Supporting Information for "Simulating Lagrangian Subgrid-Scale Dispersion on Neutral Surfaces in the Ocean"

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Text S1. Tapering scheme

We use central differences to compute the neutral slopes S_x and S_y (see equation (4)) in the discrete Eulerian model data in the experiments in Section 4. Incidentally, the computed neutral slopes can be unrealistically high, for example in the vicinity of the meridional ridge or in the mixed layer (see Text S2). The buoyancy in the mixed layer is mostly uniform, but if we compute neutral slopes in this region, small deviations in the local buoyancy field can lead to huge slopes. It is common practice in Eulerian ocean modeling to limit or turn off isopycnal/isoneutral diffusion in regions with high slopes, in order to prevent numerical instability. This practice is called 'tapering'.

Here we use a tapering scheme similar to that of Danabasoglu and McWilliams (1995) to smoothly decrease the values of the Markov-0 diffusivity tensors $\mathbf{K}_{\text{redi,approx}}$ and $\mathbf{K}_{\text{LS}}$ to zero in regions with high slopes. Similarly, for Markov-1, we use it to smoothly decrease the perturbative velocity $\mathbf{u}'$ to zero in such regions. At each timestep, we respectively multiply $\mathbf{K}_{\text{redi,approx}}$, $\mathbf{K}_{\text{LS}}$, or $\mathbf{u}'$ by a taper function f_{taper} which assumes values between 1 in regions where the isoneutral slopes are well-behaved and 0 in regions where it is unrealistically high. Danabasoglu and McWilliams (1995) choose a taper function

$$f_{\text{taper,DMW}}(S) = \frac{1}{2}\left(1 + \tanh\left[\frac{S_c - |S|}{S_d}\right]\right), \tag{1}$$

where S_c is the slope at which $f_{\text{taper}} = 0.5$ and S_d an acting distance over which f_{taper} changes steeply. If we were to multiply the perturbative velocity $\mathbf{u}'$ in the Markov-1 model (7) with such a function, this causes an exponential decay of the $\mathbf{u}'$ with an e-folding timescale of $\Delta t/\log(f(s))$. This can significantly shorten the effective decorrelation of $\mathbf{u}'$ as set by T_L. For example, in a simulation with $T_L = 20$ days and $dt = 40$ minutes, if $f(S)$ persistently equals 0.999, this causes $\mathbf{u}'$ to exponentially decay with a timescale of 28 days. In conjunction with the exponential decorrelation specified using T_L, this leads to an effective decorrelation with an e-folding timescale of 12 days. This is why we limit the slope values over which tapering happens smoothly to values that differ from S_c by at most $3S_d$. We thus use the following taper function

$$f_{\text{taper}}(S) = \begin{cases} 1 & |S| < S_c - 3S_d \\ \frac{1}{2}\left(1 + \tanh\left[\frac{S_c - |S|}{S_d}\right]\right) & S_c - 3S_d \leq |S| \leq S_c + 3S_d \\ 0 & S_c + 3S_d < |S| \end{cases} \tag{2}$$

Note that $f_{\text{taper}}(S_c - 3S_d) \approx 0.998$ and $f_{\text{taper}}(S_c + 3S_d) \approx 0.002$. In our simulations in section 4, we choose $S_c = 8 \times 10^{-3}$ and $S_d = 5 \times 10^{-4}$. With these values, tapering occurs only in a fraction of the domain, namely near the meridional ridge and in the mixed layer.

Text S2. Treatment of the mixed layer

By definition, potential temperature is approximately homogeneous in the mixed layer. As neutral surfaces appeal to the notion of a strong stratification which inhibits motion in the dianeutral direction, the concept of neutral surfaces does not apply in the mixed layer. That is why the experiments in this study focus on the ocean interior. In the experiments in section 4, particles are released well below the mixed layer. Still, since neutral surfaces in the Southern Ocean can outcrop to the surface (Marshall & Speer, 2012), particles in our model may be transported to the surface. In Figures 8 and 9, we exclude particles that fall within the mixed layer. Similarly, in the computation of the spurious dianeutral diffusivities in section 4.6, we exclude particle trajectories that at any point reach depths of $-50\,\text{m}$. The actual mixed-layer, marked by a sharp gradient in potential temperature, lies less deep, but since it varies in space, we use $-50\,\text{m}$ as a global approximation for computational efficiency.

References

Danabasoglu, G., & McWilliams, J. C. (1995). Sensitivity of the Global Ocean Circulation to Parameterizations of Mesoscale Tracer Transports. *Journal of Climate*, *8*, 2967–297.

Marshall, J., & Speer, K. (2012). Closure of the meridional overturning circulation through Southern Ocean upwelling. *Nature Geoscience*, *5*(3), 171–180. doi: 10.1038/ngeo1391

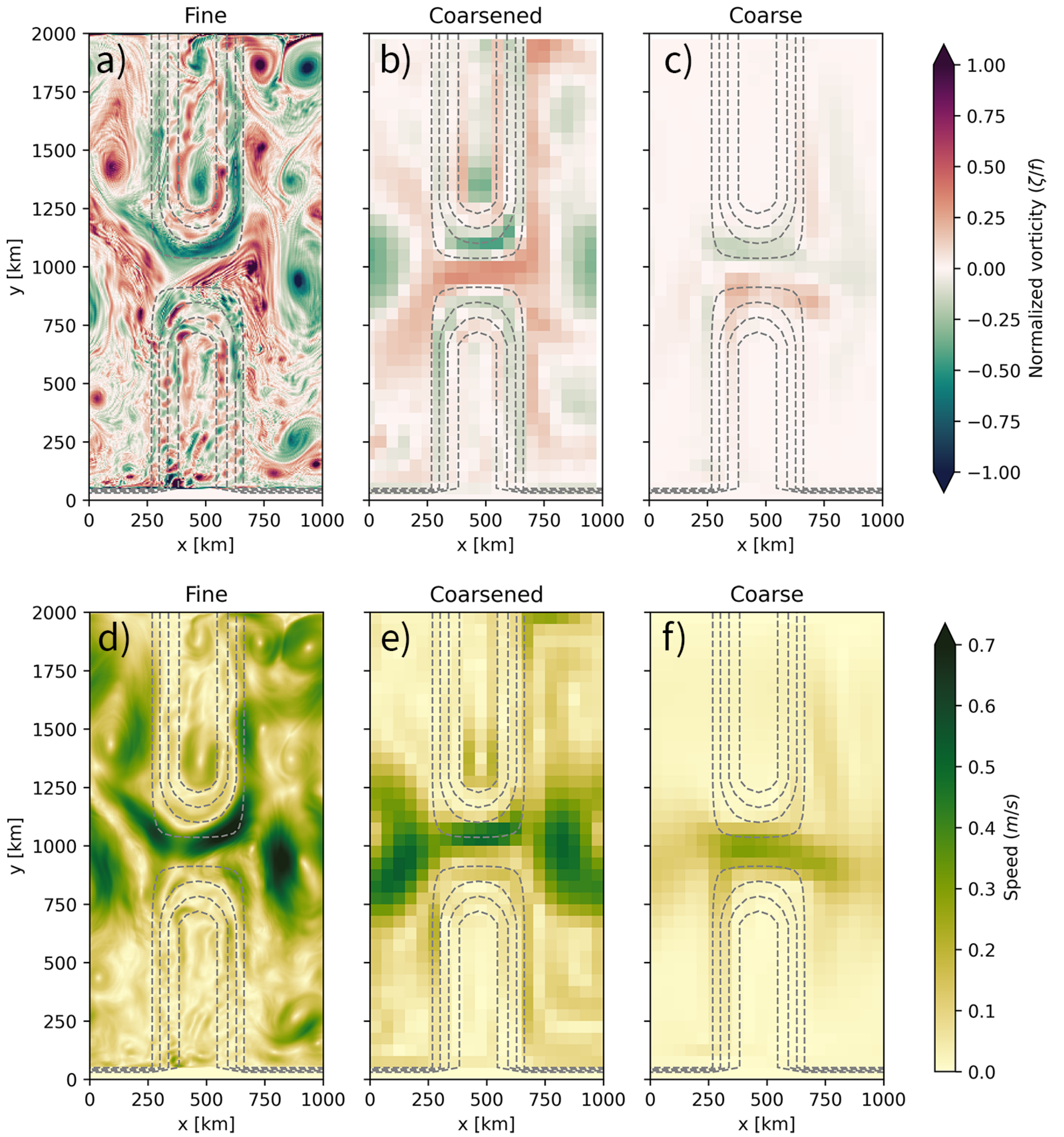

Figure S1. Snapshot of the vorticity (a-c) and speed (d-f) of the fine (a & d), coarsened (b & e), and coarse (c & f) model fields used in this study. The fine fields are daily averages, the coarsened fields are 1-year time averages and 50 kilometer spatial averages, and the coarse model is in steady state. Dashed lines indicate the position of the meridional ridge.

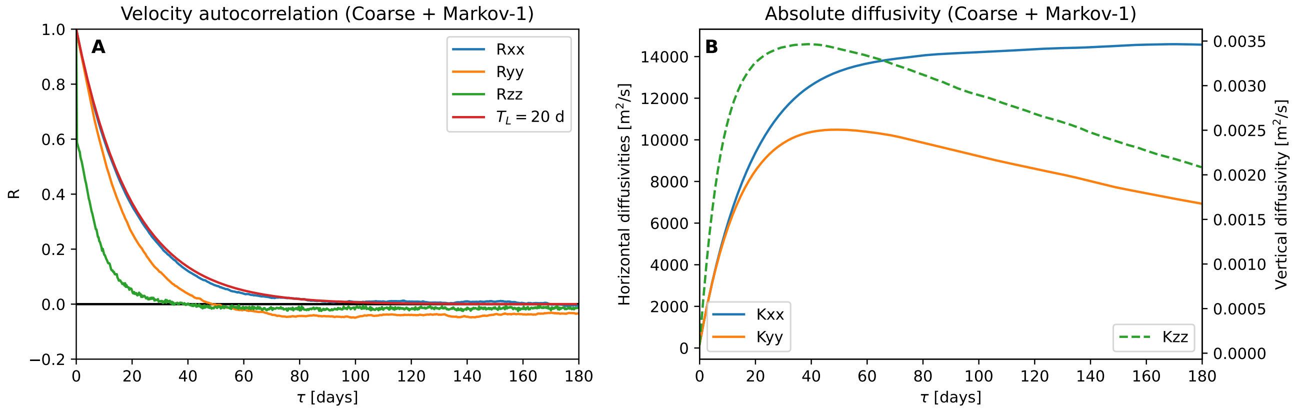

Figure S2. Lagrangian autocorrelation and absolute diffusivity produced by the Markov-1 model when applied on the coarse field (cf. Figure 5). The Lagrangian autocorrelation in the x-direction best resembles that of an exponentially decaying function with a 20-day e-folding timescale (in red for reference).

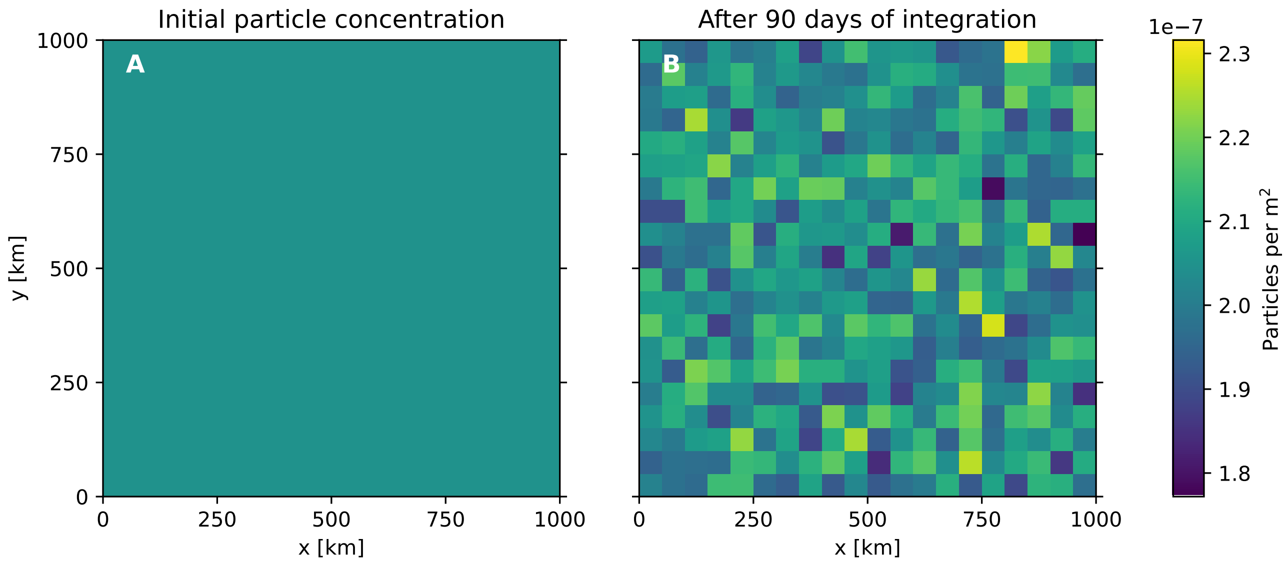

Figure S3. Concentrations of 204,800 particles before (a) and (b) after 90 days of integration using the Markov-1 model, using $T_L = 20$ days and $\nu^2 = 5.79 \times 10^{-4}\,\mathrm{m}^2\,\mathrm{s}^{-2}$. We take advantage of the periodicity of the domain and analyze all particles over one wavelength $1/k_x = 1/k_y = 1000\,\mathrm{km}$ by displacing them as $x = x \mod 1/k_x$, $y = y \mod 1/k_y$. The concentrations are computed by binning particles and dividing by the total area of curved surface per bin. Particles start out evenly spaced. From (a) it can be seen that the curvature causes only negligible differences in initial concentrations. After 90 days of integration (b), concentrations are much less homogeneous than they were initially, but there are no clear accumulation patterns coinciding with specific features of the idealized neutral surface. If that were the case, it would indicate that the well-mixed condition is violated.