Supporting Information for "Anatomy of Acoustic Emission"

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Text S1.

Slider mechanics. In Fig. S1(a) we show a schematic of the experimental system. It stands vertically in the earth gravitational field. Because of its spatial symmetry there is no net torque on the slider (center granite block). The side blocks are the laboratory frame. For $V_0 = 10\mu m$ in an experiment lasting $2000 sec$ the contact area, across the "yellow" shear support structure, equivalent to the fault contact asperities and gouge material, goes from $10 \times 10 cm^2$ to $8 \times 10 cm^2$ leading to a slow evolution of the mechanical response. The system is at normal stress of $15 MPa$ for about $350 sec$, Fig. S1(b), leading to relative motion of about $3.5 mm$. There are about 16 slip-stick events at $15 MPa$ taking approximately $22 sec$ each. We study the last 6 of these events. Approximately at $15 MPa$ the shear stress, measured by the load cell, has a time independent part of about $9 MPa$ and a time dependent part of about $1 MPa$. These two parts of the shear stress are associated with the *white* and *articulated* response. Simiar remarks apply to time domains corresponding to lower values of the normal stress.

Text S2.

Slider motion. Consider a slider, described by Brownian dynamics, that resides on an inhomogeneous substrate. For it's equation of motion we write

$$\frac{m}{\tau_0}\frac{dX}{dt} = k(V_0 t - X) - F_1 - F_2. \tag{1}$$

Source 1 involves a large number M_1 of weak contacts, strength γ_1, that continually break and re-set.

$$F_1 = M_1 \gamma_1. \tag{2}$$

Source 2 involves a number M_2 of strong contacts, strength γ_2, that sporadically break and re-set.

$$F_2 = M_2\gamma_2. \tag{3}$$

Suppose F_2 can be neglected. At some moment, $t = 0$, the system is in mechanical equilibrium at X_0 moving with velocity U

$$\frac{m}{\tau_0}U = k(-X_0) - M_1\gamma_1(X_0) \tag{4}$$

$$X_0 = -\frac{1}{k}\left(\frac{m}{\tau_0}U + M_1\gamma_1(X_0)\right). \tag{5}$$

At $t = 0 + dt$, $X = X_0 + Udt$ we have

$$\frac{m}{\tau_0}U = k(V_0dt - X_0 - Udt) - M_1\gamma_1(X_0 + Udt) = k(V_0dt - Udt) - M_1\frac{\partial\gamma_1(X_0)}{\partial X_0}Udt \tag{6}$$

from which we obtain (set terms in dt to zero)

$$U = \frac{k}{k + \Gamma_1}V_0, \quad \Gamma_1 = M_1\frac{\partial.\gamma_1(X_0)}{\partial X_0}. \tag{7}$$

From the numbers in Table I we have $U \ll V_0$. Thus $\Gamma_1 \gg k$. From Table III, $U \sim N^{-2} \to \Gamma_1 \sim N^2$. As N increases the slider slows and T_{SS} becomes longer.

Text S3.

Comparison of calculations. The general idea of the calculations being undertaken is to establish a function of the acoustic emission (AE) as surrogate for the mechanical state of the slider. We sketch the calculation in the text **A** and a simple version of extant machine learning calculations **B.**

A. From text. There are two fields to be compared to one another: (1) the on-board-displacement $X_S(t)$ and (2) the AE $\alpha(t)$.

(1). On a "tread," i.e., between two slip events on-board-displacement is put in the form

$$X_S(t) = U_n t + X_a(t), \quad t_S(n) \leq t \leq t_S(n+1), \tag{8}$$

where $t_S(n)$ is the center time of slip n measured from the behavior of X_S (here U_n is found by fitting X_S to a line). The target of the treatment is the non-trivial part of X_S, X_a, the articulated part of X_S. The average of $X_a(t)$ on the "tread" is zero.

(2) On a "tread" the AE is $\alpha(t)$.

1. The DC part of $\alpha(t)$ is removed.
2. The positive envelope of $\alpha(t)$ is formed, $\beta(t)$.
3. The cumulative sum of the positive envelope is formed

$$\mathcal{C}_n(t) = \int_0^t \beta(t)dt, \quad t_S(n) \leq t \leq t_S(n+1), \tag{9}$$

4. The cumulative sum is put in the form

$$\mathcal{C}_n(t) = W_n t + C_a(t), \tag{10}$$

where $C_a(t)$, the articulated part of $\mathcal{C}(t)$, is the target of the signal processing. The average of $C_a(t)$ on the "tread" is zero.

Comparison of the behavior of the slider with the AE involves comparison of $X_a(t)$ with $C_a(t)$. Simply watching $C_a(t)$ as time advances allows one to locate the slider in time and see where it is in the slip-slip hiatus..

B. Conventional machine learning.

There are two fields to be compared to one another as it relates to the present work: (1) the shear stress $\tau(t)$ and (2) the AE $\alpha(t)$ [we note other fields can be mapped from the AE, e.g., friction, gouge layer thickness, etc. e.g., (Johnson et al., 2021) and references therein].

(1) The shear stress on a "tread", measured with a load cell is given by

$$\tau_n(t) = k(V_0 t - X_S(t)), \quad t_S(n) \le t \le t_S(n+1). \tag{11}$$

The quantities k and $V_0 t$ are known. The shear stress on a "tread" depends on the mechanical state of the slider through $X_S(t)$.

(2) The AE on a tread is

$$\alpha_n(t), \quad t_S(n) \le t \le t_S(n+1). \tag{12}$$

The machine learning exercise relating $\alpha_n(t)$ to $\tau_n(t)$ employs a suite of features of $\alpha_n(t)$ at each time to determine the relationship between the suite of features and $\tau_n(t)$. An equation of state is therby detemined in which there is a one-to one correspondence between feature set and shear stress.

The difference between **A** and **B**. The two recipes outlined here differ trivially because in **A** the white noise is removed *a priori* and $X_S(t)$ is thus isolated from the signal that accompanies it when it is part of τ, as in **B**. These are data processing differences. There is a physical difference. The quantity $\mathcal{C}(t)$, formed from $\alpha(t)$, has been shown to be essentially equivalent to the on-board-displacement. No special relationship between $\alpha(t)$ and the on-board-displacement has been established beyond that of evidence of correlation driven by feature choice.

References

Johnson, P. A., Rouet-Leduc, B., Pyrak-Nolte, L. J., Beroza, G. C., Marone, C. J., Hulbert, C., ... Reade, W. (2021). Laboratory earthquake forecasting: A machine learning competition.

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Table S1. Numbers characterizing the slip-stick motion of the on-board- displacement, X_S.

$X_S(t)$	N	b_0	U	T_{SS}	$b = b_0 + UT_{SS}$
	MPa	mm	mm/sec	sec	mm
	6	0.0366	0.00130	5.52	0.0438
	9	0.0631	0.00070	9.09	0.0694
	12	0.0894	0.00041	13.35	0.0949
	15	0.1506	0.00020	22.6	0.1551

Table S2. Numbers characterizing $\mathcal{C}(t)$, "amp" are the units of $\mathcal{C}(t)$.

$\mathcal{C}(t)$.	N.	$\mathcal{C}(T_{SS})$	W	T_{SS}	$\mathcal{C}(T_{SS}) - WT_{SS}$
	MPa	amp	amp/sec	sec	amp
	6	24.54	4.33	5.51	0.68
	9	39.36	4.31	8.99	0.61
	12	62.99	4.68	13.21	1.16
	15	105.07	4.59	22.6	1.33

Table S3. Scaling of the characteristics of X_S and $\mathcal{C}$ with N.

$X_S(t)$	$b_0 \sim N$	$U \sim N^{-2}$	$T_{SS} \sim N$
$C(t)$	$\mathcal{C}(T_{SS}) \sim N$	$W \sim N^0$	$T_{SS} \sim N$

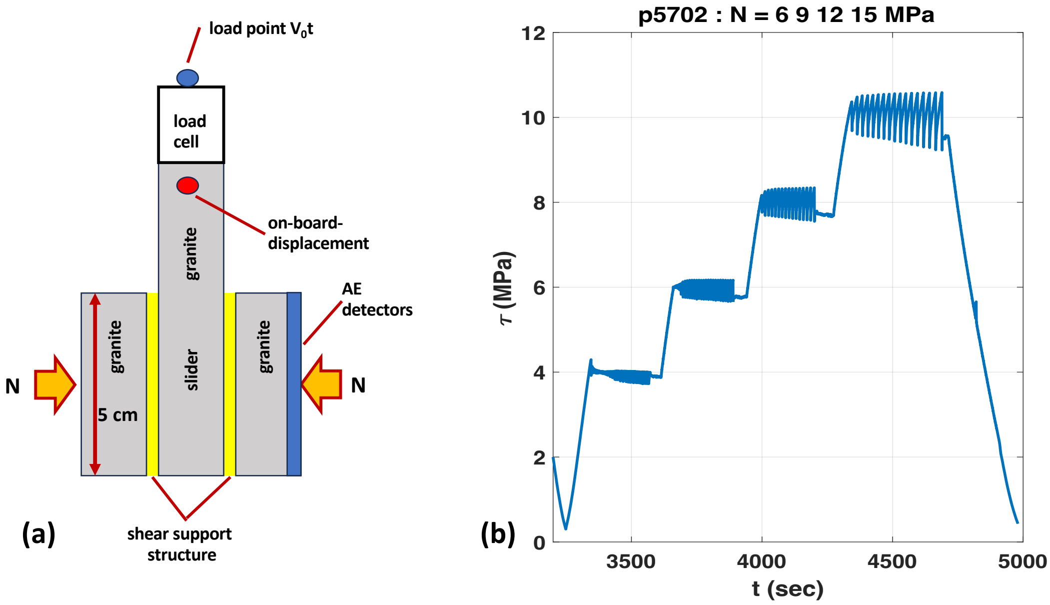

Figure S1. (a) Schematic of the p5702 experimental system. The center block is driven past the side blocks to which it is coupled by the shear support structure (yellow). The acoustic emisssion measurements are made in blue domain on right. (b) The shear stress as a function of time as N is changed from $6MPa$ to $15MPa$.

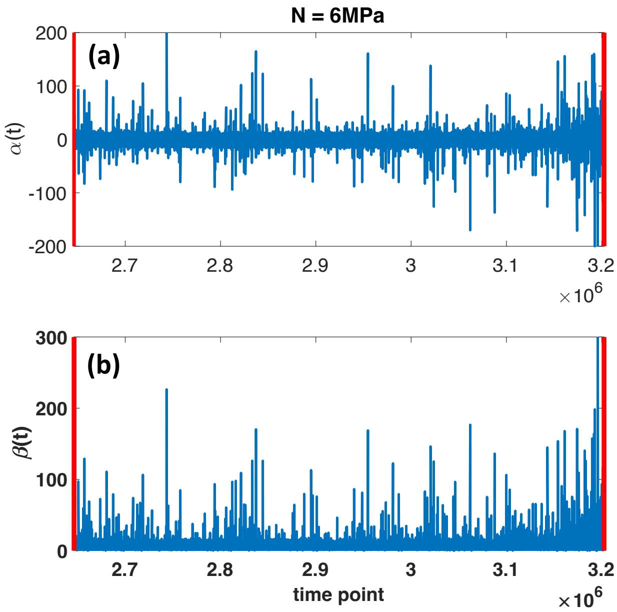

Figure S2. (a) The AE, $\alpha(t)$, for one "tread" at $6MPa$. Red boundaries are at sucessive times of large slip. (b) The upper envelope of the AE, $\beta(t)$.