

1 **Toward data-driven generation and evaluation of model**
2 **structure for integrated representations of human**
3 **behavior in water resources systems**

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6 **Key Points:**

- 7 • Automated generation of model structure from data to describe human behavior
8 in water systems.
9 • Systematic model evaluation along performance-complexity tradeoff by cluster-
10 ing models with similar behavior.
11 • Diagnostic assessment of model generalization skill using global sensitivity anal-
12 ysis of features.

Abstract

Simulations of human behavior in water resources systems are challenged by uncertainty in model structure and parameters. The increasing availability of observations describing these systems provides the opportunity to infer a set of plausible model structures using data-driven approaches. This study develops a three-phase approach to the inference of model structures and parameterizations from data: problem definition, model generation, and model evaluation, illustrated on a case study of land use decisions in the Tulare Basin, California. We encode the generalized decision problem as an arbitrary mapping from a high-dimensional data space to the action of interest and use multi-objective genetic programming to search over a family of functions that perform this mapping for both regression and classification tasks. To facilitate the discovery of models that are both realistic and interpretable, the algorithm selects model structures based on multi-objective optimization of (1) their performance on a training set and (2) complexity, measured by the number of variables, constants, and operations composing the model. After training, optimal model structures are further evaluated according to their ability to generalize to held-out test data and clustered based on their performance, complexity, and generalization properties. Finally, we diagnose the causes of good and bad generalization by performing sensitivity analysis across model inputs and within model clusters. This study serves as a template to inform and automate the problem-dependent task of constructing robust data-driven model structures to describe human behavior in water resources systems.

1 Introduction

Increasingly, water resources models combine observed data and computational experiments to support the development of theory regarding system processes (Clark et al., 2015a, 2015b), particularly those for which existing theory may insufficiently explain available observations (Karpatne et al., 2017; Schlüter et al., 2019). One such process is human behavior, which represents a significant source of uncertainty in simulation models of water resources systems (Konar et al., 2019), as humans interact with and depend on water systems in numerous ways (Lund, 2015; Schill et al., 2019). Examples include urban and agricultural water demand (Chini et al., 2017; Marston & Konar, 2017), population displacement (Müller et al., 2016), and the nonstationary behavior of individuals and institutions across multiple sectors and scales (Mason et al., 2018; Monier et al., 2018; Muneeppeerakul & Anderies, 2020). The increasing availability of multi-sectoral data describing these processes provides the opportunity to complement theory by inferring plausible models from data (Brunton et al., 2016; Montáns et al., 2019).

Many subfields of water resources have focused on the challenge of modeling human behavior, including: dynamical systems models, as in socio-hydrology (Sivapalan et al., 2012) and social-ecological systems (Berkes & Folke, 1998); hydro-economic models (Harou et al., 2009); and agent-based modeling (An, 2012). Each offers differing perspectives on which system components should be treated as exogenous, controlled, or self-organized, and which behaviors can be adequately described by data versus theory (Anderies, 2015). However, all share the goal of accurately describing observed dynamics of the system while managing the complexity of the spatial and temporal representation (Baumberger et al., 2017; Höge et al., 2018). These approaches are not necessarily exclusive, and can be connected through a common experimental framing—for example, Müller and Levy (2019) review how economic theory can be coupled with data-driven sociohydrologic modeling to support and develop theories of human influence in water systems. Similarly, agent-based modeling studies have integrated data-driven and theory-driven approaches to investigate system processes (Gunaratne & Garibay, 2017; Schlüter et al., 2019; Vu et al., 2019). By extricating the processes driving emergent and interdependent behaviors in coupled systems, data-driven models can be used beyond the integration of observations to advance theory.

65 Several recent studies highlight the value and range of applications for data-driven
66 approaches in water resources. For example, Giuliani et al. (2016) generate adaptive be-
67 havioral rules from historical climate and land use data by coordinating reservoir deci-
68 sions with downstream cropping decisions from an economic model. Similarly, Quinn et
69 al. (2018) employ policy emulation methods for coupled reservoir and irrigation decisions
70 to reduce the computational cost of exploring a range of future hydroclimate scenarios.
71 Worland et al. (2019) combine heterogeneous attributes of stream gauge networks to re-
72 construct observed flow duration curves under human influence with high accuracy us-
73 ing multi-output neural networks. Finally, Zaniolo et al. (2018) use data-driven variable
74 selection across hydroclimate indicators and observed state variables to automatically
75 design Pareto-optimal drought indices (i.e., constructing a function) to balance trade-
76 offs between complexity and performance. These studies have underscored the signifi-
77 cant potential for data-driven methods to advance model design in water systems, while
78 also identifying key challenges related to structure and complexity.

79 Model accuracy alone does not engender trust (Baumberger et al., 2017), particu-
80 larly in the case of “black-box” models (Shen, 2018), though accuracy is often the pri-
81 mary metric by which model structure is validated (Eker et al., 2018). By starting from
82 fixed model structures, many data-driven methods bypass the question of structural un-
83 certainty (Walker et al., 2003). This complicates any eventual reconciliation with avail-
84 able theory or process knowledge to support interpretation and validation (Lipton, 2018;
85 Knüsel et al., 2019; P. J. Schmidt et al., 2020). By contrast, data-driven methods for sys-
86 tem identification are capable of searching both model structures and parameters to find
87 candidate representations (Ljung, 2017). Methods have been demonstrated for systems
88 in which the target relationships are well-known, such as the double pendulum (M. Schmidt
89 & Lipson, 2009) and the Navier-Stokes equations (Rudy et al., 2017). In hydrology, data-
90 driven system identification methods have been used to infer rainfall-runoff transfer func-
91 tions (Klotz et al., 2017) and to automate the identification of rainfall-runoff model struc-
92 tures using global optimization (Spieler et al., 2020).

93 Generating model structures through data-driven system identification allows for
94 the testing of multiple model structures and parameterizations as competing hypothe-
95 ses (Beven, 2019), similar to how conceptual and theory-driven model components have
96 been compared to reduce structural uncertainty (Clark et al., 2015a, 2015b; Nearing &
97 Gupta, 2015; Knoben et al., 2020). Several specific challenges arise in the way candidate
98 models are evaluated. First, data-driven system identification typically results in a trade-
99 off between model performance and complexity (Hogue et al., 2006; Bastidas et al., 2006;
100 Pande et al., 2009). Second, additional criteria may be required for model evaluation,
101 such as interpretability and agreement with available theory (Khatami et al., 2019; Knüsel
102 et al., 2019). Opportunities remain for data-driven methods to identify model structures
103 of water resources system components for which theory is still being developed, such as
104 varied human influences. There remains a need for a general approach capable of gen-
105 erating and evaluating models of human interactions within water systems, with the si-
106 multaneous goals of accuracy and interpretability across a broad spectrum of possible
107 representations (Schill et al., 2019).

108 This work contributes an approach to model generation and evaluation for the gen-
109 eral challenge of deriving process representation and understanding from observed data
110 in water resources systems. We focus on the particular challenge of modeling human be-
111 havior, an influential system process which poses significant uncertainty in hydrologic
112 systems (Konar et al., 2019; Schill et al., 2019; Herman et al., 2020). By generating many
113 candidate models as competing hypotheses and simultaneously evaluating models for per-
114 formance and complexity, we operationalize a preference for parsimonious model struc-
115 tures in combinatorial search spaces. The structures resulting from search in broadly de-
116 fined model spaces are consolidated through systematic decomposition and diagnostic
117 assessment of plausible model sets to determine driving structure. The approach is demon-

118 strated for a case study of agricultural land use decisions in California, a complex spa-
 119 tially distributed process through which humans exert substantial influence on the sys-
 120 tem. This approach provides a foundation for future studies of model structural uncer-
 121 tainty, reconciliation with theory, and integrated systems modeling, particularly regard-
 122 ing the role of these challenges in planning and management for coupled human-water
 123 systems under uncertainty.

124 2 Methodological Background

125 We extend data-driven system identification approaches to generate and evaluate
 126 plausible model structures describing human behavior in water resources systems (Fig-
 127 ure 1). The experimental steps presented here share similarities with the problem of con-
 128 structing emulators (surrogates) of environmental systems models (Castelletti et al., 2012;
 129 Kleijnen, 2015), though with the additional goal of generating models that support the
 130 development of candidate theory regarding system processes. This requires an evalua-
 131 tion phase in which the structures of generated models are examined directly. By search-
 132 ing over the space of model structures for a given problem definition, the uncertainty as-
 133 sociated with selecting any given model can be visualized as a function of complexity and
 134 accuracy on held-out data.

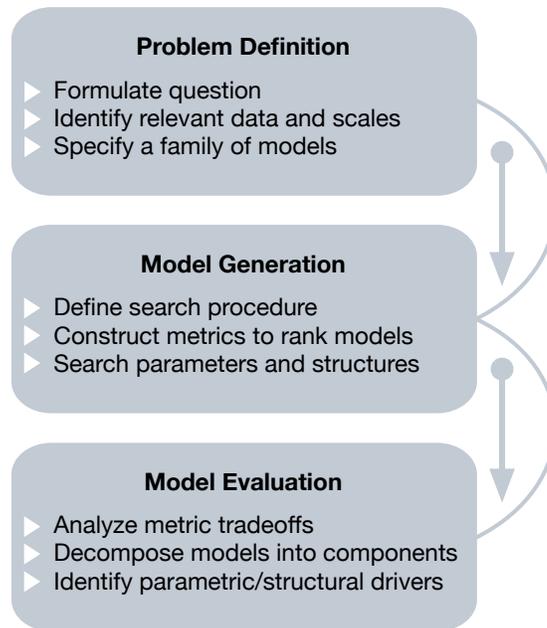


Figure 1. Flowchart of methodological steps involved in generating model structure from data.

135 2.1 Problem Definition

136 Problem definition for data-driven modeling includes the formulation of a question
 137 about the system, the collection and organization of available data at relevant spatial
 138 and temporal scales, and the specification of a family of models to answer the question.
 139 A data-driven system identification approach to problem definition can avoid human-
 140 intuited priors in the form of model structure and feature engineering, in favor of dis-
 141 covering useful constructions of both the data and the model simultaneously (Knüsel et

al., 2019). First, the heterogeneous feature types common to integrated settings and observed human behavior can be considered across spatio-temporal scales. Feature engineering is then performed by transforming the observations, typically along with some form of dimension reduction such as eigenvalue decomposition (Giuliani & Herman, 2018). Variables at incongruent spatial and temporal scales and categorical variables can also be incorporated, for example through encoding schemes (Cerdeira et al., 2018).

In formulating the question, the model ϕ must be identified to map predictor variables X (input samples) to the response variable y in a multivariate regression problem: $\phi : \mathbb{R}^n \rightarrow \mathbb{R}^1$. For modeling dynamical systems, the problem might involve learning the next state or derivative of a state variable in time given the current and previous states. The goal is to automatically reverse-engineer structure in ϕ that enables novel insights of the system (Bongard & Lipson, 2007). Discovering the optimal ϕ without pre-specifying the form of the function invokes exploration over both the structure and parameterization of ϕ . This generalized multivariate regression problem is an instance of supervised learning. However, it could alternatively be cast as a multivariate control problem, where rather than learning a model, a policy is learned based on rewards received by an agent after taking actions in an environment (Barto & Dietterich, 2004). The relationships between environmental observations and human decisions can also be framed in terms of causal inference, such as through instrumental variable and fixed effect experiments (Müller & Levy, 2019).

There are a number of model families from which functions could be drawn to perform this mapping, such as linear additive models or neural networks. Functions can be most generally encoded as trees or graphs, either of which can be used to represent a universal approximator (Breiman, 2001; Huang et al., 2006, e.g.) of highly complex, non-linear human behavior. A common approach for the automatic construction of models of arbitrary mathematical structure and complexity is to combine objects from a primitive set of basic functions (Quade et al., 2016). As an instance of a process influencing the natural system, human behavior is integrated in model computation graphs, the network representing model operations and numerical fluxes (Gupta & Nearing, 2014; Khatami et al., 2019), by defining representational nodes and specifying links. Taken together, nodes and links in a model’s graph form a natural measure of model integration (Claussen et al., 2002).

2.2 Model Generation

The model generation process involves inferring models from data. Within water resources, model inference has largely focused on parameter estimation for given model structures. This is a broad category, including deterministic data-driven models with trainable weights such as neural networks (Hsu et al., 1995, e.g.), physically-based model structures with probabilistic search procedures such as Markov Chain Monte Carlo (MCMC) (Vrugt & Beven, 2018, e.g.), and general procedures for examining parametric uncertainty in conceptual linear and nonlinear model structures such as Generalized Likelihood Uncertainty Estimation (Beven & Binley, 1992). For example, Vrugt and Beven (2018) demonstrate the evolution and training of set-length Markov chains for different experiments, using differential evolution to explore a broadly-defined parameter space and maintaining a population of models in place of explicit structural search. These approaches are thus a combination of theory-driven structure and data-driven parameterization, which enables analysis of complexity and equifinality among parameter sets (discussed in Section 2.3).

Model structure can also be generated through a number of data-driven search methods that explicitly add or subtract elements—referred to as construction and pruning methods, respectively—which originate in the fields of machine learning and evolutionary algorithms. Construction methods include decision trees, which successively add lin-

ear decision rules to accurately classify samples (Quinlan, 1986), and genetic programming, the use of a genetic algorithm to build and search over graphical model structures composed of simple mathematical elements and inputs (Koza, 1992, 1995), among others. Broadly, global optimization methods such as evolutionary algorithms have proven useful for this task (Reed et al., 2013), given the potentially non-convex or discontinuous objective surface that results from optimizing both structure and parameters simultaneously. Though the target processes may be simple, basic implementations of these methods do not explicitly minimize model complexity. With increasing interest in model interpretability in machine learning (Lipton, 2018), pruning methods for the discovery of sparsely-connected sub-networks have been introduced that reproduce or improve performance of fully-connected neural networks after they have been trained (Frankle & Carbin, 2018). In contrast, multiple objectives can be used with construction methods to evaluate model structures simultaneously for error performance and structural complexity during optimization, codifying a preference during search for simpler models that perform equally well.

Genetic programming is particularly useful for its ability to conduct global multi-objective search over model structures of arbitrary complexity, i.e., symbolic regression (Quade et al., 2016). Symbolic regression uses linear and nonlinear operators as base functions, and can, for example, learn to compose nested functions and automate the process of feature engineering. Symbolic trees can also incorporate noise (M. D. Schmidt & Lipson, 2007), can be seeded with relations of interest during optimization (M. D. Schmidt & Lipson, 2009; Chadalawada et al., 2020, e.g.), and can be strongly-typed to incorporate and handle heterogeneous data types or function outputs (Montana, 1995). Model evaluations of symbolic regression trees are generally faster than traditional feed-forward neural networks because each model evolves a sparse input representation based only on the inputs that improve performance. These factors make symbolic trees suited for iterative and exploratory model generation when using a gradient-free optimization method. The primitive set of structures for building symbolic trees determines the size of the search space, which grows combinatorially with the number of primitives (Vanneschi et al., 2010). In applications where the target functions are not known, as in the modeling of complex and highly nonlinear human behavior, the space of possible model structures can be broadened to include a large number of possible functional relationships.

2.3 Model Evaluation

Model evaluation consists of the examination of performance metrics and component-level behavior, and the identification of parametric and structural drivers. This section reviews different approaches and perspectives regarding model evaluation for data-driven system identification, recognizing that the implementation of this phase is problem-dependent, and that integrated systems models including human behavior may be difficult to validate against theoretical or conceptual results depending on their scale.

The minimization of one or more error metrics between the model and data defines its proximity to the “true” model (Haussler & Warmuth, 1993; Kearns et al., 1994; Valiant, 2013). The different methodological and philosophical details of model evaluation in these settings are reviewed by Höge et al. (2018). Since the potential for a model to overfit to training data increases with complexity, the foremost issue regarding model evaluation is the test error, the indicator of a model’s ability to generalize to unseen data by balancing model bias and variance (Friedman, 1997; Pande et al., 2009). Generalization to unseen data is also required to appropriately accommodate non-stationarity in data, a necessity when seeking to describe dynamic human behavior over long time periods (Höge et al., 2018). Finally, standard error metrics can be supplemented by additional criteria such as the information content learned from a model (Nearing & Gupta, 2015; Nearing et al., 2020), or when functional relationships are known, the evaluation of structural

error through tradeoffs between predictive and functional performance (Ruddell et al., 2019).

For data-driven model structures describing human behavior, several extensions arise that deserve consideration during the model evaluation phase. The first is model complexity, recognizing that additional components or parameters do not necessarily result in the ability to represent increasingly complex system behavior (Z. Sun et al., 2016). Instead, the goal is to find a parsimonious model, or the simplest model that still describes the data accurately. This has been identified as a challenge for heuristic approaches to data-driven system identification (Bongard & Lipson, 2007; M. D. Schmidt & Lipson, 2008; M. Schmidt & Lipson, 2009).

The second extension is model equifinality, or lack of uniqueness, which occurs when many model structures produce comparable predictions even after being tuned, trained, constrained, or optimized (Beven, 1993). This can suggest possible redundancy or oversimplification in the model, meaning that the parsimonious model may not have been found or the collected data is not diverse enough to fully represent the underlying process. For data-driven system identification this is especially challenging given the large space of possible model structures and conflicting performance metrics (Curry & Dagli, 2014). The concept of equifinality has been widely explored in hydrology and water resources (Khatami et al., 2019), as well as in agent-based models (Williams et al., 2020). However, with the exception of a recent example from the social sciences (Vu et al., 2019), the equifinality problem is rarely approached in integrated studies by global search over model structures that considers both performance and complexity during training.

Finally, when model generation results in a large number of plausible model structures, a range of diagnostic tools can be applied to further assess the common structures and parameters driving model behavior. For example, Pruyt and Islam (2015) use clustering to partition exploratory model parameterizations based on their behavior as transfer functions mapping input to output. In the absence of well-characterized uncertainty, sensitivity analysis can diagnose model prediction behavior and provide a metric by which to justify the inclusion of parameters (Pianosi et al., 2016; Gupta & Razavi, 2018; Wagener & Pianosi, 2019). Dobson et al. (2019) design a scenario resampling strategy to show the importance of contextual uncertainty in the performance of operational rules of water systems. These and similar approaches assist with the evaluation of models of human behavior in the abstract, through which key structural elements can be identified post-optimization.

3 Experiment

Figure 2 outlines the computational steps for the three experimental phases: problem definition, model generation, and model evaluation. The Problem Definition phase includes the definition of prediction tasks, feature engineering, and the specification of function primitives. The Model Generation phase includes the selection of an encoding representation and search procedure, the definition of metrics to use for evaluating models during search, and the search over candidate model structures in a multi-objective space. The Model Evaluation phase for these experiments focuses on the collection and analysis of many plausible model sets across many random trials. Clustering and sensitivity analysis techniques are employed to determine driving structure and features in different regions of the performance space.

3.1 Problem Definition

This approach is demonstrated on an application of agricultural land use change, one of the primary ways in which human decisions influence water resources systems, in addition to reservoir operations and urban consumption. Land use change represents a

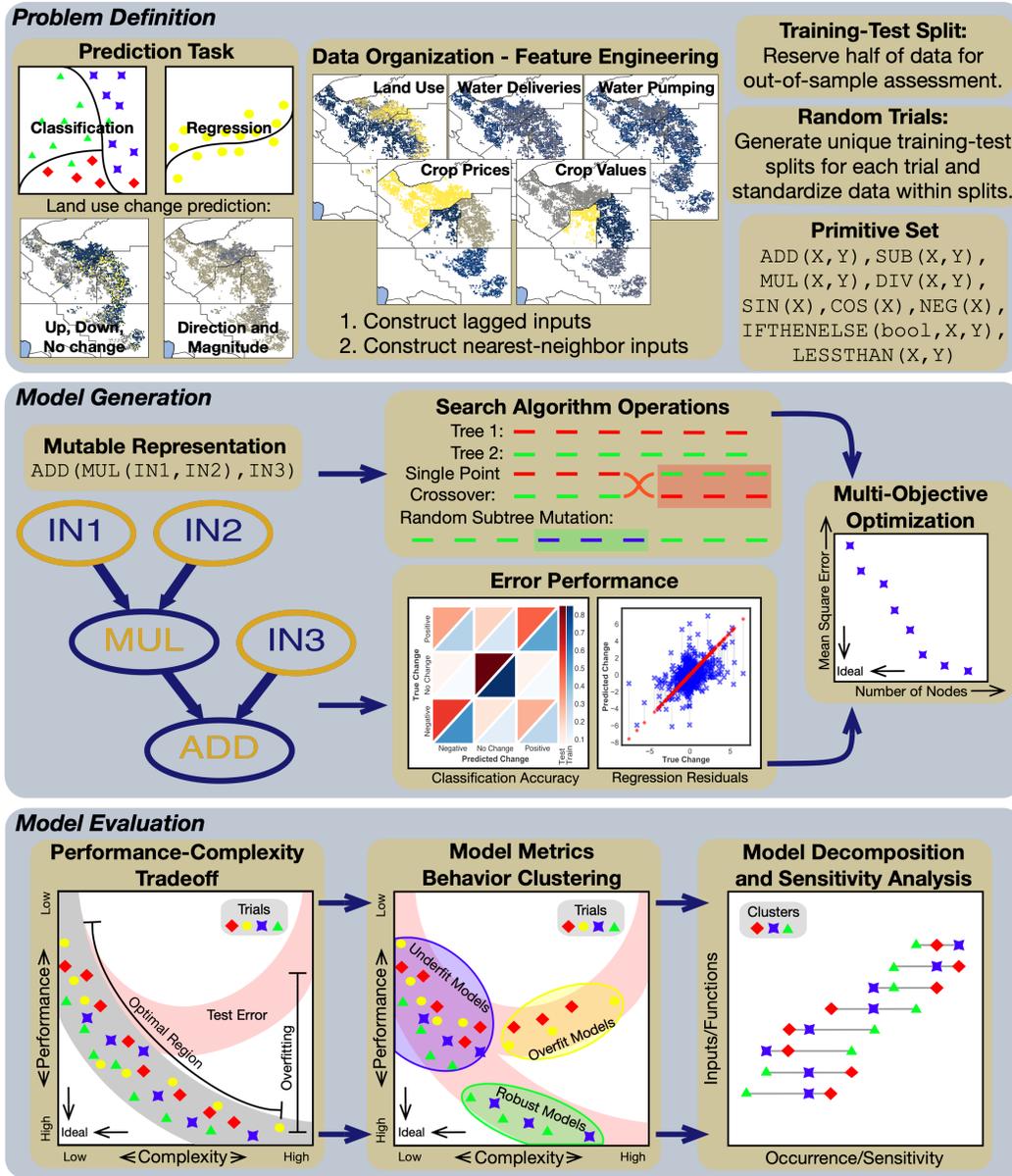


Figure 2. Schematic of experimental setup and workflow

293 complex test case in spatially distributed, heterogeneous decision-making (Groeneveld
 294 et al., 2017). Additionally, models of land use change depend on heterogeneous sources
 295 of information, such as water availability and socioeconomic factors (Nelson & Burch-
 296 field, 2017; Jasechko & Perrone, 2020; Malek & Verburg, 2020). This problem has been
 297 approached from multiple perspectives, including theoretical models based on economics
 298 and psychology (Schlüter et al., 2017), as well as statistical models (B. Sun & Robin-
 299 son, 2018), which together suggest no clear agreement on process representation (Verburg
 300 et al., 2019). Economic models of land use change have been developed at the global scale
 301 (Prestele et al., 2017; Stehfest et al., 2019) and also at the regional scale (Howitt et al.,
 302 2012, e.g.), and the integration of local and regional results into global models is cur-
 303 rently being explored (Melsen et al., 2018; Schlüter et al., 2019; Malek & Verburg, 2020).
 304 In both cases, parameters are calibrated against historical observations. However, it is

305 also acknowledged that land use decisions, like other water resources decisions, do not
 306 always follow the principle of full rationality (Groeneveld et al., 2017; Schlüter et al., 2017).
 307 By contrast, agent-based rules have also been developed for regional land use models,
 308 often ad hoc using expert judgment (Thober et al., 2017) informed by empirical stud-
 309 ies (Robinson et al., 2007). There remains an opportunity to automate this process via
 310 model generation techniques, as has been explored elsewhere in the social sciences (Gunaratne
 311 & Garibay, 2017; Vu et al., 2019, e.g.). While land use change presents a challenging
 312 test case, the methods proposed here also generalize to other aspects of human behav-
 313 ior in water resources systems, contingent on the availability of scale-appropriate datasets.

314 3.1.1 Case Study

315 This approach is applied to the problem of understanding dynamic agricultural land
 316 use patterns in the Tulare Basin region of California. In this case study, we use data-
 317 driven system identification to discover a mathematical function to predict the year-to-
 318 year change in tree crop acreage for all continuously planted square-mile sections of land
 319 in the Tulare Basin from 1974 to 2016. This is a human response variable that is of par-
 320 ticular interest for water resources management because of a strong historical trend to-
 321 wards tree crops (Figure 3) that has exacerbated groundwater overdraft, especially in
 322 times of drought (Jasechko & Perrone, 2020).

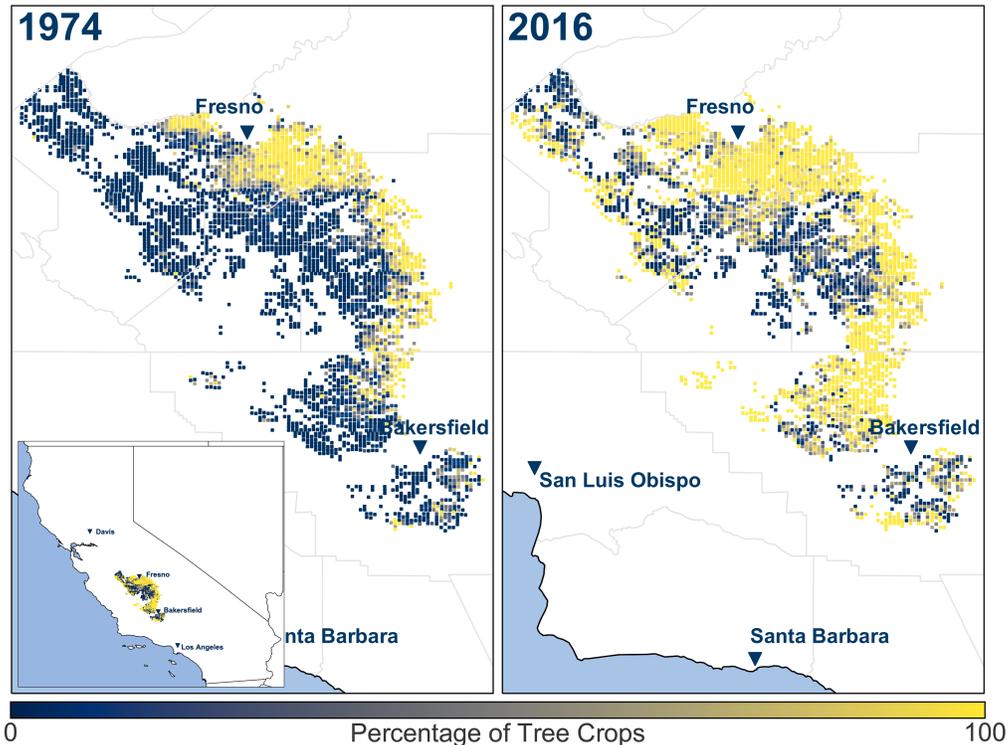


Figure 3. Historical change in crop type in the Tulare Basin, California from 1974 to 2016. Each grid cell is 1 mi², and tree crops are defined as in Mall and Herman (2019). The grey lines indicate county boundaries within which crop prices are reported annually.

3.1.2 Problem Definition

The state of the system x_t is defined as an n -tuple drawn from \mathbb{R}^n that includes the current and previous state of tree crops (a_t, a_{t-1}, \dots) and non-tree crops, the lagged change of tree-crops ($a'_{t-1}, a'_{t-2}, \dots$) and non-tree crops since the current change is being predicted, and other current and lagged information such as the current crop price, agricultural pumping, and surface water deliveries.

$$x_t := (a_t, b_t, c_t, \dots, a_{t-1}, a'_{t-1}, b_{t-1}, c_{t-1}, \dots) \quad (1)$$

where $a_t = a_{t-1} + a'_{t-1}$. Given the state of the system x_t representing all current and previous information at a given spatial index, in learning the dynamics of the system we aim to predict the annual change in acreage at the same spatial index, a'_t , as a function of previous changes, current and previous states, and other features (more information about these feature variables is given in Section 3.1.3):

$$D_{x_t} := \frac{\Delta x_t}{\Delta t} = F(x_t) \quad (2)$$

The notation D_{x_t} is used to refer to the difference in tree crops a'_t that would advance the tree crop state forward in time, $a_{t+1} = a_t + a'_t$. x_t includes lagged responses such as $D_{x_{t-1}}$, the response of the previous state at the same index. The problem of learning model structure is therefore to determine the function F that maps a given set of features to the annual change in state. x_t includes potentially high-dimensional information describing the current state and any number of previous states (Lusch et al., 2018). When the dynamics of F are unknown, general function forms are initialized randomly and trained to approximate system dynamics by learning from observed or measured data.

We explore two different prediction tasks related to this problem, regression and classification. In the regression formulation, models predict the magnitude and direction of the annual change in tree crop acreage. In the classification problem, models predict the direction of change only—positive, negative, or no change—as displayed under Prediction Task in Figure 2. Regression is generally considered a more difficult problem as functions must predict a continuous value, whereas this classification task requires predicting the most likely of three classes.

3.1.3 Feature Engineering

Feature data describing land use, water availability, and economics were organized into samples to train and test candidate model structures. Land use data was taken from the California Pesticide Use Reports, available digitally beginning in 1974 and extracted by Mall and Herman (2019). Annual crop type data are taken from 1974-2016 at the square-mile scale for over 3000 grid cells in the Tulare Basin, and the target data are partitioned into tree and non-tree crops. Water availability data were taken from the C2VSim-IWFM groundwater model representing pumping and delivery estimates for the categories of urban, agricultural, rice crop, and refuge pumping and deliveries, further details for which are described in Kourakos et al. (2019). Lastly, county-level crop prices were taken from the California County Agricultural Commissioner reports, beginning in 1980 across Tulare, Fresno, Kings, and Kern counties (USDA National Agricultural Statistics Service - California Field Office, 2019). Crop prices were adjusted for inflation using the producer price index for agriculture, based on the year 2016, published by the U.S. Bureau of Labor Statistics (U.S. Bureau of Labor Statistics, 2019). A summary of trends for this heterogeneous data set is presented in Figure 4.

Additional features were included to account for the space-time dependence of the problem. Samples were organized such that each grid-cell sample was tagged with its data,

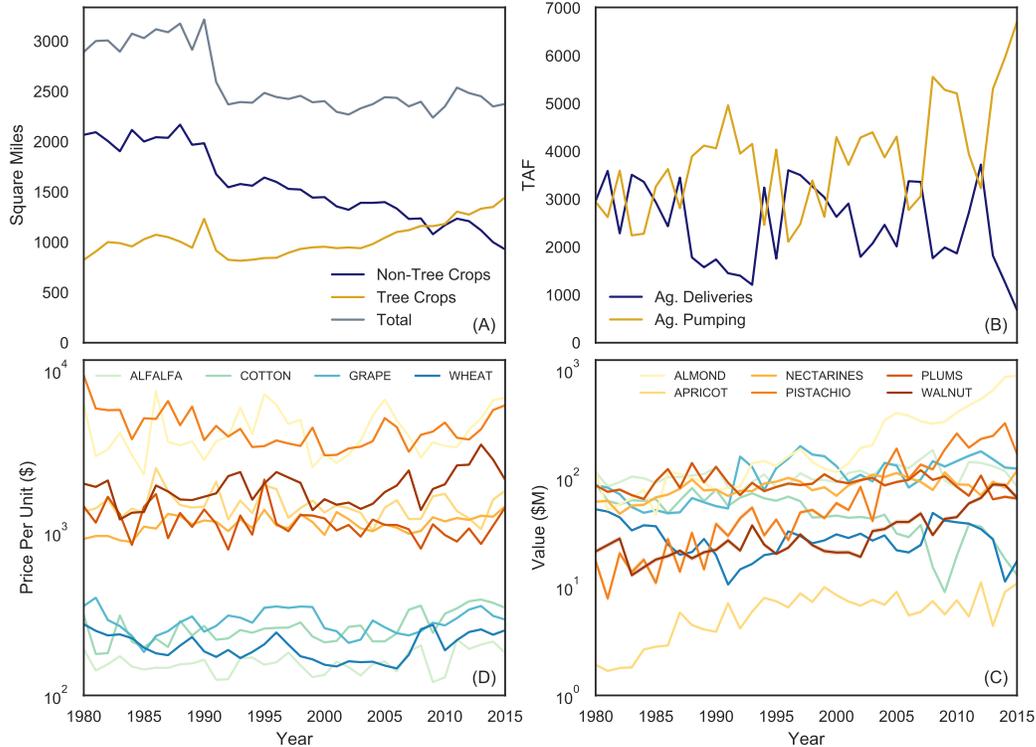


Figure 4. Historical trends in heterogeneous feature data. (A) Tree crop acreage, non-tree crop acreage, and total acreage planted; (B) Yearly total agricultural water deliveries and pumping; (C-D) Inflation-adjusted prices and total crop values for a selection of crops.

367 the previous six years of data, and the same data from each of 5 neighboring grid cells
 368 in space. Since economic information is only available from 1980 onward and spatially
 369 distributed at the county scale, this space-time extension was only implemented for land
 370 and water data. Absolute data, such as the year and location, were excluded from the
 371 set of features to avoid overfitting. The resulting dimensions of the data were on the order
 372 of 500 predictor variables and 130,000 samples. No explicit dimension reduction steps
 373 were implemented in order to maintain the interpretation of feature variables within the
 374 eventual model structures generated by this approach. Samples were split into 50% training
 375 and 50% test, and both the features and response variable were standardized to $\mathcal{N}(0, 1)$.
 376 Other than the bias introduced by constructing variables representing temporal lags and
 377 spatial neighborhoods, no empirical or theoretical priors were provided to inform the search.
 378 This spatiotemporal construction process also adds redundancy into the feature set, and
 379 we rely on the model search (Section 3.2.2) to navigate this redundancy to identify the
 380 most informative features while retaining interpretability.

381 3.1.4 Model Structural Elements

382 In addition to the feature variables, the primitive set of functions composing the
 383 feasible model structures must also be specified. The primitive set includes the math-
 384 ematical relationships detailed in Table 1.

385 To include relational and logical operators in addition to mathematical operators
 386 in the primitive set, the functions are strongly typed, meaning that intermediate vari-
 387 ables must match data types for the input and output of each component function. Con-

Functions

[float] = add([float],[float])	[float] = sin([float])
[float] = subtract([float],[float])	[float] = cos([float])
[float] = multiply([float],[float])	[float] = negative([float])
[float] = divide([float],[float])	[bool] = less_than([float],[float])
[float] = if_then_else([bool],[float],[float])	

Constants

(1,[bool])	(RandInt(0,100)/10.,[float])
(0,[bool])	(RandInt(0,100)/1.,[float])

Table 1. The primitive set functions and constants, as defined for both regression and classification experiments. The space of feasible models is constrained by strong typing. The function $\text{RandInt}(a, b)$ generates a uniform random integer on (a, b) .

388 stants are also defined as either boolean or floating point values as indicated in Table
 389 1 and appear as terminal nodes in an expression, as do the model inputs (features). Con-
 390 stants are drawn from a distribution, though the resulting model is deterministic after
 391 the constants have been generated. However, the distributions themselves can be included
 392 in the primitive set, allowing the automatic construction of stochastic models (M. D. Schmidt
 393 & Lipson, 2007). In addition, search over the model space can be biased by providing
 394 a specific set of operators, inputs, or constants as seeds (M. D. Schmidt & Lipson, 2009).
 395 By defining the primitive set and input space in this way, we ensure that search over the
 396 model space covers a broad general space of models, including linear and higher-order
 397 combinations of inputs and discontinuous functions.

3.2 Model Generation

3.2.1 Search Objectives

400 For the regression problem, the performance objective used to train model struc-
 401 tures is the mean squared error (MSE), a commonly-used error metric that emphasizes
 402 larger residuals. A baseline performance value for MSE on the response variable—standardized
 403 to $\mathcal{N}(0, 1)$ —is 1.0, which results from using the average prediction (zero) for every sam-
 404 ple. For a given regressor $F : \mathbb{R}^n \rightarrow \mathbb{R}^1$:

$$MSE_{train} := \text{ave}_{x_t \in X_{train}} (\hat{D}_{x_t} - D_{x_t})^2 \quad (3)$$

405 In the classification experiment, the multi-class output is addressed via ensemble
 406 learning, a common method in genetic programming studies (Espejo et al., 2010). The
 407 performance objective is the percent of misclassified samples. This is equivalent to $1 -$
 408 *Accuracy*, where accuracy is the percentage of classes predicted correctly. A baseline per-
 409 formance for misclassification percentage for this application is approximately 0.54, which
 410 results from predicting the most common class (no change) for every sample. The mis-
 411 classification percentage can be calculated using the Hamming loss, $l(\hat{y}, y)$, which takes

412 the value 1 for predictions that do not match the response and 0 otherwise. For a given
 413 classifier $F : \mathbb{R}^n \rightarrow \{Negative, No\ Change, Positive\}$:

$$MCP_{train} := ave_{x_t \in X_{train}} l(\hat{D}_{x_t}, D_{x_t}) \quad (4)$$

414 Though the three classes are relatively balanced in this experiment $\{Negative \sim$
 415 $25\%, No\ Change \sim 47\%, Positive \sim 28\%\}$, this simple accuracy metric might pro-
 416 mote models that perform well on only a subset of classes. This can be a problem par-
 417 ticularly when classes are not equally represented in the training set (Provost & Fawcett,
 418 2001). Multi-class metrics such as the macro/micro-averaged F1-measure (Lipton et al.,
 419 2014) and receiver operating characteristic (ROC) curve (Fawcett, 2006) can account for
 420 class imbalance by weighting measures based on individual class accuracies. However,
 421 we find that for this problem, alternate metrics do not significantly change the rank or-
 422 der of models within each class (see Supplemental Material). In regard to improving re-
 423 gression metrics, the water resources field has thoroughly considered how error metrics
 424 for natural process models can incorporate available process knowledge (Gupta et al.,
 425 2009; Khatami et al., 2019; Lamontagne et al., 2020, e.g.,). These approaches are also
 426 relevant in scenarios lacking process knowledge but with known statistical relationships
 427 in the error signals.

428 A second objective, model complexity, is formulated and optimized concurrently
 429 with the performance objectives above using multi-objective optimization. The complex-
 430 ity metric is taken to be the representation length, a commonly used surrogate for com-
 431 putational or algorithmic complexity of a model (Vanneschi et al., 2010), which in this
 432 case is the number of elements (nodes) in the ordered list representing the model. The
 433 complexity value is normalized by the maximum depth of recursive function calls in Python
 434 (90) to roughly match the scale and precision of the performance objectives.

435 3.2.2 Search Algorithm

436 The search over candidate model structures and parameterizations employs a cus-
 437 tomized genetic programming algorithm, an evolutionary approach that encodes math-
 438 ematical expressions in a tree structure to support symbolic regression. Modular com-
 439 ponents of the algorithm were drawn from the package Distributed Evolutionary Algo-
 440 rithms in Python, or DEAP (De Rainville et al., 2012). As depicted in the Model Gen-
 441 eration panel of Figure 2, mutation and crossover operators act on ordered representa-
 442 tions of models, where each tree is flattened into an ordered list of elements, to gener-
 443 ate new structures from promising candidates and explore the model space during op-
 444 timization. The mutation operator adds a randomly initialized sub-tree of depth 1-2, rep-
 445 resenting a random addition into the model element list. Single-point crossover randomly
 446 selects a location along paired model element lists and exchanges the elements beyond
 447 this location to generate a new model, an example of which is depicted in Figure 5. Mu-
 448 tation explores the model space by introducing new model structures, and crossover ex-
 449 ploits the attributes of current models by testing new combinations of existing model struc-
 450 tures. The mutation and crossover operations can result in invalid models according to
 451 the strong typing criteria, where intermediate data types among tree operations do not
 452 match; these models are discarded before evaluation.

453 During training, the performance and complexity objectives are both minimized.
 454 This has two implications: (1) the minimum complexity (maximum interpretability) model
 455 is preferred among two models with the same performance, (2) if the space of possible
 456 models is searched exhaustively, the resulting tradeoffs between models should be the
 457 minimum complexity model for a given level of performance. The algorithm follows a
 458 $\mu + \lambda$ evolution strategy, which allows parents to persist in the population. At each gen-
 459 eration, a number of offspring μ are generated from λ parents in the population by ap-

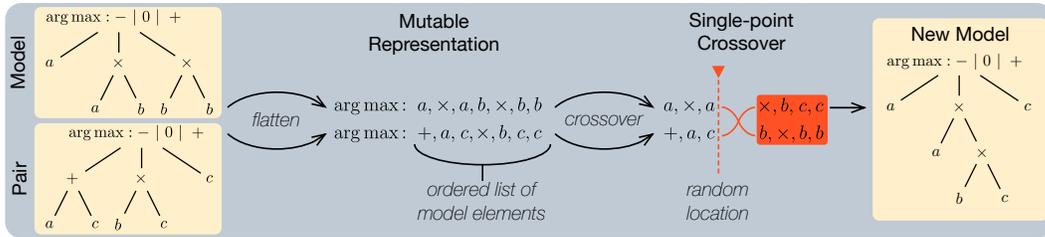


Figure 5. Detailed view of crossover operations, expanded from Figure 2. Two models from the population are used to generate a new model by splitting and recombining the ordered list representations at a random location, a process repeated throughout the search. Mutation operates similarly by adding a random sub-tree at a random location in a single model.

460 plying mutation and crossover with a given probability. The population is updated by
 461 applying deterministic crowding selection analogous to NSGA-II (Deb et al., 2000) to
 462 the collection of individuals $\mu + \lambda$, selecting λ individuals to be used as parents in the
 463 next generation. The use of deterministic crowding for selection is intended to promote
 464 diversity within populations by spacing out models along the Pareto front. This ensures
 465 that no single model dominates in all objectives and is therefore used to generate all new
 466 individuals in the next generation. Separately from the population, an archive of Pareto-
 467 approximate model structures is maintained and updated through strict non-dominated
 468 sorting of the archive and population together in each generation, with no crowding dis-
 469 tance selection applied. This archive represents the best approximation of the Pareto front
 470 at each iteration of the optimization (including the final result), and allays degradation
 471 known to occur in populations when using deterministic crowding for selection.

472 Experiments were run using the UC Davis College of Engineering HPC1 Cluster
 473 with 96 processors, employing DEAP package support for distributed computing. Each
 474 population of models contains 96 individuals, and each tree is initialized randomly with
 475 depth 1-3. Trials run for a maximum of 20,000 generations with a stagnation convergence
 476 criterion of 2,500 generations, which will stop the algorithm if performance improvements
 477 are not detected during this time. Performance improvements can be found throughout
 478 the optimization, but can become exceedingly small as models start to overfit. As in many
 479 high-dimensional sampling problems, it is not possible to prove that the global optimum
 480 has been reached. Though the algorithm is likely to comprehensively sample low-complexity
 481 models, the size of the primitive set (number of inputs, constants, and functions) dic-
 482 tates that the sampling coverage of possible models decreases at least factorially with
 483 additional model primitives (Knuth, 2011). Combinatorial expansion reflects the curse
 484 of dimensionality, and complicates the search for medium- and high-complexity models,
 485 though more efficient algorithms are an active area of research (Hadka & Reed, 2013; Vrugt
 486 & Beven, 2018; Conti et al., 2018, e.g.). This complexity increases the likelihood of op-
 487 timization trials getting stuck in local minima as trees grow, and emphasizes the impor-
 488 tance of appropriately defining the model space during problem definition. To account
 489 for this stochasticity in optimization, 21 randomized trials are performed, which includes
 490 the initialization of the train-test split. The code to reproduce this study can be found
 491 at DOI: 10.5281/zenodo.3887360.

492 This algorithm configuration may generate spurious structure and/or redundant
 493 features within the same model. The Supplemental Material includes more details about
 494 the feature variables and their correlation. However, the algorithm performs variable se-
 495 lection to some extent when feature variables that lead to improved objective performance
 496 are introduced through mutation or crossover, suggesting an informative relationship.
 497 Even with correlated features, we expect that over the course of many iterations of the

498 mutation operator, and multiple random seeds, the most informative features will oc-
 499 cur most often in the resulting sets of models. This stochasticity in model structural iden-
 500 tification reinforces the need for multiple trials, ensemble averaging across optimization
 501 trials during model evaluation, and summary statistics describing high-complexity re-
 502 gions of the model space, as any one model structure by itself may be subject to feature
 503 redundancy.

504 3.3 Model Evaluation

505 Following the model training, candidate structures are evaluated in three ways: trade-
 506 offs between performance objectives, model behavior in the metric space, and decom-
 507 position and sensitivity of the underlying structure and features. The approach to model
 508 evaluation taken during this phase depends on modeling decisions during problem def-
 509 inition and model generation. In these experiments, the feature data and primitive set
 510 together define a combinatorially large space of possible models, creating substantial un-
 511 certainty that must be acknowledged in the analysis that follows.

512 3.3.1 Performance-Complexity Tradeoff

513 After evaluating performance on the test set, models are placed in a three-dimensional
 514 performance-complexity tradeoff, as illustrated under Model Evaluation in Figure 2. Along
 515 the Pareto front, training error within a given trial will strictly decrease as complexity
 516 increases. However, as complexity of the model increases, test error can diverge from train-
 517 ing error if the model overfits. If error performance changes relatively little across a broad
 518 range of model structures, this is an indicator of equifinality. To investigate this outcome
 519 further, candidate models can be clustered into groups with similar behavior. Specifi-
 520 cally, k-means clustering is used to separate models according to training error, test er-
 521 ror, and complexity.

522 3.3.2 Model Decomposition and Sensitivity Analysis

523 The collection of Pareto-optimal sets of models constitutes a new high-dimensional
 524 data set of structured model components and their associated performance metrics. Among
 525 many network analysis tools for structural and dynamic analysis of graphical models,
 526 model decomposition is a very simple initial step. The driving structural properties of
 527 each model—number of metrics, attributes, inputs, functions, and constants—are linked
 528 to their behavior cluster as described above. Each model is also tested for its sensitiv-
 529 ity to individual features and their interactions using Sobol sensitivity analysis with the
 530 Python package SALib (Herman & Usher, 2017). The goal of this sensitivity analysis
 531 is to determine whether the different clusters of model behavior are influenced by dif-
 532 ferent feature variables, for example if certain features appear primarily in overfit mod-
 533 els. To perform this step, each model is re-evaluated with 1000 samples scaled by the
 534 cardinality of its unique feature set to ensure sufficient coverage of the sample space. For
 535 example, if a model has 5 unique inputs, the model would be tested with 5000 samples
 536 for each unique input to appropriately characterize pairwise and total-order sensitivi-
 537 ties in the Sobol method.

538 4 Results

539 4.1 Model Performance-Complexity Tradeoff

540 Figure 6 shows the tradeoff between model performance and complexity across the
 541 Pareto set of candidate model structures for both (a) regression and (b) classification
 542 experiments. Each point represents the performance (MSE) on the test data, while the
 543 gold background shading shows the distribution of performance for the same set of mod-

544 els on the training data. Figure 6 highlights four different regions: Parsimonious, Over-
 545 fit, Equifinal, and Dominated model clusters. These designations are subjective, but sep-
 546 arate the models for discussion according to their primary evaluation characteristics. Dur-
 547 ing each trial, initial structure building occurs in the Parsimonious cluster in both Fig-
 548 ure 6a and 6b. The Overfit clusters in Figure 6 are highlighted as the regions where mod-
 549 els begin to rely on spurious structure discovered later in the trial. The Equifinal clus-
 550 ter in Figure 6a represents a region where multiple model structures exist at roughly the
 551 same level of performance. The Dominated cluster in Figure 6b represents models that
 552 are both relatively complex and do not generalize well to unseen data.

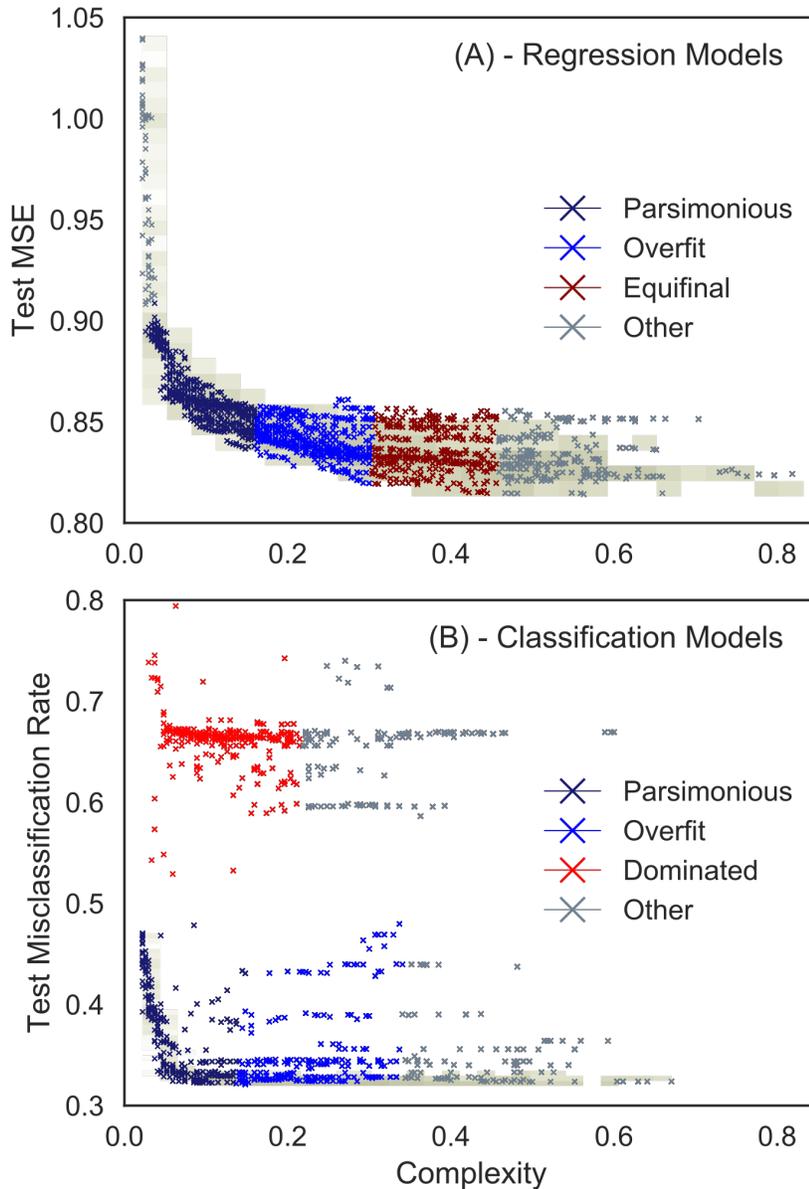


Figure 6. Tradeoff between performance (test error) and complexity for model structures across (A) all regression trials and (B) all classification trials. Light gold shading indicates the distribution of the same models evaluated on the training data. Models are clustered according to their behavior in this three-dimensional space (training error, test error, and complexity).

553 These results indicate several points. First, regression trials in Figure 6a exhibit
 554 better robustness to test data, with most models remaining within the region of the train-
 555 ing error displayed in the gold background. Classification experiments show diminish-
 556 ing returns to increasing complexity much faster than regression experiments. The progress
 557 of the optimization trials is determined by the model structures developed in the Par-
 558 simonious clusters; insufficient exploration may explain why significant overfitting oc-
 559 curs in Figure 6b. Equifinal model structures are observed in both cases, as many mod-
 560 els with increasing complexity demonstrate similar performance.

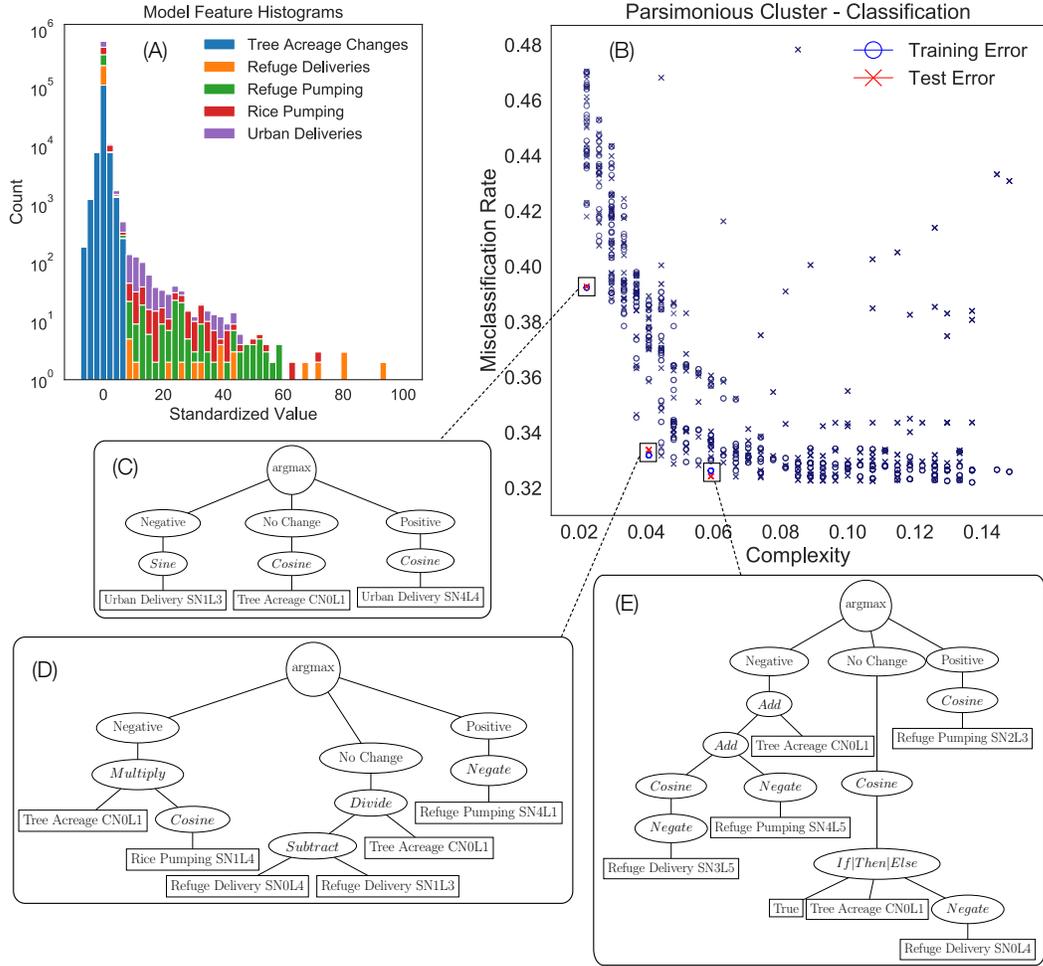


Figure 7. (a) Histograms of standardized feature data ($\frac{x-\mu}{\sigma}$) represented in the models; (b) Training and test error for models in the Parsimonious cluster; (c-e) selection of models from the Parsimonious cluster. Feature constructions are annotated as {State/Change | Neighbor 0–5 | Lag 1–5}. In (a), some of these distributions are asymmetric even after standardization; skewed features such as refuge deliveries and pumping occur more infrequently than the relatively balanced tree acreage changes. In (c-e), the arg max operator returns the class {*Negative*, *No Change*, *Positive*} of maximum value for a given sample.

561 The classification results in Figure 6b show model structures with a variety of macro-
 562 scopic behavior that can be investigated further. We proceed with the classification re-
 563 sults to determine the drivers of model behavior, and also to examine the structure of
 564 three models selected from the Parsimonious cluster that perform well on both training

565 and test data in Figure 7. These three classification models depend on a variety of fea-
 566 ture variables and structural elements. Figure 7a displays a histogram of standardized
 567 feature data represented in the models to understand any patterns shared among the dis-
 568 tributions of feature variables selected by the algorithm for these three model structures.
 569 While the models occasionally rely on sparse, skewed feature distributions such as non-
 570 agricultural water use, they mainly rely on tree acreage changes. Specifically, all three
 571 models use the acreage change in the previous timestep (lag-1) and same location, indi-
 572 cating that decision-making agents are informed by past decisions. Additionally, the
 573 tree acreage change feature tends to occur closer to the output of each model structure
 574 (Figure 7c-e), and as a result is less modified than other features by the sequence of arith-
 575 metic operations in each model.

576 4.2 Feature Occurrence and Sensitivity

577 Large differences among models regarding the selection of other feature variables
 578 indicate that some of these structural components may be spurious. The distribution of
 579 features chosen by the algorithm might be a result of their different spatiotemporal res-
 580 olutions. For example, the lack of consensus on the use of economic data could be due
 581 to its coarser resolution in space and limited coverage in time, or the inability of the search
 582 method to find informative features beyond the lag-1 tree acreage change. To investi-
 583 gate this further, we aim to identify the structural drivers separating robust models in
 584 the Parsimonious and Overfit Clusters from models that do not generalize well (i.e., the
 585 Dominated cluster). First, we start by analyzing the occurrence of features and function
 586 primitives among models in each cluster, displayed in Figure 8.

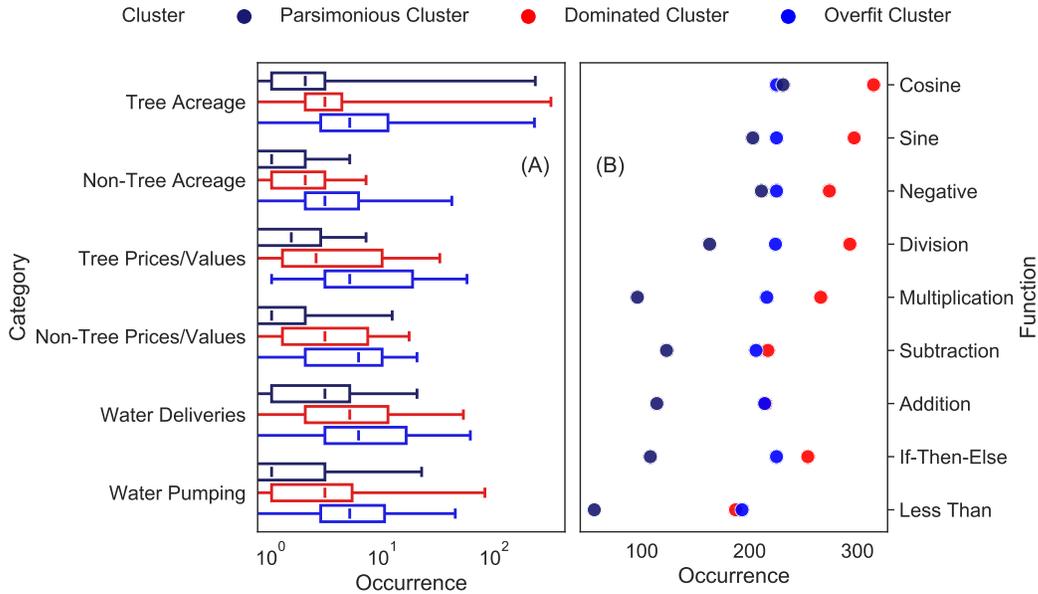


Figure 8. Occurrence of feature variables and function primitives among classification models. (A) Distribution of occurrence by feature category; (B) occurrence of functions. The former is a distribution because each feature category contains multiple feature variables, while the functions are not grouped.

587 Figure 8a shows the distribution of feature occurrence counts in each model cluster,
 588 where the features are grouped into categories (y-axis). The boxplots and ranges sug-
 589 gest several key points. All model clusters show a dependence on the group of inputs re-

lated to tree acreage data (all lagged and neighboring states and values for tree crops). The lag-1 tree acreage change in the same location (categorized under Tree Acreage) appear in every model across all clusters, indicated by the range of the whiskers at the top of Figure 8a. The Overfit cluster contains more instances of features from each category as compared to the Dominated and Parsimonious clusters, suggesting a higher level of feature complexity overall. Lastly, the largest differences in feature usage between the Parsimonious cluster and the Overfit cluster is in the tree and non-tree prices/values and water pumping feature categories.

Figure 8b shows the occurrence count of function primitives for models in each cluster; the primitives are not categorized into groups, so the values are a single count rather than a distribution. The Overfit cluster exhibits a more even distribution of function occurrence across primitives than the Parsimonious and Dominated clusters, suggesting an increase in the diversity of function primitives relative to the Parsimonious cluster. Both the Overfit cluster and Dominated cluster learn a dependence on the two conditional primitives. Finally, models in the Dominated cluster contain more instances of nearly every function type, particularly deviating from the Overfit and Parsimonious clusters for single-input functions, suggesting a higher level of functional complexity and feature transformations than either the Overfit or Parsimonious clusters.

Figure 8a-b together indicate that robustness to test data may be extended for models in the Parsimonious cluster by increasing reliance on feature complexity versus functional complexity. This contrast may also explain why additional complexity in two- and three-input functions for combining features is warranted over single-input functions that merely transform individual features. However, feature occurrence alone does not explain which features drive model output. Model responses to feature variable changes are quantified using Sobol sensitivity analysis. Results for total sensitivity indices are presented in Figure 9 as empirical cumulative distributions. The sensitivities are presented for two categories of feature variables, tree acreage and non-tree acreage, across the three clusters of classification models.

Figure 9 shows that over 60% of the tree acreage features (including lagged and neighboring feature occurrences) in models from the Overfit cluster have a total-order sensitivity index near zero, meaning that these features have a negligible effect on the class prediction. Both the Overfit and Dominated models show lower sensitivity to both categories of features relative to the Parsimonious cluster, indicating that the best-performing models are driven by a wider range of features. In the case of tree acreage inputs, over 70% of features in the Overfit cluster show small sensitivities ($S_T < 0.2$) compared to less than 50% for the model structures from the Dominated and Parsimonious clusters. However, at least 20% of tree acreage inputs to both the Overfit and Dominated models are high ($S_T > 0.8$), illustrating a high reliance on fewer feature variables, which may reduce the ability of these models to generalize out-of-sample. Conversely, both the Overfit and Dominated models do not show the same high sensitivities to non-tree acreage data that appear in the Parsimonious models.

This result confirms the conclusion from Figure 8 that previous tree acreage states and changes are a main driver for this problem. The results also indicate a partition in the information important to the decision problem; since crop switching requires specializing and alternate scheduling, it is perhaps unexpected that over 60% of non-tree crop features had negligibly small impacts on the class prediction. Similarly, there were very few features with sensitivity indices greater than 0.6 among the Overfit or Dominated models. Sensitivity testing was only applied to features selected during the generation of each model, so the distributions of sensitivity indices are not affected by the frequency with which a feature is included in each model cluster.

Finally, the average total-order sensitivity indices within each feature category and variable construction are displayed across model clusters in Figure 10. Parsimonious mod-

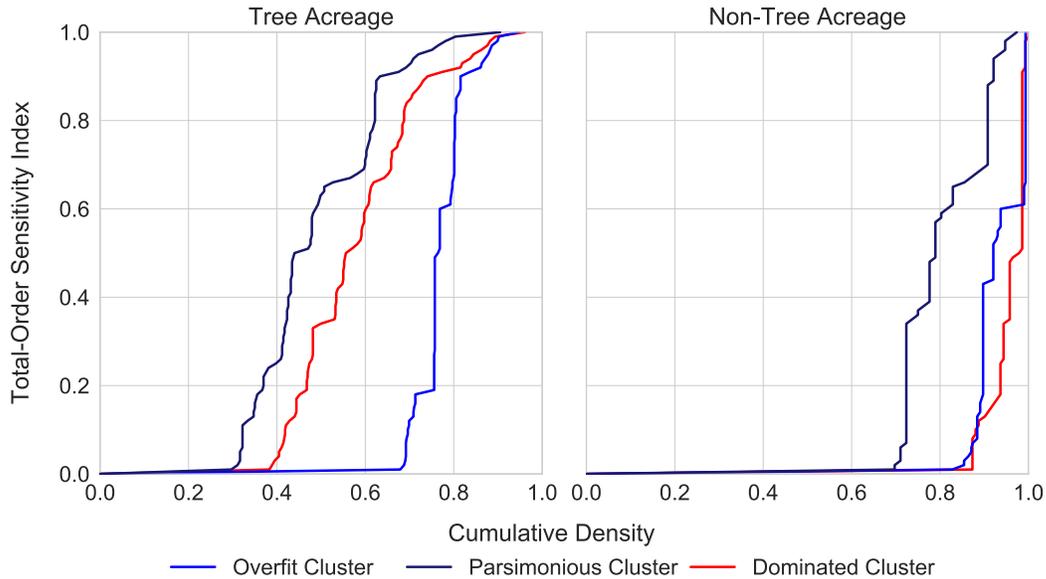


Figure 9. Empirical cumulative distribution of total-order sensitivity indices for two categories of feature variables: tree acreage and non-tree acreage, separated by model cluster (color). Only the feature variables appearing in each model were included in the sensitivity analysis.

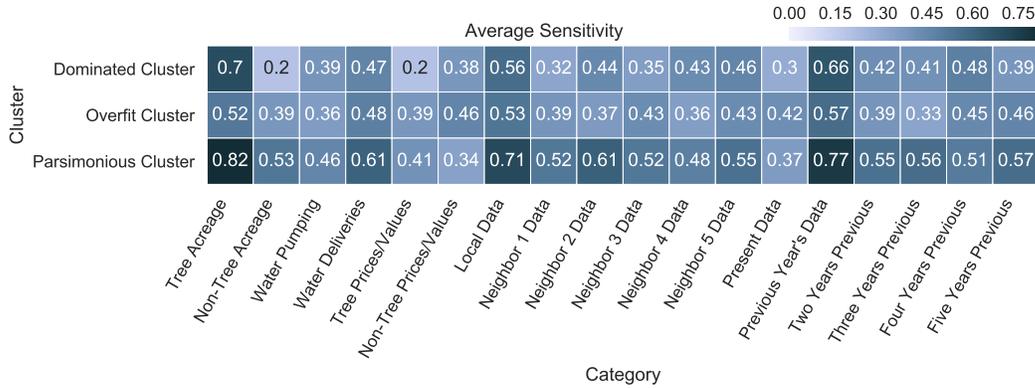


Figure 10. Average total-order sensitivity indices of feature variables across input categories for each cluster of model structures. In the feature grouping labels, “data” refers to the combination of state, change, temporal lags, and spatial neighbors for each type of feature.

642 els demonstrate elevated sensitivities across many of the feature categories and construc-
 643 tions. Since Parsimonious models are less complex than either Overfit or Dominated mod-
 644 els, that their predictions are highly affected by a wide range of input variables makes
 645 intuitive sense, as Parsimonious models explain a similar fraction of total variance with
 646 fewer features. Parsimonious models are also particularly sensitive to tree acreage fea-
 647 tures from the previous year, as shown in prior figures. Though we might expect high
 648 bias in these Parsimonious models, bias seems to be minimized fairly quickly by selec-
 649 tive inclusion of feature variables.

650 Models in the Dominated cluster share high average sensitivity for only some fea-
 651 ture categories but not others, as do models from the Overfit cluster to a lesser degree.
 652 Models from the Overfit cluster exhibit relatively equal sensitivities across all feature cat-
 653 egories as compared to the range of sensitivities represented in the Parsimonious and Dom-
 654 inated clusters. This, combined with the function occurrence result from Figure 8b, sug-
 655 gests that the Overfit models avoid becoming overly sensitive to individual features from
 656 any one category by over-engineering the function structure, which likely leads to their
 657 improved generalization ability over the Dominated models. This result demonstrates
 658 how averaging sensitivity to certain categories of features within model clusters can re-
 659 veal the extent to which models should be sensitive to feature data given target prop-
 660 erties, such as model robustness to unseen data. However, averaging across the set of mod-
 661 els may obscure the sensitivities of individual models, the distribution of which is bet-
 662 ter shown in Figure 9.

663 5 Discussion

664 There is a distinct need for integrated systems models when descriptions of the phys-
 665 ical system are incomplete without consideration of the human component (Konar et al.,
 666 2019; Herman et al., 2020). This must include representations (Schill et al., 2019) and
 667 feedbacks (Calvin & Bond-Lamberty, 2018) that may not be implemented in existing model
 668 structures. This study proposes methods to automate the exploration of model struc-
 669 ture along the canonical tradeoff between performance and complexity to describe hu-
 670 man behavior. In this illustrative case study focused on agricultural land use and wa-
 671 ter demand, enumerating the range of model performance with increasing model com-
 672 plexity by drawing structures from a general, unconstrained space provides context for
 673 any prior-informed solutions that might arise in the same context. The relative perfor-
 674 mance demonstrated here thus forms a basis for the analysis of model structural uncer-
 675 tainty (Walker et al., 2003) by considering model structures as competing hypotheses
 676 (Beven, 2019), which could be compared alongside theory-based models.

677 Generating candidate model structures includes automatic feature selection and
 678 requires no prior knowledge of the system’s mechanics, constraints, or information re-
 679 quirements beyond the basic provision of feature data and primitives (Bongard & Lip-
 680 son, 2007; M. Schmidt & Lipson, 2009), though informing and bounding search through
 681 process understanding and structural priors (Knüsel et al., 2019), constrained problem
 682 framings (Dobson et al., 2019; Müller & Levy, 2019, e.g.), and structured generation schemes
 683 (Chadalawada et al., 2020; Spieler et al., 2020, e.g.), and using advanced interpretation
 684 tools post-search (Worland et al., 2019; Quinn et al., 2019, e.g.) could uncover more spe-
 685 cific emergent phenomena in the data and resulting models. However, framing model struc-
 686 tural experimentation according to this generic framework enables a baseline contextu-
 687 alization of the complex integrated systems problem. In this way, a data-driven approach
 688 to generating and evaluating model structure can support the design of integrated sys-
 689 tem models such as agent-based or hydro-economic models.

690 This case study was encumbered by two primary sources of difficulty: (1) algorithmic
 691 search in combined parametric-structural model spaces, and (2) heterogeneous fea-
 692 ture data across multiple temporal and spatial scales. First, the search space of candi-
 693 date model structures grows combinatorially with the number of features and primitives,
 694 making it extremely unlikely to identify unique optimal solutions. In this study, the sud-
 695 den failure to improve in performance past a given level of complexity in the classifica-
 696 tion experiment (Figure 6b), a saturation often interpreted as convergence, could be driven
 697 by a structural boundary beyond which improvements could not easily be found. Since
 698 search effectiveness is partially determined by the size of the model space, available the-
 699 ory regarding target or related processes can be used to plausibly constrain model gen-
 700 eration, reinforcing the need for process knowledge alongside data in data-driven anal-
 701 ysis (Karpatne et al., 2017; Knüsel et al., 2019). Additionally, studies have argued for

702 an upper limit on the description length of a model (Vanneschi et al., 2010) as done in
 703 Chadalawada et al. (2020), though this limit is difficult to identify *a priori*. Hybrid meth-
 704 ods, such as evolutionary strategies to approximate a gradient, are promising for tractable
 705 search in combined model-parameter spaces (Conti et al., 2018; Miikkulainen et al., 2019),
 706 as well as approaches that asynchronously tune parameters and structure (Frankle & Carbin,
 707 2018). However, even when appropriately complex models can be identified, their often
 708 black-box nature does not guarantee interpretability. The results presented here indi-
 709 cate how increasing equifinality as a function of complexity can inhibit interpretability.
 710 Diminishing returns to model accuracy as complexity increases highlight the importance
 711 of parsimony as a key model evaluation and selection mechanism. More strategic anal-
 712 ysis can be done to interpret the underlying logic behind model predictions, such as ex-
 713 plaining the importance of features and structure in neural networks (Montavon et al.,
 714 2018; Worland et al., 2019, e.g.), and using sensitivity analysis to explicate structural
 715 dependence in space and time (Quinn et al., 2019, e.g.).

716 Second, the performance-complexity tradeoff of candidate model structures is tied
 717 to the choice of feature variables at the appropriate scale, and observed with the nec-
 718 essary accuracy, to generate acceptable test performance (Höge et al., 2018). This is also
 719 the case when the relations that would model such data do not exist or are not included
 720 in the primitive set (Kearns et al., 1994). This study incorporates land use and economic
 721 data across multiple decades and at a relatively fine spatial resolution to derive a sin-
 722 gle decision model, a task which may be better served by developing an ensemble of func-
 723 tions across the spatial region. Additionally, while the feature engineering applied to the
 724 data helps discern the importance of correlations in space and time, it also obfuscates
 725 the resulting model structures by increasing the interdependence among features. This
 726 could be resolved in future work with dimension reduction techniques (Giuliani & Her-
 727 man, 2018; Cominola et al., 2019), potentially at the cost of feature interpretability. The
 728 feature data itself may not provide the right signal to adequately model the underlying
 729 process in this setting, due to noise in measurement or observation error, or the choice
 730 of inadequate features. However, examining multiple problem formulations allows the
 731 comparison of relative performance, as in the regression and classification experiments
 732 in this study; while classification is the easier problem, it shows higher potential for over-
 733 fitting and may be underrepresenting the complexity in the data. Many-class classifica-
 734 tion could provide a middle ground between these two tasks, as well as the incorpora-
 735 tion of metrics that more realistically reflect model accuracy across classes, such as weight-
 736 ing by class prevalence (Provost & Fawcett, 2001; Lipton et al., 2014) or adding process-
 737 informed definitions of model error as objectives (Gupta et al., 2009; Lamontagne et al.,
 738 2020). Using heterogeneous data to identify the model structure of integrated systems
 739 is not simple or straightforward, but the explanation of decisions made by complex be-
 740 havioral agents based on multiple sources of information is enabled by the methodolog-
 741 ical template presented here.

742 6 Conclusion

743 This study develops an approach to the inference of model structures and param-
 744 eterizations from data describing human behavior in water resources systems. Three phases
 745 are considered: problem definition, model generation, and model evaluation, demonstrated
 746 on a case study of land use decisions in the Tulare Basin, California. No priors are as-
 747 sumed on the model search space beyond the function primitives and feature data, in-
 748 cluding some feature engineering to build a high-dimensional dataset reflecting land use,
 749 water use, and crop prices. Results indicate a tradeoff between model performance and
 750 complexity, with substantial equifinality in model structures that require additional di-
 751 agnostic analysis. To this end, model structures are clustered according to similar be-
 752 havior, and driving structural features are diagnosed by considering function importance
 753 and input sensitivity. Specific challenges arise due to identifying spatially distributed de-

754 decisions from heterogeneous, multi-sectoral data, generally preventing the identification
 755 of a single “best” model from the performance-complexity tradeoff. This provides a ba-
 756 sis for analyzing structural uncertainty under broadly-defined problem contexts, and a
 757 possible path forward for the generation of model components from observed data to sup-
 758 port integrated representations of human actors in water systems.

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 764 Julianne Quinn for helpful comments on a preliminary draft. Data and code are avail-
 765 able at DOI:10.5281/zenodo.3887360.

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