Supporting Information for "Control of the oxygen to ocean heat content ratio during deep convection events"

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Derivation of 1-D convective model

1. Heat budget and mixed layer depth

First, we consider the heat budget and the mixed layer depth (MLD) of the water column where stratification is controlled entirely by the temperature gradient. The initial potential temperature profile is $T_0(z)$ ($z \leq 0$, $z = 0$ is the surface). $T_0(z)$ needs to be monotonically decreasing with depth to remain stably stratified. As the water in the mixed layer cools down, MLD, indicated as $H(t)$, will increase. Potential temperature in the mixed layer is uniform and its value is set equal to the initial profile at the base of the mixed layer, $T(t) = T_0(-H)$. When cooling is applied at the surface, the heat loss at the surface equals the time rate of change in the heat content so that:

$$Q(t) = \rho_0 C_p \frac{d}{dt}\left\{T(t)H(t) + \int_{-H_{max}}^{-H(t)} T_0(z)dz\right\} \tag{S1}$$

where ρ_0 and C_p are the reference density and specific heat of sea water, H_{max} is the total depth of the water column ,and $Q(t)$ is the surface heat flux ($Q < 0$ for cooling). It can be further simplified as

$$Q(t) = -\rho_0 C_p H(t) \frac{dT_0}{dz}\frac{dH}{dt}, \tag{S2}$$

This leads to an evolution equation for the MLD:

$$\frac{dH^2}{dt} = -\frac{2Q(t)}{\rho_0 C_p}\left(\frac{dT_0}{dz}\right)^{-1}. \tag{S3}$$

2. Evolution of of the oxygen flux

Next we apply similar principle to the evolution of dissolved oxygen. The diffusive oxygen gas flux is parameterized as the product of gas transfer velocity (G) and the air-sea disequilibrium of oxygen,

$$F = -G\delta O_2(t), \tag{S4}$$

where $\delta O_2(t) = O_2(t) - O_{2,sat}(T(t))$ is the oxygen saturation state in the mixed layer assuming constant salinity. We first consider the oxygen budget of the water column where the air-sea oxygen flux equals to the changes in the total oxygen content.

$$F = \frac{d}{dt}\left\{O_2(t)H(t) + \int_{-H_{max}}^{-H(t)} O_{2,0}(z)dz\right\}, \tag{S5}$$

with $O_{2,0}(z)$ being the initial O_2 profile.

$$F = H(t)\frac{dO_2}{dt} + (O_2(t) - O_{2,0}(-H))\frac{dH}{dt}. \tag{S6}$$

The first term in the RHS of Eq.S6 is the change in the mixed layer O_2 content, and the second term is the entrainment of subsurface O_2 from below the mixed layer. This model is relevant to the winter-time condition, and the biological O_2 consumption is omitted. The biological effects, however, are implicitly represented through the relative depletion of subsurface O_2. When the entrainment mixes subsurface O_2 into the surface layer, it can cause undersaturation of the surface water. The oxygen budget can then be transformed into the budget equation for the oxygen saturation, $\delta O_2(t)$. Eq.S2 and S6 can be combined with the temperature dependence of oxygen solubility where $A = \partial O_{2,sat}/\partial T$:

$$H(t)\frac{d\delta O_2}{dt} = -\left\{\delta O_2 - \delta O_{2,0}(-H)\right\}\frac{dH}{dt} + F - \frac{A\,Q(t)}{\rho_0 C_p}. \tag{S7}$$

3. Case 1: small η limit

When assuming that the deepening of the mixed layer is relatively slow compared to the air-sea equilibration of the diffusive gas transfer ($\delta O_2 \sim 0$), Eq.S7 is dominated by the diffusive gas exchange. The total heat loss equals the heat content change in the mixed layer:

$$I_Q = \int_0^t Q(t')dt' = \rho_0 C_p \left\{T_0(-H)H(t) - \int_{-H(t)}^0 T_0(z)dz\right\}. \tag{S8}$$

The total oxygen uptake is equal to the change of δO_2 ($\delta O_2 = 0$ in the mixed layer at the end of each time step in this case) plus the change due to the cooling-induced solubility increase.

$$I_{O_2} = \int_0^t F(t')dt' = -\int_{-H(t)}^0 \delta O_{2,0}(z)dz + A\frac{I_Q}{\rho_0 C_p}. \tag{S9}$$

The seasonal O_2-OHC ratio for this convective event is

$$\frac{I_{O_2}}{I_Q} = \frac{1}{\rho_0 C_p}\left\{\frac{-\int_{-H(t)}^0 \delta O_{2,0}(z)dz}{T_0(-H)H - \int_{-H(t)}^0 T_0(z)dz} + A\right\}. \tag{S10}$$

The first term represents DO enrichment in the mixed layer, and is determined by the initial temperature and δO_2 profiles. The second term is the solubility effect. If we further simplify the problem by assuming that these profiles are linear, then,

$$I_Q = \frac{1}{2}\rho_0 C_p k_T H^2, \tag{S11}$$

$$I_{O_2} = \frac{1}{2}k_{\delta O2}H^2 + A\frac{I_Q}{\rho_0 C_p}, \tag{S12}$$

where $k_{\delta O2}$ and k_T are the vertical gradient of $\delta O_{2,0}(z)$ (assuming $\delta O_{2,0}(0) = 0$) and $T_0(z)$. The seasonal and interannual O_2-OHC ratios then share the same form as

$$\frac{I_{O_2}}{I_Q} = \frac{dI_{O_2}}{dI_Q} = -\frac{1}{\rho_0 C_p}\left(\frac{k_{\delta O2}}{k_T} - A\right), \tag{S13}$$

4. Case 2: large η limit

When δO_2 is dominated by the entrainment of subsurface water, and the effect of air-sea gas exchange does not affect the δO_2 in the mixed layer, the integral δO_2 balance is set by the entrainment of subsurface waters and the cooling-induced solubility increase.

$$\delta O_2 H(t) - \int_{-H}^{0} \delta O_{2,0}(z)dz = -\frac{A}{\rho_0 C_p}I_Q. \tag{S14}$$

This equation can be used to diagnose the air-sea O_2 flux and its relationship to the heat flux. First δO_2 is diagnosed from Eq.S14 driven by the cooling and the deepening of mixed layer. Then, the diagnosed δO_2 can be used to determine the air-sea O_2 flux through Eq. S4.

$$F(t) = -\frac{G}{H(t)}\left\{\int_{-H(t)}^{0} \delta O_{2,0}(z)dz - \frac{A}{\rho_0 C_p}I_Q\right\}. \tag{S15}$$

A simple solution can be obtained by assuming linear initial profiles and a constant heat flux Q ($I_Q = Qt$). Eq. S3 can be solved for H(t) finding that:

$$H(t) = \sqrt{\frac{-2\,Q\,t}{\rho_0\,C_p\,k_T}}. \tag{S16}$$

By combining Eq.S15 and S16 we obtain the equation for the total oxygen uptake

$$I_{O_2} = \frac{G}{3}(\frac{2k_T}{\rho_0 C_p})^{1/2}(\frac{k_{\delta O2}}{k_T} - A)(-Q)^{1/2}t^{3/2}. \tag{S17}$$

The seasonal O_2-OHC ratio can be written as:

$$\frac{I_{O_2}}{I_Q} = -\frac{G}{3}\left(\frac{-2k_T\,t}{\rho_0\,C_p\,Q}\right)^{1/2}\left(\frac{k_{\delta O_2}}{k_T} - A\right). \tag{S18}$$

Assuming the duration of the cooling is constant, the interannual O_2-OHC ratio simplifies to

$$\frac{dI_{O_2}}{dI_Q} = -\frac{G}{3}(\frac{k_T}{2\rho_0 C_p})^{1/2}(\frac{k_{\delta O2}}{k_T} - A)\frac{t}{(-Qt)^{1/2}}. \tag{S19}$$

If we assume that the variation of the heat flux is mainly controlled by the change of sensible and latent heat, which are proportional to the surface wind speed, we can write the surface wind speed (U) as a linear function of surface heat flux:

$$U = U_0 + \beta Q, \tag{S19}$$

where U_0 is a constant and β is the linear regression coefficient. In our model simulations, G is proportional to the square surface wind speed, thus is a quadratic function of the surface heat flux. We have

$$G = \alpha(U_0 + \beta Q)^2, \tag{S20}$$

where α is the closure coefficient. Under such assumption, Eq. S19 becomes

$$\frac{dI_{O_2}}{dI_Q} = -\frac{\alpha(U_0 + \beta Q)(U_0 + 5\alpha Q)}{3}(\frac{k_T}{2\rho_0 C_p})^{1/2}(\frac{k_{\delta O2}}{k_T} - A)\frac{t}{(-Qt)^{1/2}}. \tag{S21}$$

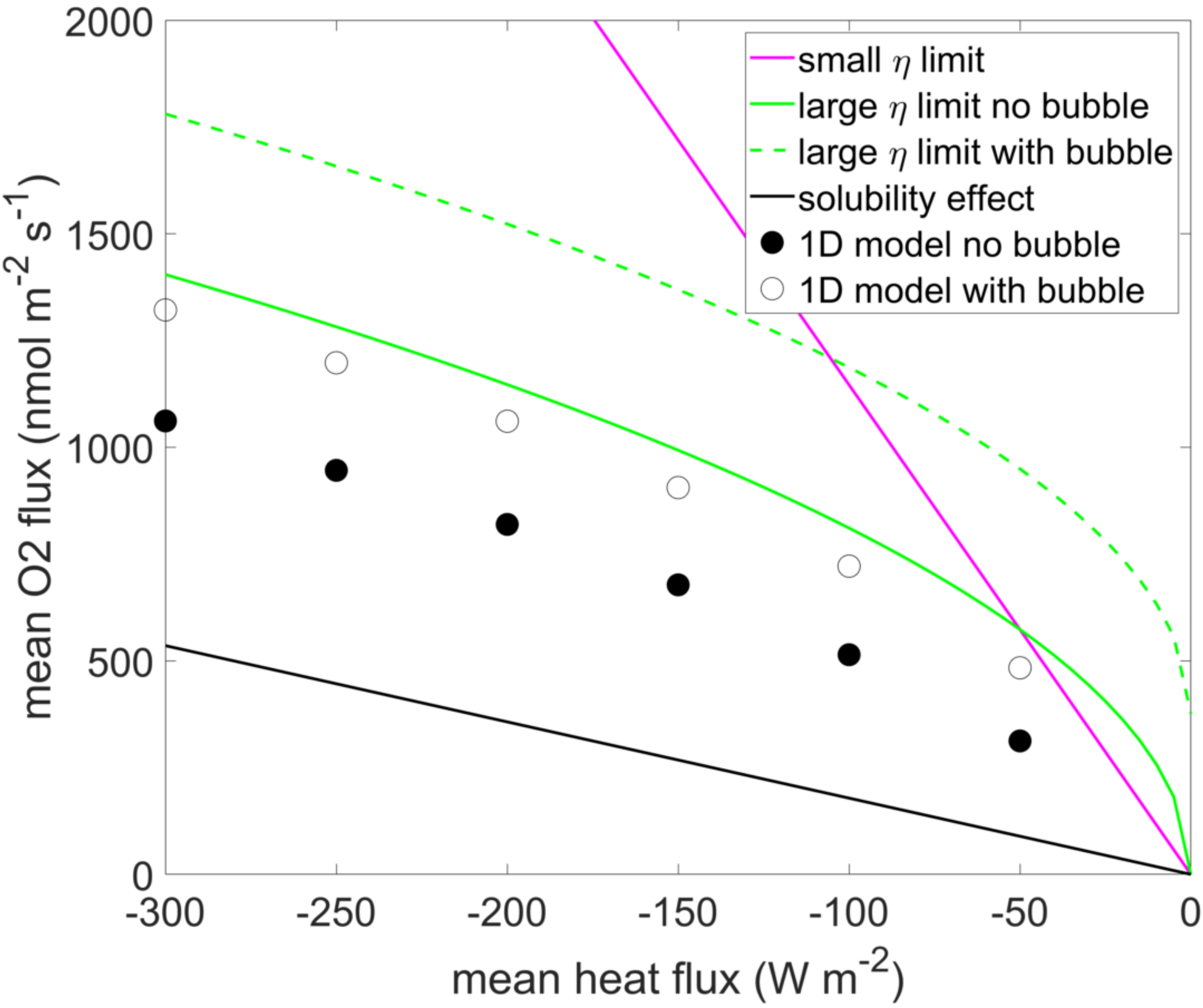

Figure S1. Numerical solution of the 1-D convective adjustment model under different cooling rates and for different extreme cases. Both the runs with bubble injection (open circles) and the runs with (solid circles) are included.

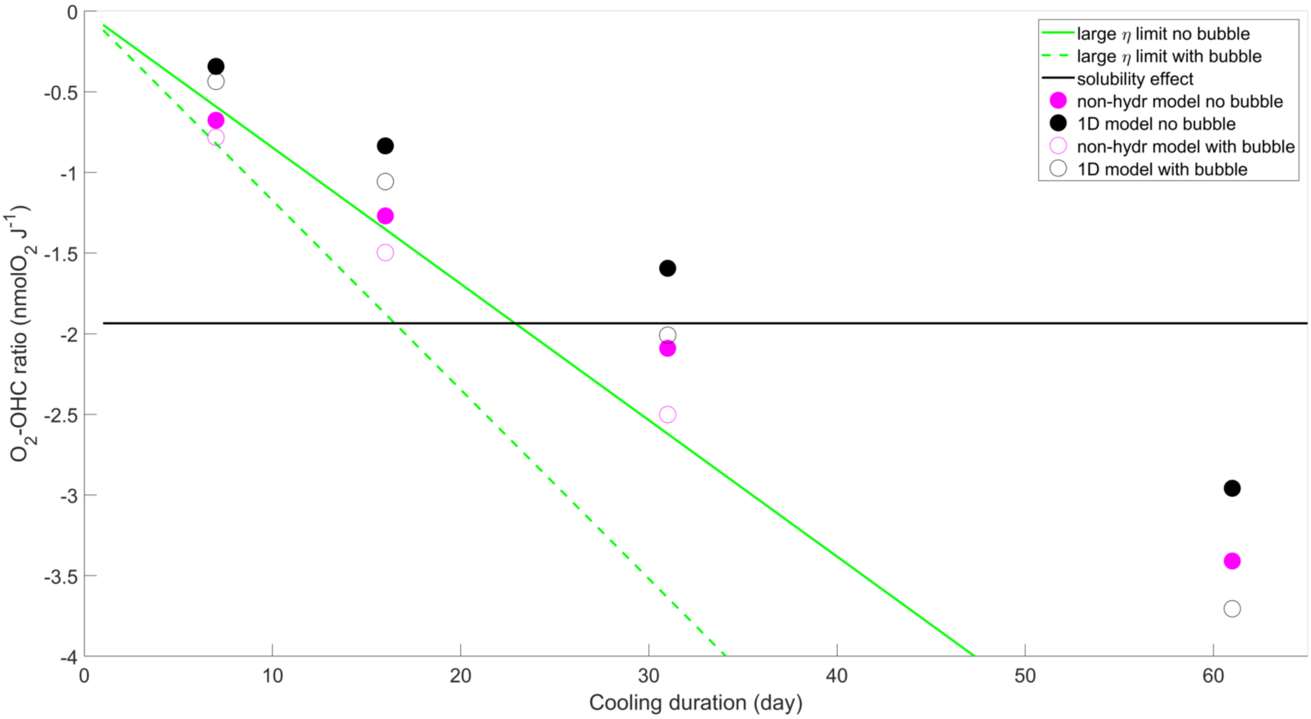

Figure S2. The O_2-OHC ratio as a function of cooling duration from the non-hydrstatic simulations compared with the solutions of the 1-D convective adjustment model. Both the runs with (open circles) and without (solid circles) bubble injection are included.

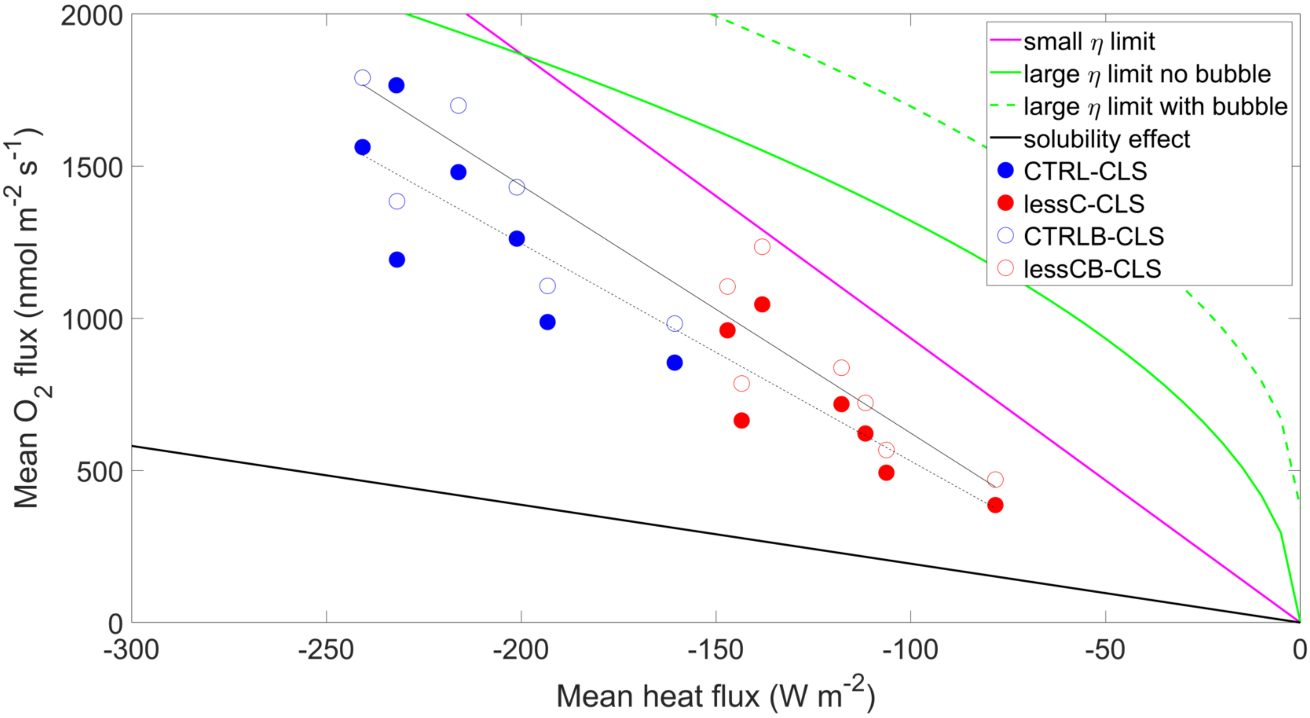

Figure S3. Mean air-sea oxygen flux as a function of mean surface heat flux over 7 different winters (from the 2000-2001 to the 2006-2007 convective seasons) in the CLS from the regional simulations compared with the theoretical predictions under different assumptions. Both the runs with (open circles) and without (solid circles) bubble injection are included. The thin solid and dashed black lines are the linear fittings for the 16 points with ($CTRLB$ and $lessCB$) and without ($CTRL$ and $lessC$) bubble injection, respectively.