{v}_{t+1}}{\tilde{v}_t}, & t \le T-1. \end{cases}
$$

Using this, the groundwater stock G_{t-1} can be transformed into:

$$
\begin{aligned}
G_{t-1} = &\left[\prod_{\tau=1}^{t-1}\left(1-\tilde{\Phi}(\tau)\right)\right]G_0 \\
&-\left\{\tilde{\Psi}(t-1)S_{t-1} + \tilde{\Psi}(t-2)\left(1-\tilde{\Phi}(t-1)\right)S_{t-2} + \cdots + \left[\tilde{\Psi}(1)\prod_{\tau=2}^{t-1}\left(1-\tilde{\Phi}(\tau)\right)\right]S_1\right\} \\
&+\left\{1 + \left(1-\tilde{\Phi}(t-1)\right) + \cdots + \left[\prod_{\tau=2}^{t-1}\left(1-\tilde{\Phi}(\tau)\right)\right]\right\}R \\
&-\left\{\tilde{\Lambda}(t-1) + \left(1-\tilde{\Phi}(t-1)\right)\tilde{\Lambda}(t-2) + \cdots + \left[\prod_{\tau=2}^{t-1}\left(1-\tilde{\Phi}(\tau)\right)\right]\tilde{\Lambda}(1)\right\}.
\end{aligned}
\tag{D.5}
$$

In addition, (D.1) can be transformed into:

$$
g_{it} = \gamma_{it}^{**}(S_t, G_{t-1}) = \frac{\hat{a}_i}{2b} + \frac{\tilde{\Lambda}(t)}{N} + \frac{\tilde{\Phi}(t)}{N}G_{t-1} - Z_{it} + \frac{1}{N}\left(1 - N\varepsilon_i + \tilde{\Psi}(t)\right)S_t, \tag{D.6}
$$

where

$$
Z_{it} = \begin{cases} 0, & t = T \\ \dfrac{d\beta}{2b}\left[\dfrac{\tilde{\rho}_{t+1} + N\tilde{\varphi}_{t+1}}{\tilde{v}_{t+1}}\Theta_i(\bar{S}) + \dfrac{Nd^2\beta\tilde{\rho}_{t+1}^2 - N\tilde{v}_t\tilde{\varphi}_{t+1} - 2b\tilde{v}_{t+1}\tilde{\rho}_{t+1}}{N\tilde{v}_t\tilde{v}_{t+1}}\Theta(\bar{S})\right], & t \le T-2. \end{cases}
$$

Substituting (D.6) into the aggregate instantaneous net benefit $\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)$ and extracting only the terms with $S_1^2, \dots, S_T^2$ by using (D.5), we get:

$$
\begin{aligned}
-\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2 S_t^2 + \frac{\tilde{\Phi}(t)\left(Nd - b\tilde{\Phi}(t)\right)}{N}&\left[\tilde{\Psi}(t-1)^2 S_{t-1}^2 + \tilde{\Psi}(t-2)^2\left(1-\tilde{\Phi}(t-1)\right)^2 S_{t-2}^2\right. \\
&\left.+ \cdots + \tilde{\Psi}(1)^2\prod_{\tau=2}^{t-1}\left(1-\tilde{\Phi}(\tau)\right)^2 S_1^2\right].
\end{aligned}
\tag{D.7}
$$

If we take the expected value of $E_0[\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)]$, the terms with σ^2 are generated by replacing $S_1^2, \dots, S_T^2$ in (D.7) with σ^2. They give $\tilde{\Xi}_1, \dots, \tilde{\Xi}_T$ in Proposition 5.