Bathymetric influences on Antarctic ice-shelf melt rates

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Abstract

Ocean bathymetry exerts a strong control on ice sheet-ocean interactions within Antarctic ice-shelf cavities, where it can limit the access of warm, dense water at depth to the underside of floating ice shelves. However, ocean bathymetry is challenging to measure within or close to ice-shelf cavities. It remains unclear how uncertainty in existing bathymetry datasets affect simulated sub-ice shelf melt rates. Here we infer linear sensitivities of ice shelf melt rates to bathymetric shape with grid-scale detail by means of the adjoint of an ocean general circulation model. Both idealised and realistic-geometry experiments of sub-ice shelf cavities in West Antarctica reveal that bathymetry has a strong impact on melt in localised regions such as topographic obstacles to flow. Moreover, response of melt to bathymetric perturbation is found to be non-monotonic, with deepening leading to either increased or decreased melt depending on location. Our computational approach provides a comprehensive way of identifying regions where refined knowledge of bathymetry is most impactful, and also where batymetric errors have relatively little effect on modelled ice sheet-ocean interactions.

1 Introduction

The bathymetry of the ocean exerts a leading order influence on ocean circulation, both at global and regional scales (e.g., Roberts & Wood, 1997; D. Marshall, 1995; Hughes & Killworth, 1995; Gille et al., 2004). It plays a key role in regulating exchanges between the Antarctic continental shelf and the deep ocean (e.g., Walker et al., 2013; Thoma et al., 2008; Graham et al., 2016; Thompson et al., 2018) and in setting circulation patterns on the continental shelf (e.g., Padman et al., 2010; Jacobs et al., 2011; Arneborg et al., 2012; Cochran & Bell, 2012; De Rydt et al., 2014; Rosier et al., 2018; Wählin et al., 2020). Its role in ice sheet-ocean interactions is accentuated by the fact that a large part of the Antarctic ice sheet rests well below sea level (Bentley et al., 1960), with a sizable portion of its margins terminating in large floating ice shelves. These ice shelves slow the speed of fast-flowing ice streams through buttressing (Thomas & Bentley, 1978; Thomas, 1979). Therefore the collapse or retreat, melting and associated thinning of ice shelves, while having a limited direct effect on sea level (Jenkins & Holland, 2007), can result in increased grounded ice loss from the continent (Shepherd et al., 2004) – a loss which may be amplified due to a positive feedback involving the geometry of sub-ice sheet topography known as the Marine Ice Sheet Instability (Schoof, 2007; Joughin et al., 2014).

The circulation of water under ice shelves is of great importance in the Amundsen and Bellingshausen Seas, West Antarctica, where intrusions of warm, salty Circumpolar Deep Water (CDW) from the Antarctic Circumpolar Current occur (Jacobs et al., 1996; Jenkins et al., 1997; Thoma et al., 2008; Arneborg et al., 2012; Jenkins et al., 2016; Zhang et al., 2016), promoted in part by continental shelf geometry in these regions (Pritchard et al., 2012). Regional atmospheric forcing and sea-ice states lead to stable stratification of the water column that limits mixing of this dense water with cool surface layers (Petty et al., 2013), allowing higher rates of ice-shelf mass loss than elsewhere in Antarctica (Jenkins, 2016). CDW-driven ice-shelf melt is not strictly limited to the Amundsen and Bellingshausen Seas (Gwyther et al., 2014; Greene et al., 2017), and climate modelling suggests it could become more widespread around Antarctica under climate change scenarios (Hellmer et al., 2012). The ability of this warm, dense water to drive ice-shelf melt depends to a large extent on how it is steered or blocked by bathymetry on the continental shelf and within the cavity.

Despite considerable efforts devoted to improving Antarctic-wide estimates of bed topography (see most recently Morlighem et al. (2020)), our knowledge of bathymetry in large parts of the marine margins of the ice sheet is highly uncertain. Direct observations of the ocean seafloor near Antarctica are beset by difficulties such as remoteness and sea ice cover (Nitsche et al., 2007). Collecting bathymetric data under floating ice shelves is even less practical. Autonomous submersibles capable of measurements under floating ice shelves are only beginning to be deployed. With a ∼300 m swath, they provide relatively small coverage of sub-ice-shelf cavity bathymetry (e.g., Jenkins et al., 2010). Airborne gravity sensing offers an alternative means of bathymetric measurement (e.g., Tinto & Bell, 2011; Millan et al., 2017); however, gravimetric inversions are subject to errors related to resolution and geologic uncertainty. Seismic observations of the bed do not rely on lithology assumptions, but as they are generally ground-based, data-gathering is expensive and often limited to point estimates (e.g., Rosier et al., 2018).

Previous studies have addressed this uncertainty in the context of a physical ocean model by considering idealised bathymetries (De Rydt et al., 2014; Zhao et al., 2018) or testing different bathymetry products (Schodlok et al., 2012; Goldberg et al., 2019). To date, no modelling study has investigated the melt response to the full range of uncertainty in sub-ice shelf bathymetry. Here, we aim to provide a better understanding of this uncertainty by estimating the sensitivity of ocean-driven ice-shelf melt rates to bathymetry in a West Antarctic sector.

Previously, Losch & Heimbach (2007) developed a method to calculate the sensitivity of circulation metrics (e.g., the strength of meridional overturning or zonal mass transport) to ocean bathymetry using the adjoint of the Massachusetts Institute of Technology general circulation model (MITgcm). In general, adjoint models generate linearized sensitivities of model outputs to an arbitrarily large set of input parameters (Wunsch, 1996), providing a computationally efficient means for investigating the impacts of grid-scale uncertainties. To avoid tedious "by-hand" differentiation of a complex ocean general circulation model, Losch & Heimbach (2007) made use of *algorithmic differentiation* (AD) software, which has been used extensively with the MITgcm (Heimbach et al., 2005; Wunsch et al., 2009). However, this adjoint model involving bathymetry sensitivities was not applied to sub-ice shelf circulation.

In this paper, we "revive" the adjoint model infrastructure for treating bathymetry as an uncertain input variable, and employ this framework to investigate the impacts of bathymetric uncertainty on ice-shelf melt rates. Two important technical improvements are (i) the use of an open-source AD tool to generate the adjoint model, and (ii) improved treatment of the implicit free-surface solver in generating the adjoint model. These are summarized in Section 2, where we briefly discuss our methodology, including our adjoint approach and our updates to the MITgcm code base (with further details in Appendix A). We apply our framework to an idealised domain and analyse the resulting sensitivities (Section 3). We then carry out a study of the Crosson and Dotson ice shelves in the Amundsen Sea Embayment (Section 4), and conclude with discussion in Section 5.

2 Methodology

2.1 Modelling of ice-ocean interactions

We simulate sub-ice shelf circulation with the MITgcm, an open-source general purpose finite-volume code which solves the hydrostatic primitive equations on the rotating sphere governing ocean flow (J. Marshall et al., 1997). (The code has nonhydrostatic capability but it is not used in this study.) Since its inception, code "packages" representing modularized parameterizations, numerical algorithms, and separate climate components have been introduced. One such package, `SHELFICE` (Losch, 2008), allows for circulation in cavities beneath ice shelves that may be many hundreds of meters deep. `SHELFICE` also calculates melt rates and the associated heat and salt fluxes at the ice-ocean interface based on under-ice ocean properties using a viscous sublayer parameterization (Holland & Jenkins, 1999). In this study we use the velocity-dependent form of the melt parameterization (Dansereau et al., 2014). The ice-ocean model has successfully run the Ice Shelf Ocean Model Intercomparison Experiment (ISOMIP; Holland et al. (2003)), the experimental setup of which forms the basis for our first experiment.

2.2 Discretization of bathymetry in the MITgcm

The vertical discretization of bathymetry in MITgcm is distinct from other aspects of discretization in the model, and given the nature of this study it deserves mention. To allow for varying bathymetry but avoid dramatic steps due to the prescribed vertical level thicknesses, a *partial cell* discretization is implemented (Adcroft et al., 1997), where bottom cells can be partially fluid-filled with fraction h_f, down to a minimum specified thickness $h_{f,min}$. This means that horizontal cell faces are partially fluid-filled as well, which is important as cell faces determine volume and tracer transport. Due to memory requirements, bathymetry is represented as piecewise-constant (as opposed to piecewise-linear), meaning fluid fractions at cell faces are a function of depth at adjacent cell centers (see Fig. 1(a)). This choice has implications for algorithmic differentiation of bottom sensitivity, as discussed below.

2.3 Adjoint model

An ocean model may be conceptualised as a mathematical function that maps an input vector $\mathbf{x}_{in}$ onto an output vector $\mathbf{x}_{out}$. The input vector $\mathbf{x}_{in}$ consists of the discretized initial conditions for the oceanic state, as well as all inputs required to integrate the partial differential equations that govern the circulation of the ocean, including discretized input fields for surface (forcing) and bottom (bathymetry) boundary conditions. $\mathbf{x}_{out}$ consists of all prognostic model output (generally of a much higher dimension than that of $\mathbf{x}_{in}$), or diagnostic functions thereof, including scalar-valued metrics. It is often of interest to know how perturbations in $\mathbf{x}_{in}$ affect $\mathbf{x}_{out}$, or how they affect quantities that depend on $\mathbf{x}_{out}$ (sometimes referred to as "objective functions" or "quantities of interest"). An example application of an adjoint model might be investigating how Atlantic meridional overturning is sensitive to global patterns of precipitation (Pillar et al., 2016; Smith & Heimbach, 2019).

The *sensitivity vector*, i.e. the gradient of the quantity of interest with respect to $\mathbf{x}_{in}$, could be determined by perturbing separately each element of $\mathbf{x}_{in}$ and observing the model response (formally, inferring a directional derivative); however, such an approach for computationally intenstive models and input vectors of high dimension is impractical. However, forming the *adjoint* of the model (or, more precisely, the adjoint of its Jacobian) provides an alternative (Errico, 1997), enabling calculation of the sensitivity vector at a computational cost that does not depend on the dimension of $\mathbf{x}_{in}$.

Differentiation of the ocean model can be carried out at the equation level (Sirkes & Tziperman, 1997), though this approach requires a separate code that must be updated when the ocean model is modified. Another method – and the one used in this work – is Algorithmic Differentiation (AD), which uses a software tool to automate differentiation of the model at the discrete (code) level. In this study, two different AD tools are used: *Transformations of Algorithms in Fortran* (TAF; Giering et al. (2005)) and OpenAD (Utke et al., 2008). Both are source-to-source tools, meaning code is generated in the native language (as opposed to operator-overloading). Both tools have been used to generate the MITgcm adjoint; TAF, a commercial product, has been used more extensively with the MITgcm, while OpenAD is a more recent open-source tool.

While AD presents great benefits in differentiating complex numerical codes and keeping the adjoint code in synchronization with the parent numerical code, some degree of manual intervention is generally required. In the present study changes to the adjoint generation were necessary to facilitate efficient computation, the foremost dealing with the way in which MITgcm evolves the ocean free surface. These and other details as discussed in detail in Appendix A.

3 Idealised Experiment

To gain insight into how bathymetry modulates the interaction between ocean circulation and ice shelf melt, we first examine sensitivity of melt to bathymetry in an idealized domain, which is a slightly modified version of the computational domain used in the Ice Shelf Ocean Model Intercomparison Project (ISOMIP; Holland et al. (2003)). In the MITgcm implementation of the standard ISOMIP setup, the ocean circulates within a closed rectangular domain with a flat bathymetry of 900 m depth. A zonally-uniform ice-shelf draft slopes meridionally from 700 m depth to 200 m depth over about 450 km, and is constant north of this point. We use a resolution of 30 m in the vertical, 0.3° zonally, and 0.1° meridionally. A full description can be found in Losch (2008). We modify the ISOMIP domain by introducing a zonally-constant ridge in the bathymetry just south of the point of deepening of the ice shelf. The meridional expression is a half-cosine "bump" with a width of 2° latitude and a height of 200 m above the uniform seafloor (Fig. 2(a)), and we refer to our experiment as "ISOMIP-bump". This bathymetry is inspired by bathymetric ridges identified under a number of Antarctic ice shelves (e.g., Jenkins et al., 2010; Wei et al., 2019), which are found to strongly control the transport of relatively warm water within ice shelf cavities (De Rydt et al., 2014; Dutrieux et al., 2014).

Our adjoint experiment is as follows: the ISOMIP-bump model is run forward in time for 2 model years, and the spatial integral of the melt rate in the final time step is evaluated as our quantity of interest J:

$$J = \sum_i d_i m_i, \tag{1}$$

where d_i and m_i are the area of, and melt rate within, horizontal cell i. The adjoint model accumulates sensitivity of J with respect to bathymetry back in time along the 2-year simulation trajectory and thus depends on the state of the entire 2-year run, not just the final state. Thus, to mitigate impacts of equilibration, we begin the model run from a "spun-up" state rather than a quiescent one. The model is thus first spun-up for 3 years, and the resulting state forms the initial conditions for our 2-year forward and adjoint run.

The melt rate at the final time in the adjoint experiment (Fig. 2(b)) is broadly similar with that of Mathiot et al. (2017) (their Fig. 2), although our peak melt rate is larger, and there is a "tongue" of melt rates bisecting the accretion region over the ridge. The barotopic circulation also differs slightly with respect to the standard ISOMIP experiment: rather than a broad cyclonic gyre, there is a narrow anticyclonic anomaly on the north side of the ridge (Fig. 2(b)). Barotropic flow is primarily along the ridge, crossing it primarily near the eastern and western boundaries, similar to what has been shown in a simplified two layer model (Zhao et al., 2018). Zonally-averaged temperatures (Fig. 2(a)) suggest slightly cooler waters at depth just south of the ridge as opposed to the northern flank. The slightly larger melt rates as compared to Mathiot et al. (2017) could reflect the fact that our simulation has not yet reached steady-state – indicating that the presence of the ridge increases the time to reach a new steady-state.

The adjoint-derived sensitivities are shown in Fig. 3(a). Positive values indicate locations where raising the seafloor will increase integrated melt, and negative values indicate where lowering the seafloor will increase melt. There are distinct broad-scale patterns in the sensitivities, particularly over the ridge itself. Across much of the zonal extent of the ridge there is negative sensitivity, indicating a lowering of the ridge would increase melt. Near the eastern boundary, however, there is a region with strongly positive sensitivities. Northward of the ridge where both bathymetry and ice draft are constant, there is a broad dipole pattern, with positive sensitivities toward the center and negative toward the east. In our investigation below we focus on these four regions (labelled in Fig. 3(a)); foregoing close analysis of regions with negligible influence on melt (such as southward of the ridge), and regions where there is strong spatial variability in the sensitivity, such as the western edge of the ridge.

In order to ensure that adjoint sensitivity patterns did not arise from issues involving Algorithmic Differentiation, both AD tools (OpenAD and TAF) were used to generate sensitivities. (A similar approach was taken in in Heimbach et al. (2011).) The differences in the sensitivities, likely arising from numerical truncation, were negligible, and are not shown.

3.1 Finite-amplitude perturbations of bathymetry

As with any adjoint-based study, it is important to verify the adjoint-derived sensitivities by perturbing the input, or *control*, field in the forward model, i.e. by estimating finite-difference approximations to the gradients that the adjoint model calculates. In the MITgcm this type of "gradient check" is more challenging when dealing with model bathymetry than with other control variables, as demonstrated in Fig. 1(b): finite perturbations of bathymetry can change grid structure, for example by adding new cells to, or removing cells from, the domain. Neither operation is differentiable, and hence linearized sensitivities may not reflect model responses to perturbed bathymetry. Additionally, bathymetric perturbations may not be as anticipated, as thicknesses of cells will be adjusted by the model initialization to ensure no partial cell is thinner than $h_{f,min}$.

These challenges aside, we implement finite perturbations to bathymetry in order to test the results from the adjoint model, but our experiment design is intended to minimize the above complications. Rather than perturb values in individual cells, we apply perturbation *patterns*. We carry out experiments with four separate perturbation patterns, naturally selected in regions of high sensitivity, where bathymetric perturbations exhibit the greatest control on melt-rates, as shown in Fig. 3. The patterns have a Gaussian profile:

$$\delta R(\phi, \lambda) = \delta R_0 \exp\left(-\frac{(\phi-\phi_0)^2}{L_\phi^2} - \frac{(\lambda-\lambda_0)^2}{L_\lambda^2}\right) \tag{2}$$

where ϕ and λ are latitude and longitude. ϕ_0, λ_0, L_ϕ and L_λ vary with experiment but the location and radii of the perturbations can be seen from Fig. 4 for each region. Different values of δR_0 are considered as described below.

For a given depth perturbation δR, the linear response to J predicted by the adjoint is

$$\delta J = \sum_i \delta J_i = \sum_i (\delta R_i)(\delta^* R_i), \tag{3}$$

where δR_i is the finite perturbation to bathymetry in ocean column i and $\delta^* R_i = \frac{\partial J}{\partial R_i}$ is the bathymetric sensitivity in i as calculated by the adjoint. If the adjoint model is accurate, Eqn. (3) should be fairly accurate for small values of δR_i. This is the case for $\delta R_0 = 0.1$ m (Fig. 3(b)). Positive and negative perturbations are considered in regions 1 and 2; in regions 3 and 4 only positive perturbations are examined as negative perturbations would lower bathymetry beyond the extent of the computational grid. For larger perturbations ($\delta R_0 = 10$ m), linear sensitivities give fairly accurate predictions in regions 2, 3 and 4; in region 1 (the center of the ridge), the linear approximation underestimates the response. Closer inspection reveals that, when bathymetry is perturbed in the center of the ridge, a number of fluid-containing cells become empty. Similarly, when regions 1 and 2 are negatively perturbed with $\delta R_0 = 10$ m, an even larger number of previously empty cells become fluid-filled. These non-differentiable changes could explain the underestimates.

These perturbation experiments provide insight into the mechanisms that cause the sensitivity patterns produced by the adjoint model. In these experiments, bathymetric perturbations cause circulation changes that are evident in the perturbed barotropic stream function field, shown in Fig. 4. A bathymetric rise in region 3 induces an anticyclonic region just to the west of the rise and a broad cyclonic region to the east (Fig.

4(c)). A rise in region 4 induces a similar pattern but with the relative sizes and strengths of the cyclonic and anticyclonic regions reversed, in part due to zonal boundary constraints (Fig. 4(d)). The pattern is reminiscent of the interaction between a jet and a topographic rise (Huppert & Bryan, 1976; Holland et al., 2003), with the broad cyclonic cell in this region (Fig. 2(b)) generating the background flow. As this cell transports water away from the cold outflow from the cavity before it circulates back toward the ridge, it is likely that perturbations which strengthen/oppose this circulation will increase/decrease melt, explaining the sensitivity pattern north of the ridge.

On the ridge itself, there is a similar response to bathymetric bumps in regions 1 and 2 (Fig. 4(a,b)), although complicated by the varying background topography. In the case of a raised bump on the eastern ridge, the leading effect on the circulation is a southward shift of the warm jet travelling eastward along the ridge, increasing warm-water transport into the cavity, and increasing melt. A depression in the center of the ridge has a similar effect.

While these results are highly idealized, they are nonetheless instructive regarding bathymetric influence on melt in ice-shelf cavities with topographic obstacles: (1) bathymetry in areas "protected" by the obstacle play a relatively small role in controlling melt; (2) the height of the obstacle has a strong influence on melt, but the direction, or sign, of the influence may depend on the location along the ridge and related to the background flow that is set up by the geometry; and (3) bathymetry oceanward of the obstacle can influence melt as well, by controlling the circulation that brings warm water toward the ice-shelf cavity. These insights inform the interpretation of sensitivities in simulations with realistic bathymetry.

4 Realistic experiment: Dotson and Crosson ice shelves

The Dotson and Crosson Ice Shelves are relatively small but strongly thermally-forced ice shelves in the Amundsen Sea Embayment of West Antarctica (Fig. 5(a)). Recently, these ice shelves, as well as the ice streams that flow into them, have been the subject of focused glaciological and oceanographic study (e.g., Randall-Goodwin et al., 2015; Goldberg et al., 2015; Miles et al., 2016; Gourmelen et al., 2017; Jenkins et al., 2018; Lilien et al., 2018). Moreover, ice-ocean interactions under these ice shelves have significance for biological productivity in the Southern Ocean: levels of carbon sequestration in the highly productive Amundsen Polynya are thought to be connected strongly to ice-shelf melt volume (Gerringa et al., 2012; Yager et al., 2012). A recent modelling study by Goldberg et al. (2019) showed that the choice of bathymetric product has a significant influence on the melt rates modelled for these ice shelves. Therefore, it is an ideal region in which to examine the sensitivity of melt to bathymetry.

4.1 Model configuration

Our ocean model configuration is based on that of Goldberg et al. (2019). We use the MITgcm with the `SHELFICE` package and with ice-shelf draft and bathymetry based on Millan et al. (2017). At ocean-facing boundaries we impose conditions on temperature, salinity and velocity from a regional simulation by Kimura et al. (2017). However, there are important differences with the configuration of Goldberg et al. (2019), which are largely influenced by practical considerations concerning the performance of the OpenAD-generated adjoint. Adjoint models generally require more computing time than the forward models from which they derive, requiring in some cases recomputation to avoid intractable memory requirements (Griewank & Walther, 2008). The 4-year simulations conducted by Goldberg et al. (2019) ran for approximately 32 hours on 48 cores on the Research Councils UK (RCUK) ARCHER supercomputer (discounting queueing times in between batches), meaning an adjoint experiment might require up to several weeks' wall-clock execution time leading to large delays in our investigations and potentially irresponsible energy usage. (This scaling is based on the timings of experiments in this study and not a rigorous analysis of OpenAD performance.) Thus, modifications were made to reduce computational expense and facilitate adjoint computation.

A 2-km grid was used as opposed to a 1-km grid, and the time step increased from 150 to 300 seconds. Additionally, a larger horizontal eddy viscosity, $\nu_H = 300\ \mathrm{m^2 s^{-1}}$, was imposed, for the following reason. The ocean adjoint model is a distinct numerical code – related to the forward ocean model but with its own stability constraints, arising in part from the chosen quantity of interest, which informs the boundary and initial conditions of the adjoint model. It is often the case that the adjoint of a nonlinear forward model produces sensitivity patterns with sharp spatial gradients, which grow in amplitude over time because the model lacks the nonlinear feedbacks to damp them, resulting in numerical instabilities. Hoteit et al. (2005) showed that a stabilization of the adjoint may be achieved with a larger value of ν_h for the adjoint model, while retaining a smaller eddy viscosity in the forward model, but such a capability for the OpenAD-MITgcm adjoint is not yet available. We point out that our chosen value for ν_h is comparable to the ice-ocean interaction study of Dansereau et al. (2014), which also used the `SHELFICE` package of MITgcm.

Additionally the open boundary conditions of our computational domain, which represent interactions with the Antarctic Circumpolar Current (i.e. the ocean-facing boundary conditions), were made time-constant rather than time-varying as in Goldberg et al. (2019). As discussed in Section 4.3, this better enables the assessment of the timescale of adjustment to boundary conditions. Velocity, temperature and salt conditions from Kimura et al. (2017) were averaged over 2011, allowing for a shorter experiment.

Finally, the Millan et al. (2017) bathymetry was adjusted over a region of approximately 90 km^2 close to the junction between Crosson and Dotson Ice Shelves, where the Kohler range extends into the ice-shelf cavity (Fig. 5(a)). In this area, the Millan bathymetry suggests a significant ridge with a peak less than 300 m below sea level. Without modification, this ridge would lead to very thin ocean columns in our model, effectively limiting ocean transport to the narrow region between the ridge and Bear Peninsula. However, observed melt rate patterns Gourmelen et al. (2017); Goldberg et al. (2019) show high melt rates in this location, suggesting a more extensive connection between the ice shelves than the bathymetry product would allow. Furthermore, recent glider and float observations in this region show that this ridge may be lower than suggested by the gravimetry (Dutrieux et al., 2020). Our modification of this bathymetry in this region allows a wider area for ocean flow while still maintaining a ridge at the Dotson-Crosson junction. While our modification is not observationally grounded, our adjoint computation (described below) gives an indication of the impact of this modification. If circulation in this region were negligible, such assessment might not be possible.

Our adjoint experiment largely mirrors that of the ISOMIP-bump experiment. The Dotson-Crosson model is run for 1 model year, and then the sensitivity of the objective function J – the spatial integral of melt – with respect to bathymetry is computed. As in our idealized experiment, we begin with a spun-up state of the model, which is steady due to time-invariant forcing. The realistic experiment was carried out only with the OpenAD-generated adjoint model. Even with the adjustments discussed above, the required simulation run-time was still well over the limits for a single job on the available HPC resources. Therefore, further modifications were required to enable OpenAD to restart the adjoint simulation over subsequent jobs. These technical modifications are referred to as resilient adjoints and are described in Appendix B.

4.2 Results

Relevant aspects of the forward model are depicted in Fig. 5. Despite the lower resolution and higher viscosity compared to the configuration used by Goldberg et al. (2019), the melt rate patterns are similar. Broadly consistent with observation-based inferences (Randall-Goodwin et al., 2015), there is a strong outflow at the western margin of Dotson Ice Shelf – though in our model outflow is less confined to the margin, potentially due to high viscosities or horizontal resolution. The total melt rate is approximately 81.5 Gt/yr (Fig. 7), similar to that found by Randall-Goodwin et al. (2015) for Dotson ice shelf alone in January 2011. Meltrates in the simulation domain are insensitive to bathymetry under much of the Dotson Ice Shelf (Fig. 6), with the exception of the connection with Crosson Ice Shelf and over the small ridge at the entrance of the ice shelf (Fig. 5(a)).

The sensitivity pattern over the outer ridge bears similarities to the idealized ISOMIP-bump experiment – with negative sensitivities in the centre of the ridge, indicating a lowering would increase melt, and positive sensitivities at the margins. The most coherent pattern of sensitivity oceanward of Dotson is in the eastern side of the trough entering the cavity. The negative sensitivities downslope and positive sensitivities upslope imply that a steepening of the trough margin would amplify the geostrophically driven flow of warm water to the ice shelf, and thus increase melting. This result is corroborated by recent observational and experimental work which highlights the critical role of topography in steering heat to Antarctic ice shelves (Wåhlin et al., 2020).

Under Crosson Ice Shelf, there are fairly weak but extensive positive sensitivities, indicating raising of the bed would increase melt, which at first seems counter-intuitive. This could arise because the cavity column depth is relatively small (on average, the column depth under Crosson is $\sim$150 m less than under Dotson), meaning a shallower column would bring inflowing CDW closer to the ice shelf. Oceanward of Crosson, there are coherent areas of negative sensitivity, correlating with localized bathymetric highs, indicating that lowering in these regions would increase melt. However, this is not a consistent pattern, as there is a region along the front with positive sensitivities, indicating that in this shallow-bedded region, raising the bed would actually increase melt rates.

4.3 Equilibration of adjoint sensitivities

Although the adjoint model represents a differentiation of all physical processes, this does not guarantee that the adjoint run should capture the dominant linear adjustments associated with bathymetric influence of melt. This is because these adjustments operate over an intrinsic time scale (e.g. Heimbach & Losch, 2012), and it is difficult to know *a priori* if the adjoint run encompasses this scale.

The nature of our adjoint run allows us to evaluate whether this adjustment is captured *a posteriori*. The bathymetry field in the ocean model ultimately affects the model through the partial cell factors h_f (*cf.* Section 2.2), and related factors h_f^w and h_f^s, the fluid-filled portion of cell faces at the southern and western sides of bottom cells. This dependency among the cell factors is set in the initialization of the model. Thus, if the *adjoint sensitivity* fields corresponding to these variables are relatively steady as the adjoint model steps backward in time, then bathymetric sensitivities are *converged*: they would not change significantly with a longer run. In physical terms, this would imply that the length of the simulation is on order of the time scale of adjustment to perturbations or greater.

Fig. 7 shows the Euclidean norm of the $\delta^* h_f$ field, the adjoint sensitivity of h_f, as the adjoint model evolves, which it does backward in time (from month 12 to 0). Similar time series are shown for adjoint fields corresponding to the h_f^w and h_f^c fields. $\delta^* h_f^w$ and $\delta^* h_f^c$ norms have roughly steadied by the end of the adjoint run (month 0), while $\delta^* h_f$ is steadily growing. However, $\delta^* h_f$ only makes a small contribution to bathymetric sensitivity over this time period. These results suggest the immediate effect of changing bathymetry is on transport, with a timescale of about a year for the present model. However, partial cell volume, which affects, among other things, the heat content at depth, might have strong impacts on melt rate over much longer time scales, not considered here.

We point out that our ability to evaluate adjoint equilibration in this manner is due to our use of time-invariant controls. In adjoint experiments involving time-varying controls, such as wind forcing or time-evolving boundary conditions (e.g., Heimbach &

Losch, 2012), the adjoint sensitivity would not be expected to asymptotically approach a "steady state" in reverse-time.

4.4 Impact of bathymetry product uncertainty

As demonstrated in Goldberg et al. (2019), one application of adjoint sensitivities is in estimating the impact of an alternative data product on the quantity of interest. Recently, a new bathymetric product for Antarctica became available, BedMachine (Morlighem et al., 2020), which differs from that of Millan et al. (2017). In particular, there are large differences within the ice shelf cavities, especially for Dotson (Fig. 8(a)), as the bathymetry of Millan was later updated by using the methodology described in An et al. (2019), for which the Direct Current shift in the gravity data is not assumed to be spatially uniform.

In a similar fashion to the idealized finite perturbation experiments in section 3.1, we use the difference between bathymetry products as a bathymetric uncertainty estimate and input to Eqn. (3). This formula results in an estimated 10 Gt/yr uncertainty in Dotson and Crosson melt-rates due to bathymetric error resulting purely from the differences in these two products. Of course, this estimate is only a first order approximation as it assumes that this linear term dominates any higher order (i.e. nonlinear) effects. From our idealized experiments, we can expect this 10 Gt/yr uncertainty may be an underestimate. Still, it is informative to examine which areas of the ice-shelf cavities actually contribute to this increase. This can be seen from Fig. 8(b), which shows

$$\delta J_i = (\delta R_i)(\delta^* R_i) \tag{4}$$

i.e. the summand of Eqn. (3), for this combination of bathymetric perturbation and adjoint sensitivity. Despite the extensive differences in bathymetry under Dotson between the products, there are only a few regions where this difference matters, which are elucidated by the sensitivity pattern in Fig. 6. Most prominently, the representation of the ridge near the front of Dotson, which is far less pronounced in the BedMachine product, accounts for 4.3 Gt/yr difference in melt-rates (Fig. 8(b)).

4.5 Sensitivity of grounded ice loss to ocean bathymetry

Understanding the impact of ocean bathymetry on sub-ice shelf melt rates is important due to the impact of melting on the loss of buttressing and grounded ice volume. The experiments above focus on melt rate as a target quantity of interest, rather than grounded ice volume. To comprehensively estimate sensitivity of grounded ice volume to ocean and sub-ice sheet bathymetric uncertainty would require the adjoint to a fully coupled ice sheet-ocean model, which does not presently exist.

Nevertheless, with our current framework we can begin to explore pathways of sensitivity from ocean model inputs to ice-sheet state-related quantities of interest. In mathematical terms, we seek the total sensitivity of ice sheet volume (as our quantity of interest) to bathymetry, that is, $\frac{\partial V}{\partial R_i}$ where V is grounded ice volume and R is bathymetry in location i. We emphasize that this quantity is distinct from sensitivity of grounded volume to under-ice bathymetry, which directly controls ice flow and dynamic thinning (see (e.g., Goldberg et al., 2015), their Figure 7(b)); rather, the pathway of influence considered here is through control on melt rates, which in turn impact ice-shelf buttressing. Thus, for ocean bathymetric grid points, R_i, we may write:

$$\frac{\partial V}{\partial R_i} = \sum_k \frac{\partial V}{\partial m_k} \frac{\partial m_k}{\partial R_i} \; . \tag{5}$$

where m_k is ocean melt rate in cell k and $\frac{\partial V}{\partial m_k}$ is the ice-sheet model derivative of grounded volume with respect to melt in cell k. While calculating sensitivity of grounded ice volume to melt is beyond the scope of an ocean model, an ice-sheet model framework to do this does exist (e.g., Goldberg & Heimbach, 2013). If these sensitivities can be found, then a new quantity of interest for the ocean model can be defined:

$$J_{gv} = \left(\nabla_{\mathbf{m}} V\right)^T \mathbf{m} \equiv \sum_k \left(\frac{\partial V}{\partial m_k}\right) m_k, \tag{6}$$

Note that if the first term in the inner product is external to the ocean model, then the gradient of J_{gv} with respect to R_i, ocean bathymetry in location i, is equivalent to the expression on the right hand side of Eqn. (5). A different way of seeing this is that the product "projects" patterns of ice sheet volume sensitivities to melt rates onto melt rate sensitivities to ocean bottom topography.

In Goldberg et al. (2019), an *ice-sheet* adjoint model was used to find the sensitivity of grounded volume of Smith Glacier, the glacier that feeds Dotson and Crosson Ice Shelves, to ice-shelf melt rates (Fig. 9(a)). These ice-melt sensitivities are used to construct the quantity of interest J_{gv} and sensitivities with respect to ocean bathymetry are found. This result is shown in Fig. 9(b). The most striking feature of this result is the similarity of the pattern to that of Fig. 6, the sensitivity of melt to bathymetry (R^2 of 0.93; see also Fig. 9(c)). Comparing Eqns. (1) and (6), the quantities of interest effectively differ only in a weighting of melt rate by grounded ice volume sensitivities. Thus the similarity in Figs. 9(b) and 6 suggests that only *total*, or spatially integrated, melt can be strongly affected by bathymetry; whereas melt rate *patterns* are controlled by other factors such as ice-shelf geometry (Goldberg et al., 2019).

We point out this sequence of adjoint sensitivity calculations, in which ice-sheet sensitivity is passed to an ocean model adjoint, which is in turn used to find ocean sensitivity, is a simplified representation of a coupled adjoint ice-ocean model. In a properly coupled model, the ocean provides melt rates to the ice sheet, while the ice sheet provides ice-shelf drafts to the ocean model, with these fields being continually updated. Ideally, in a coupled adjoint model melt sensitivities would be passed to the ocean adjoint model and ice-draft sensitivities to the ice adjoint model with the same frequency. (In our study, ice-draft sensitivities were not calculated, but our framework could be easily modified to do so.) Moreover, if the ocean and ice models are not on the same grid (as is the case with our ocean model and the ice-sheet model used by Goldberg et al. (2019)), a coupled model would interpolate the melt rates to the ice-sheet grid. Strictly, the term $(\nabla_{\mathbf{m}} V)^T$ in the definition of J_{gv} should be right-multiplied by the adjoint of this interpolation operator. This was not done in our calculation, rather the ice-sheet adjoint sensitivity was interpolated to the ocean grid directly. Still, our results present a useful preliminary assessment of the controls of ocean bathymetry on ice-sheet volume, and can potentially inform more comprehensive assessments using coupled ice sheet-ocean models.

5 Discussion and Conclusions

In this study we have applied an algorithmic differentiation (AD) framework to an ocean general circulation model in order to determine the sensitivity of ice-shelf melt rates to ocean bathymetry. A similar framework of inferring bottom topography sensitivities has been applied before (Losch & Heimbach, 2007), in a coarse-resolution global-scale model. Here, we extend this computational framework to a regional domain that includes circulation in sub-ice shelf cavities in order to assess the impact of uncertainty in bathymetry, a quantity which cannot be measured under ice-shelves by ship-based methods, on melt rates. Additionally, we have made technical improvements by avoiding the differentiation by the AD tool of the Poisson solver for the implicit free surface and facilitating the use of the tool in high performance computing environments (Appendices A and B). We have done so using an open-source AD tool.

Results from both the idealized and realistic simulations show how bathymetry near and underneath ice-shelves modulate melt-rates. Ocean-ward of an ice shelf, troughs leading to the ice front act as a guide for incoming warm ocean waters. Specifically, we show that steepening the trough in front of the Dotson ice shelf would increase melting as a result of increasing the geostrophic inflow. These results provide a complementary perspective to the idealized simulations, observations, and experimental results shown in Wåhlin et al. (2020).

Underneath ice shelves, it is well known that ridges or sills hinder the inflow of warm, dense waters into cavities (Dutrieux et al., 2014; De Rydt et al., 2014; Slater et al., 2019; Zhao et al., 2018). However, the spatial details of how these obstacles impact ice shelf melting are in some instances counter-intuitive. For example, the sensitivities in our idealised ISOMIP-bump experiment identified locations where *raising* the level of a sub-ice-shelf ridge led to increased melt. These results were proven to be robust in forward experiments, and they were mirrored in our Dotson-Crosson regional simulation. Thus, while bathymetric obstacles do play a strong role, they do not simply serve as a "dam" to hold back dense warm waters; rather, an obstacle's impact on melt must be assessed in the context of the broader ocean circulation and topographic steering of that circulation.

When calculating sensitivities to bathymetry, the MITgcm adjoint is subject to non-differentiable operators, and may underestimate response to some perturbations (*cf.* Fig. 3(b)) – though more work is needed to determine under what conditions and scales the predicted melt response to bathymetric perturbations is valid. Nevertheless, our idealized experiments suggest the adjoint is able to identify locations and regions where topography "matters". Losch & Heimbach (2007) reach a similar conclusion with their study. They attribute this to low model resolution, though based on our idealised experiments this limitation might apply to high-resolution studies as well.

Regardless, such experiments provide utility to observations of sub-shelf bathymetry which seek to aid modelling of ice-ocean interactions. High-resolution studies of ice-shelf bathymetry (for instance, through gravity analysis and seismic inversion) are possible, but are very limited in scope. As our understanding of sub-shelf bathymetry evolves, our adjoint-based method could be adapted to identify candidate locations where high resolution observational campaigns can be most impactful – for instance, by assessing the potential information gain in important quantities of interest, as in Loose et al. (2020). Additionally, patterns of spatial variability in sensitivity (such as that seen on the flank of Dotson trough) could inform requirements for airborne gravity surveys (in terms of aircraft speed and altitude) to ensure such variability is captured.

A major use of the MITgcm adjoint model is for improved assimilation of oceanographic data (e.g., Wunsch & Heimbach, 2007; Wunsch et al., 2009). However, it is unlikely that an adjoint ocean model can be used to estimate sub-ice shelf bathymetry by assimilating spatial observations of melt rates, for two reasons. Firstly, as demonstrated in our idealised and realistic experiments, there are extensive regions under ice shelves where melt rates are not sensitive to bathymetry. Thus two very different bathymetry products (such as the Millan and BedMachine datasets) could give very similar melt rates. Secondly, sub-shelf circulation seems to "filter" the effects on melt rate, such that while bathymetry has a strong impact on total melt, its effect on melt rate patterns may be weaker – effectively limiting the information contained in spatially resolved melt patterns (Gourmelen et al., 2017). It may be possible, nevertheless, to "fine tune" our knowledge of bathymetry in regions that are known to strongly impact melt rates.

Our study was spatially limited in that only Crosson and Dotson ice shelves were modelled – but it was also *temporally* limited, with time-invariant conditions representing far-field heat content and thermocline depths. In reality, the depth of CDW on the Amundsen shelf and elsewhere in Antarctica varies both seasonally and interannually (e.g., Thoma et al., 2008; Jenkins et al., 2016; Webber et al., 2017), and it is possible that this variability could impact sensitivity of melt to bathymetry. Therefore, the results in Section 4 should be viewed as a preliminary exploration of bathymetric sensitivity of ice-shelf melt for Antarctic ice shelves. Our methodology must be applied to simulations of ice-ocean interactions that are longer-term, more spatially extensive, and validated against observations of ice-shelf melt (Rignot et al., 2013; Gourmelen et al., 2017; Jenkins et al., 2018) in order that the impacts of ocean bathymetry, and our uncertainty of it, upon ice-shelf melt can be fully evaluated. The full potential of this work may be unleashed in fully coupled forward and adjoint ocean-ice sheet calculations, in which ice sheet volume sensitivities to ocean bathymetric uncertainties may be more comprehensively studied.

Appendices

A Modifications to the MITgcm adjoint

The MITgcm, and in particular a configuration using the `SHELFICE` physics package for an Antarctic ice shelf, has been differentiated algorithmically (Heimbach & Losch, 2012), and so no additional modifications were required for applications to ice sheet-ocean interactions. However, there are technical issues in using bathymetry as a control variable. For instance, fluid fractions at grid cell faces (see Section 2.2) are based on the minimum fraction of adjacent cells, leading to potential non-differentiability. We adopt the approach of Losch & Heimbach (2007) of "smoothing" the min/max functions, but we note that this feature has not been used outside of bathymetric sensitivity studies.

Another computational challenge in treating bathymetry as a control variable lies with the implicit solve for the free surface at each time step (J. Marshall et al., 1997). The model solves the linear system $\mathbf{A}\eta = \mathbf{b}$ for η, where η is the free surface at the next time step, and $\mathbf{b}$ is a field arising from the baroclinic step of the model. $\mathbf{A}$ is a linear, self-adjoint operator on η and the propagation of sensitivity from η to b can be calculated analytically:

$$\delta^*\mathbf{b} = \mathbf{A}^{-1}\delta^*\eta, \tag{7}$$

where $\delta^*\eta$ is the *adjoint sensitivity* of η and likewise for $\mathbf{b}$. This formulation is standard in the MITgcm for adjoint based sensitivity analyses of any control variable except for fluid depth. However, the operator $\mathbf{A}$ depends on ocean column depth, which in the present study is a control variable, and therefore the backward-propagation of sensitivities from η to $\mathbf{A}$ must be considered as well. Losch & Heimbach (2007) dealt with this issue by allowing the AD tool to differentiate the linear solver code; however, as it is an iterative solver, this approach requires storing intermediate variables at each solver iteration during every time step of the forward model, which hinders performance and does not scale well to high dimensional problems. Losch & Heimbach (2007) recommend, but do not implement, using the approach of Giles et al. (2002), which augments Eqn. (7) with

$$\delta^*\mathbf{A} = -\delta^*\mathbf{b}\ \eta^T. \tag{8}$$

In this work we implement this approach, obviating the need for the AD tool to differentiate the implicit solver.

B Resilient Adjoints

Simulation of large models requires the use of high performance computing (HPC), generally with defined job time limits. The MITgcm has a restart capability allowing to circumvent these limits: the "state" of the model is periodically saved to file, and new jobs can begin from this time stamp by reading the saved state. To restart the adjoint model, simulations must save both the forward and adjoint states – a capability referred to as *resilient adjoints*. A similar capability was previously implemented with TAF as *the Divided Adjoint* (DIVA).

Here we provide an overview of resilient adjoints, a strategy that enhances the default checkpointing scheme used by OpenAD. Checkpointing approaches store the state of the primal (forward) computation and reduce the amount of memory that is required to compute adjoints. By default, OpenAD uses binomial checkpointing for the time-stepping loop (Griewank & Walther, 2000). Consider a computation consisting of l timesteps, with c the number of checkpoints that can be stored. Figure 10 (top) illustrates binomial checkpointing for $l = 10$ and $c = 3$.

A two-level checkpointing approach can build upon this approach by converting the time stepping loop into a loop nest containing l_2 outer iterations and l_1 inner iterations where $l = l_2 \times l_1$ (Aupy et al., 2014). The inner loop uses binomial checkpointing as before; the outer loop uses periodic checkpointing. The left part of Figure 10 (bottom) illustrates two level checkpointing for $l_2 = 5$, $l_1 = 10$ and $c_1 = 3$. The resilient adjoints capability enhances two level checkpointing by storing to disk the adjoint state computed at the end of each outer level iteration. To restart a computation at the granularity of an l_2 timestep then, only the stored l_2 state checkpoints and the last adjoint checkpoint, if any, are required.

Acknowledgments and Code/Data Availability.

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BedMachine data is available from https://nsidc.org/data/nsidc-0756 . Output availability information for Kimura et al. (2017) is given in their publication. All model output used to produce figures for this manuscript is provided as supporting information.

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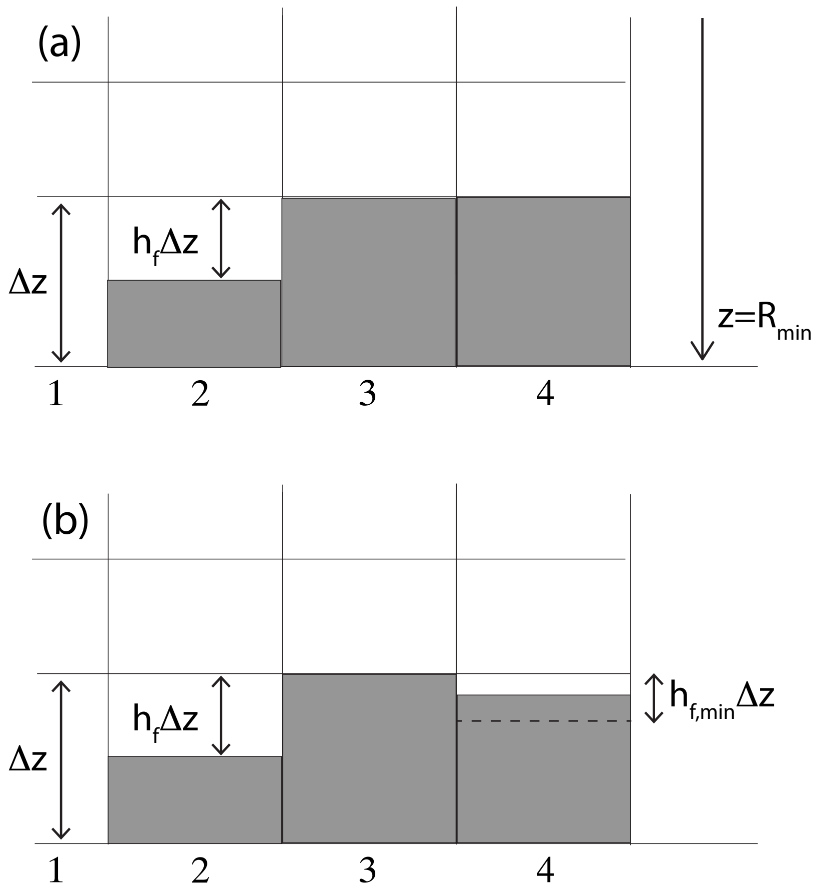

Figure 1. (a) A schematic (adapted from http://mitgcm.org/) of the representation of bottom topography in MITgcm. The white regions within cells contain fluid. In column 1, all cells are fluid-filled and the bathymetry is R_{min}. The bottom cells of Columns 3 and 4 are non-fluid-containing, and in these columns the bottom elevation is $R_{min} + \Delta z$. In Column 2, the bottom cell is a partial cell, and bathymetry is $R_{min} + (1 - h_f)\Delta z$. The interface between the bottom cells of Column 1 and Column 2 has height $h_f \Delta z$, and there is no interface between the bottom cell of Column 2 with any cell in Column 3. (b) A perturbation to bathymetry is made, indicated by gray shading in to bottom cell of Column 4. Depending on the size of the perturbation, ocean model initialisation may lower bathymetry further so that the liquid-containing portion of the bottom cell is $h_{f,min}\Delta z$; or it may restore bathymetry to that of (a).

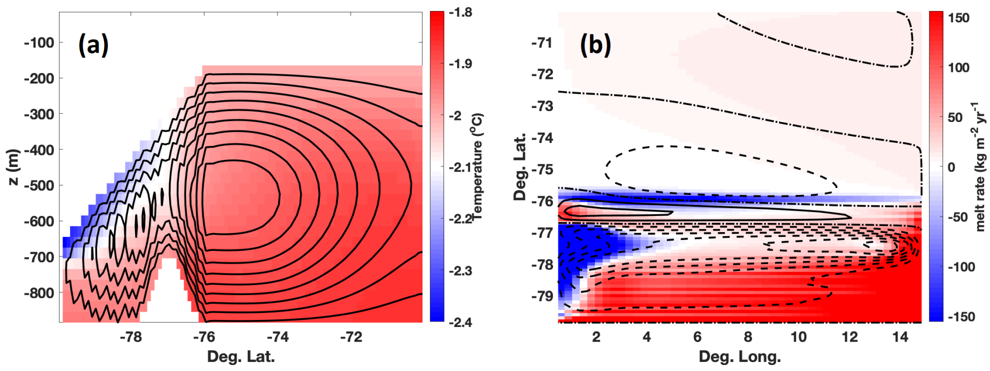

Figure 2. (left) Zonally averaged temperature (shading) and overturning stream function (contours) in the modified ISOMIP experiment. The profile of the "ridge" is apparent between -78° and -76° Latitude. (right) Melt rate at the termination of the experiment (shading; negative values indicate accretion) and depth-integrated stream function (contours; dashed lines where negative).

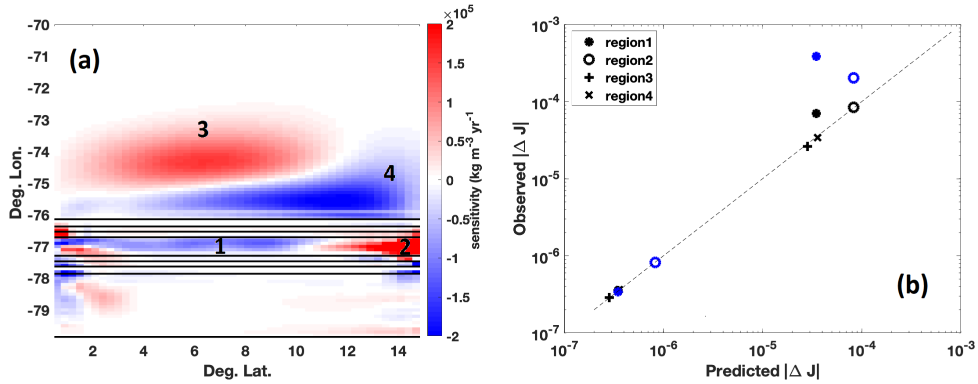

Figure 3. (left) Domain bathymetry (contours; 50m isolines) and sensitivity of spatially-integrated melt at model termination to bathymetry (shading); value of sensitivity in a cell indicates gradient of melt with respect to elevation in the cell, where positive (negative) values indicate regions where raising (lowering) the bottom will increase melt. (right) Comparison of perturbed objective function (in Gt/a melt) with value predicted by linearized sensitivities, as described in Section 3.1. Blue markers indicate negative perturbations while black markers indicate positive ones. Small values (less than 10^{-6} Gt/a) indicate perturbations scaled by 0.1m and large values (greater than 10^{-5} Gt/a) indicate perturbations scaled by 10m. Though the *sign* of the observed ΔJ is not given, it is in all cases the same as the prediction.

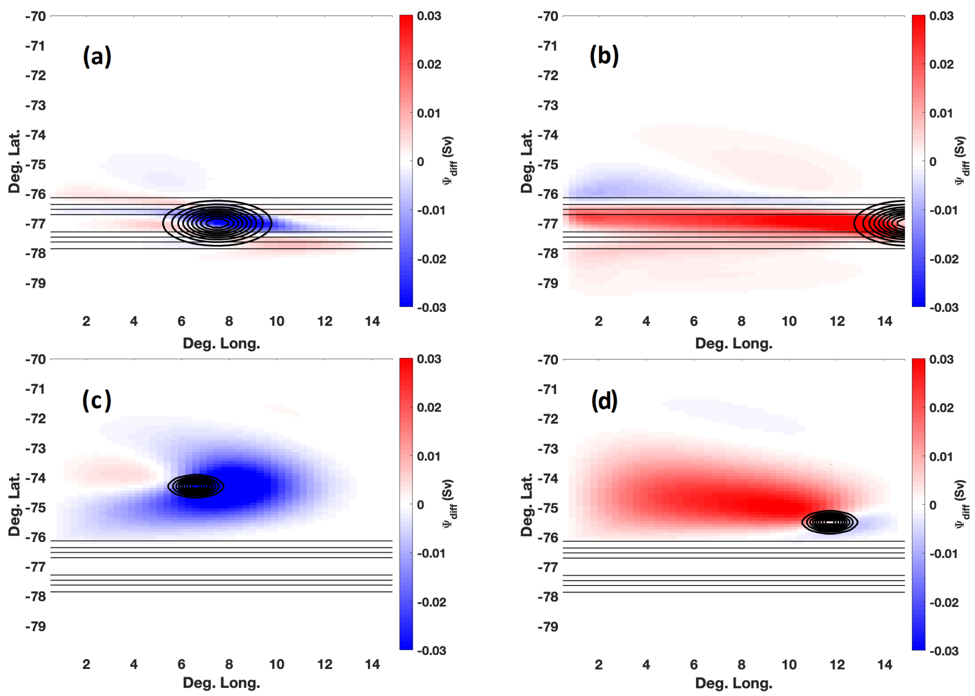

Figure 4. Perturbed beds (thick contours), background bathymetry (thin contours; 50m isolines) and corresponding perturbed stream functions (shading) in different regions of high sensitivity in Fig. 3 (a) through (d) correspond to finite perturbations in locations (1) through (4) in Fig. 3(a), respectively. Bathymetric peturbations plotted with δR=10 (Eqn. 3) and 1m isolines.

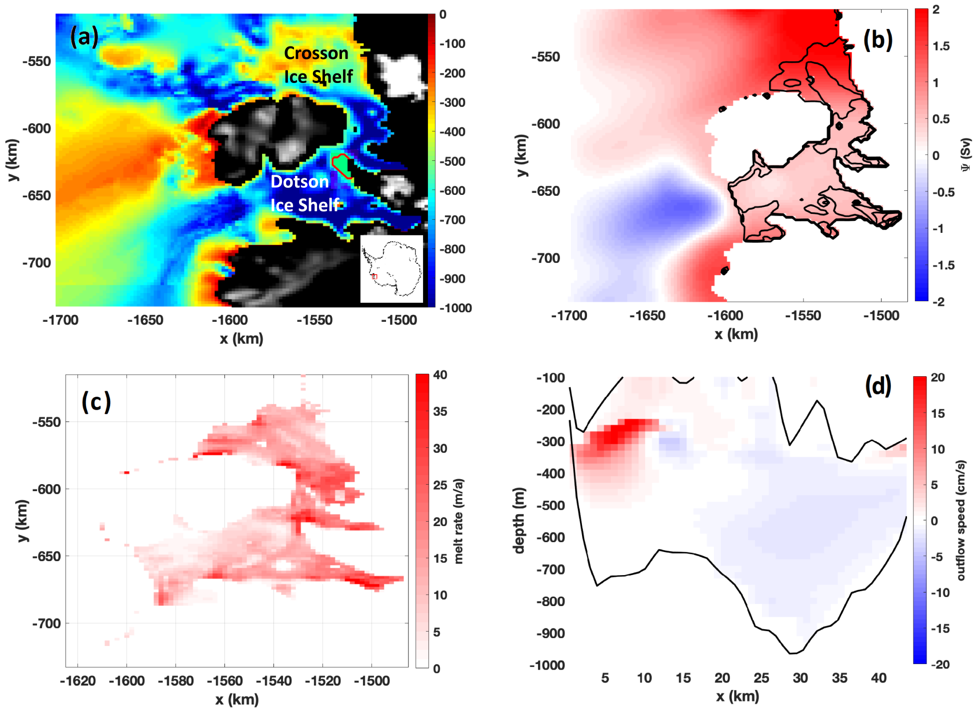

Figure 5. (a) The bathymetry of Millan et al. (2017), used in our adjoint experiment. Black and white shading indicates topography above sea level. X and Y coordinates refer to a Polar Stereographic projection. The red contour near the junction of Dotson and Crosson ice shelves indicats where bathymetry has been modified from Millan et al. (2017) as discussed in Section 4.1. (b) The barotropic stream function corresponding to the initial steady state of the ocean model. (c) Under-ice shelf melt rate corresponding to the steady state. (d) Outflow at the opening to the Dotson Ice Shelf cavity cf. Randall-Goodwin et al. (2015), their Figure 7(a)).

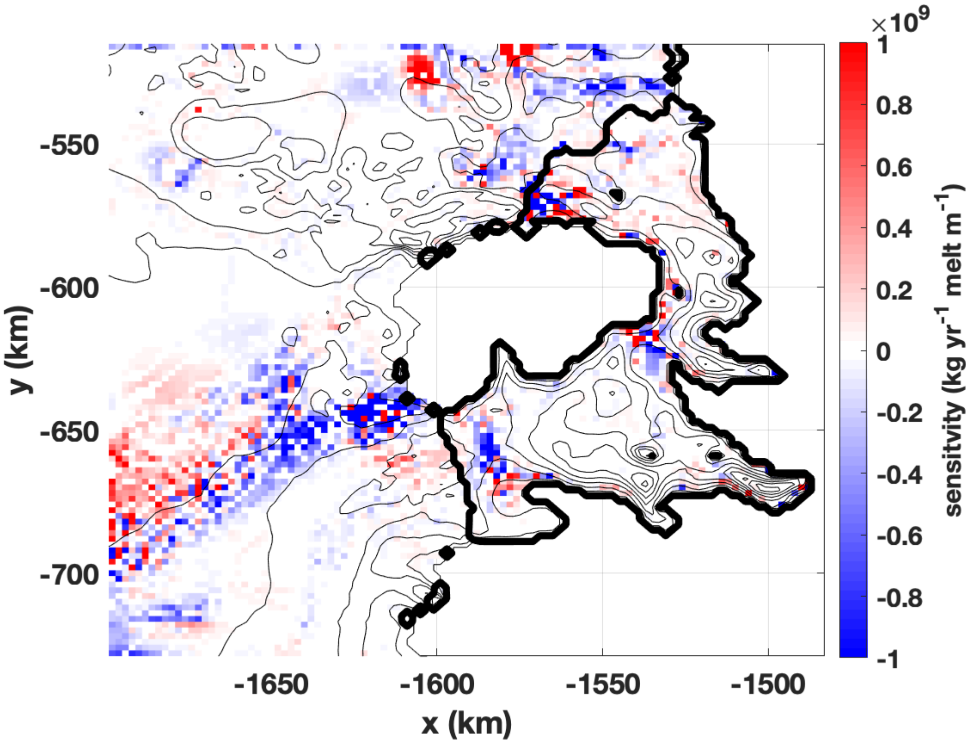

Figure 6. Sensitivity of total (area-integrated) melt to bathymetry in Dotson-Crosson experiment; interpretation is as in Fig. 3(a).

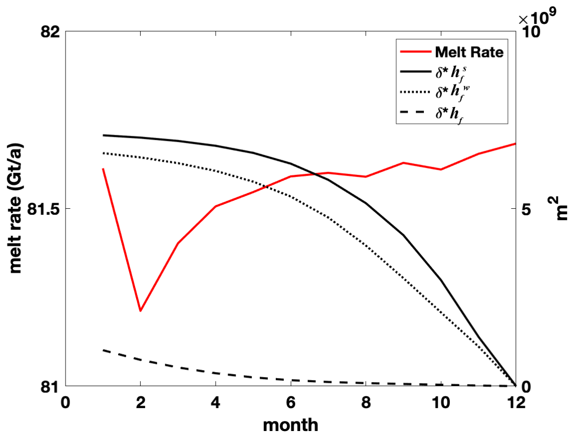

Figure 7. Time series of melt volume and bathymetric sensitivity factors in our simulation of Dotson and Crosson ice shelves. Note sensitivity fields computed from the adjoint model evolve backward in time.

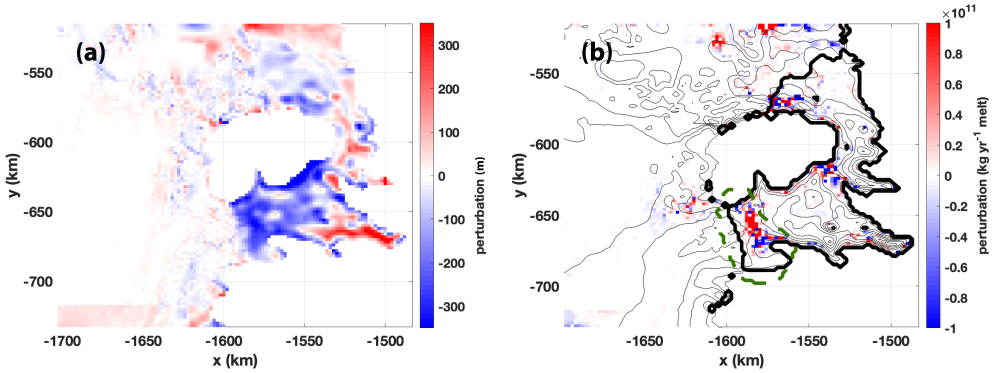

Figure 8. (a) Difference between BedMachine bathymetry and Millan bathymetry within the ocean model domain. The rectangular region in the bottom left of the figure is due to the Millan data set not extending to the edge of the domain. (b) The product of this difference and the sensitivity of melt with respect to bathymetry. The region enclosed by the dashed contour accounts for approximately 4.3 Gt/yr of the linear-predicted melt rate difference.

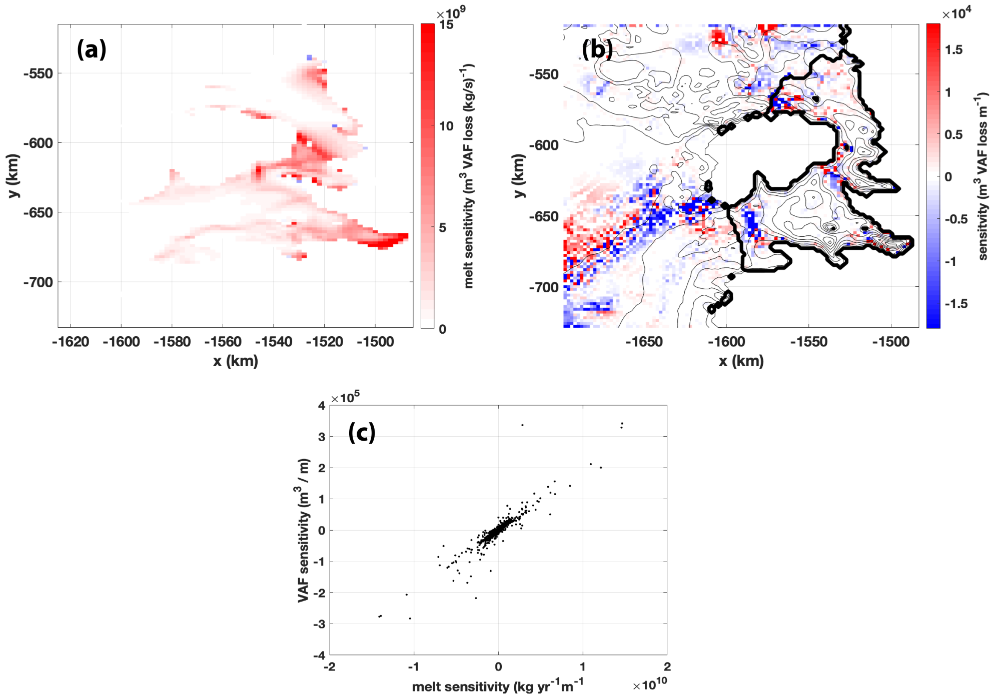

Figure 9. (a) Sensitivity of grounded ice volume to ice-shelf melt (adapted from Goldberg et al. (2019), their Fig. 3(b)). (b) Sensitivity of the objective function given by Eqn. (6) to bathymetry. (c) Cell-by-cell correspondence of grounded volume sensitivity to melt-rate sensitivity.

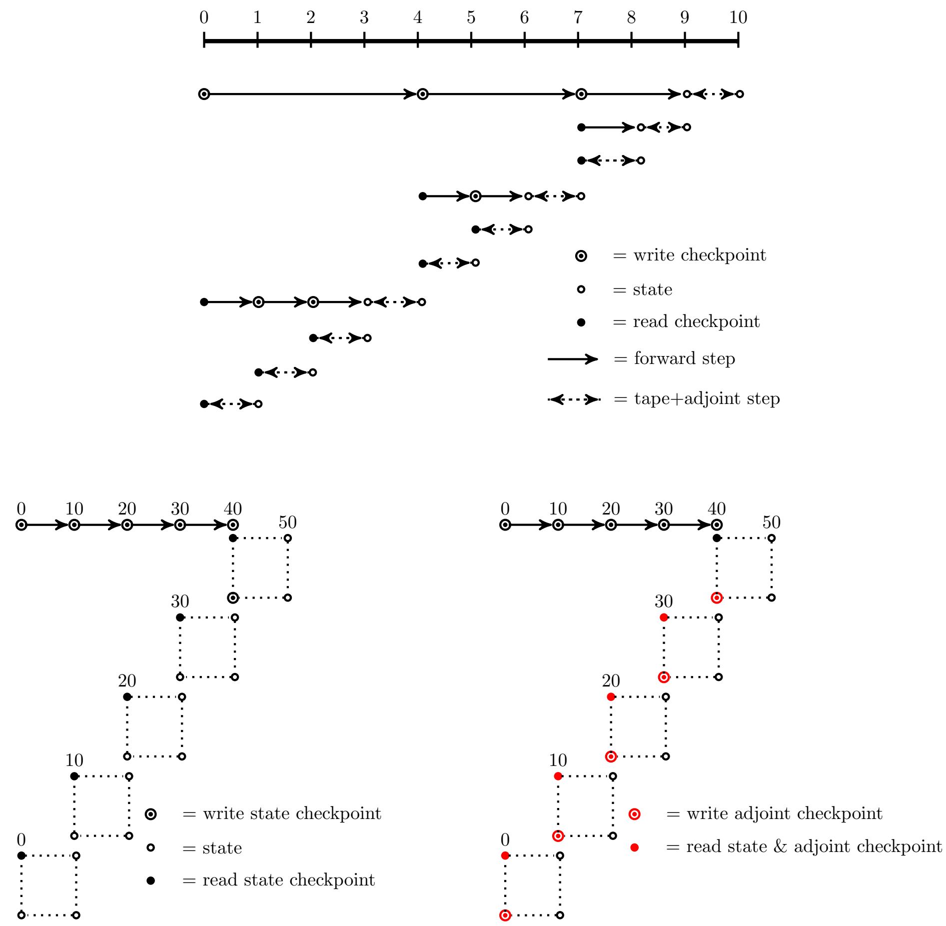

Figure 10. Top: Binomial checkpointing schedule for $l = 10$ time steps and $c = 3$ checkpoints. Bottom Left: Two level checkpointing schedule for $l = 50$ with $(l_2 = 5)$ outer level iterations and $(l_1 = 10)$ inner level iterations. Periodic checkpointing is used in the outer level and binomial checkpointing shown by the dashed box is used at the inner level. Bottom Right: Enhanced two level checkpointing schedule with support for resilient adjoints through the writing and reading of the adjoint state at the outer level.