Nonlinear and time-dependent equivalent-barotropic flows with topography

L. Zavala Sansón (lzavala@cicese.mx)
Department of Physical Oceanography, CICESE, México (www.cicese.mx/∼lzavala)

Abstract

A nonlinear, time-dependent model for an equivalent-barotropic flow is examined. The model is solved numerically to investigate the equivalent-barotropic dynamics of experimental and observational examples in comparison with a purely barotropic flow.

Some antecedents

The motion of an equivalent-barotropic flow varies in magnitude at different vertical levels while keeping the same direction. The governing equations at a specific level are identical to those of a homogeneous flow over an equivalent depth, determined by a pre-defined vertical structure. The idea was proposed by Charney (1949) [1] for modelling a barotropic atmosphere. More recently, steady, linear formulations have been used to study oceanic flows, especially the Antarctic Circumpolar Current [2,3].

1 Physical model

Quasi-2D, shallow-water flow on a β-plane with fluid depth (h, y), velocity $(\boldsymbol{v}, w)$ and vertical structure $P(z)$ [4]:

$$\begin{aligned} \boldsymbol{v}(x,y,z,t) &= P(z)\boldsymbol{v}_s(x,y,t), \\ p(x,y,z,t) &= P(z)p_s(x,y,t), \end{aligned} \qquad (1)$$

where subindex s indicates surface values. Vertical scales:

$$F(x,y) = \int_{-h}^{0} P(z)dz, \qquad G(x,y) = \int_{-h}^{0} P^2(z)dz \qquad (2)$$

Horizontal velocity components in terms of transport function:

$$u_s = \frac{1}{F}\frac{\partial \psi}{\partial y}, \qquad v_s = -\frac{1}{F}\frac{\partial \psi}{\partial x} \qquad (3)$$

Vertical velocity:

$$w(x,y,z,t) = \left(\int_z^0 P(z')dz'\right) \nabla \cdot \boldsymbol{v}_s \qquad (4)$$

In this study $P(z)$ is exponential with reference depth z_0:

$$\boxed{P(z) = e^{z/z_0},} \qquad (5)$$

Vorticity equation including wind-stress forcing $\boldsymbol{\tau}$ and bottom friction with coefficient R:

$$\boxed{\frac{\partial \omega_s}{\partial t} + J\left(\frac{\omega_s + f}{F}, \psi\right) = \nu \nabla^2 \omega_s + \frac{1}{\rho_0}\left(\nabla \times \frac{\boldsymbol{\tau}_s}{F}\right) - R\left(\frac{1}{F} - \frac{1}{z_0}\right)\omega_s}$$

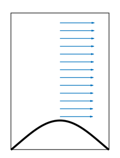
with $\boxed{\omega_s = -\frac{1}{F}\nabla^2\psi + \frac{1}{F^2}\nabla F \cdot \nabla \psi.}$

Error in the nonlinear terms associated with separation (1):

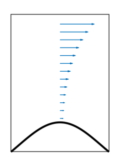
$$\max\left\{1 - \frac{G}{F}\right\}. \qquad (6)$$

Limits:

$z_0 >> h \Rightarrow F \approx h(x,y)$ $\to$ homogeneous



$z_0 << h \Rightarrow F \approx z_0$ $\to$ shallow layer



2 Simulations of vortices around seamounts

- Strongly nonlinear, cyclonic vortices, f-plane (rotation period $T = 30$ s), no external forcing.
- Comparison with laboratory experiments with barotropic vortices performed in the Coriolis platform (Grenoble, France) [5].
- Aim: to illustrate how the vortex drift is modified according to z_0 (*i.e.* the effect of the vertical structure).

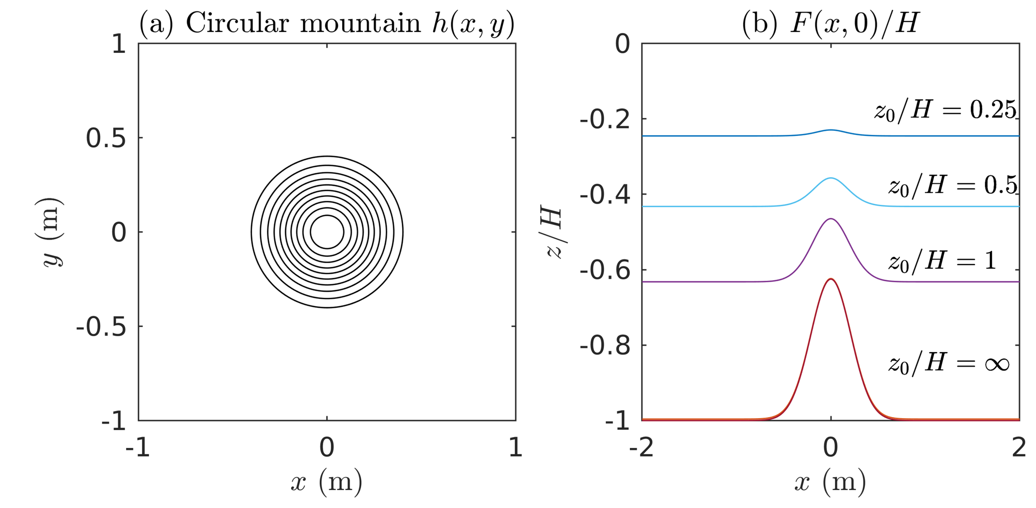


Figure 1: (a) Topography contours of a Gaussian mountain in a fluid with maximum depth H. (b) Equivalent depth profiles $F(x,0)/H$ calculated for different z_0.

$$\boxed{\omega_s(x,y)P(z), \text{ depth } h(x,y) \quad \equiv \quad \omega_s(x,y), \text{ depth } F(x,y)}$$

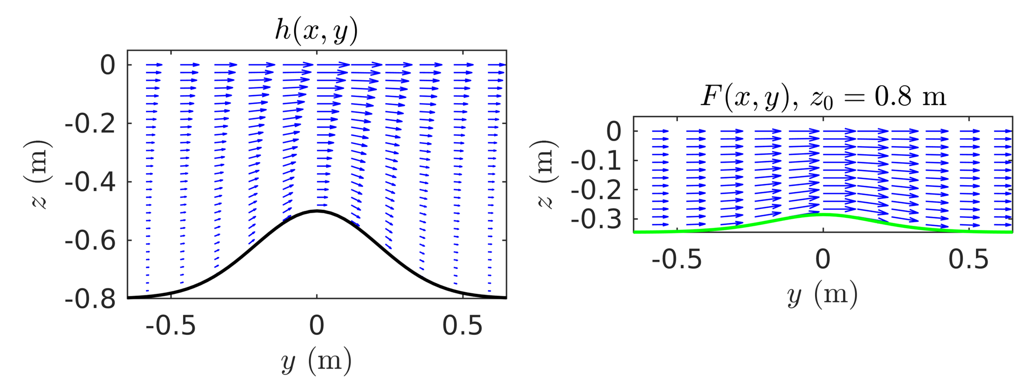


Figure 2: Vertical velocity profile for $z_0/H = 0.8$. Left: Flow with vertical structure $P(z)$ and depth $h(x,y)$. Right: Uniform flow with depth F(x,y). The surface flow in both cases is exactly the same.

The homogeneous vortex (panel a) drifts around the mountain in a clockwise direction, and negative vorticity is formed over the summit due to squeezing effects of fluid columns. As z_0 decreases the turns are reduced (b-c). For the lowest z_0 the flow is nearly 2D (panel d).

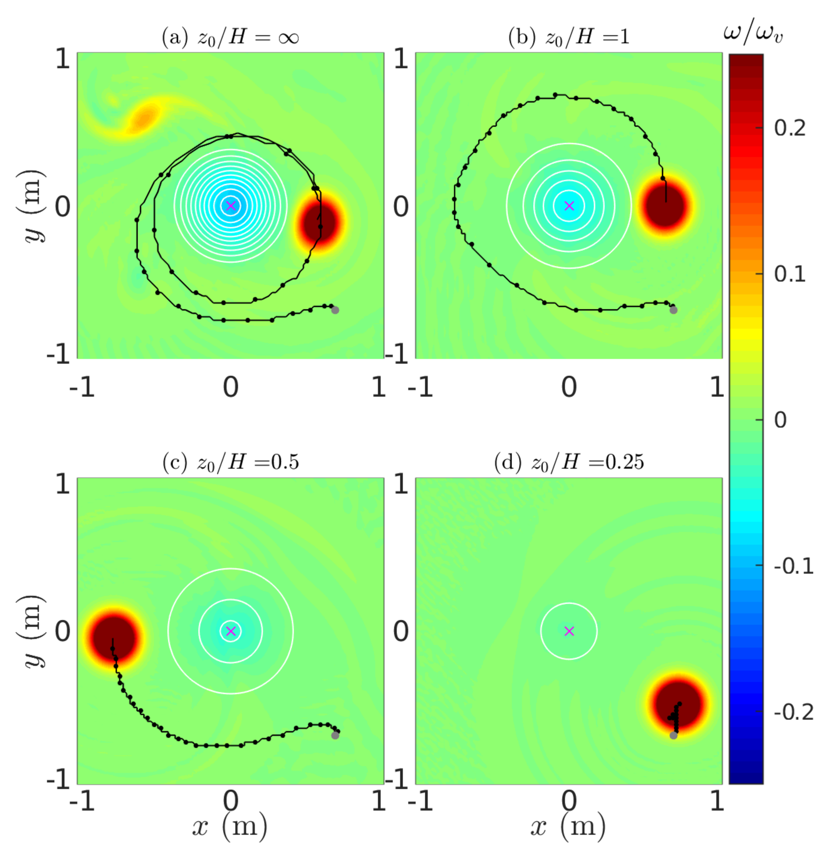


Figure 3: Relative vorticity $\omega_s(x,y)$ at $t/T = 31.5$ in simulations with different z_0.

3 Vortex generation in the Gulf of Mexico

The Campeche cyclone is a semi-permanent mesoscale circulation in the southern Gulf of Mexico (Fig. 4). According to averaged current-meter measurements along the water column, it is one of the few oceanic systems that presents a barotropic vertical structure [6]. The generation of the cyclone under the equivalent-barotropic dynamics is examined.

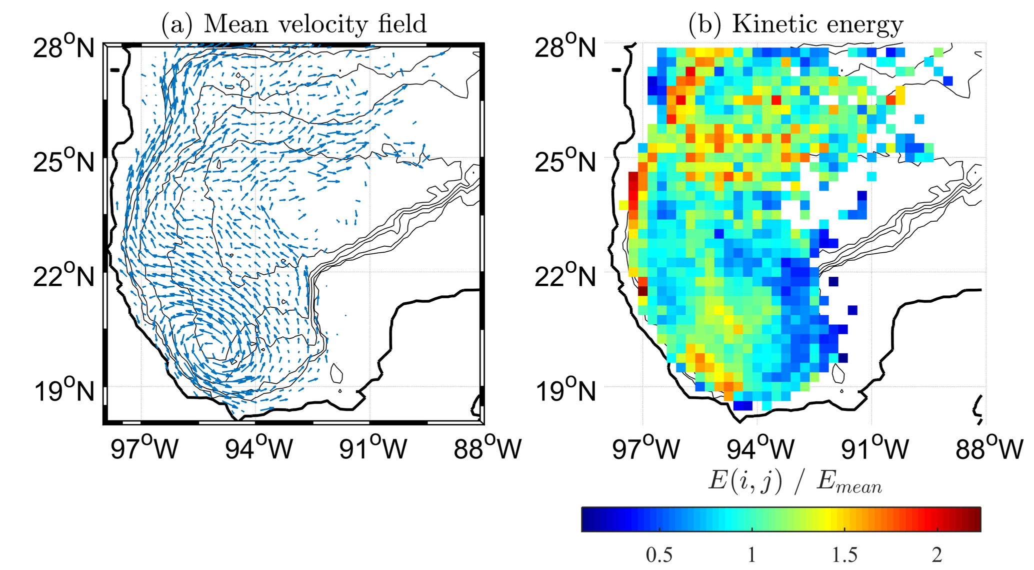


Figure 4: (a) Velocity field in the western Gulf of Mexico calculated from 441 surface drifters during a 7-year period [7]. Maximum value: 0.4 m/s. (b) Kinetic energy per geographical bin (0.25°)

3.1 Simulations using realistic topography

- Western jet-like wind-stress applied over regions with $h > 200$ m (flow over the shelves is minimized).

$$\boldsymbol{\tau}_s = (\tau^x, \tau^y) = \left(-\tau_0 \sin\left[\frac{\pi}{2L}(y+L)\right], 0\right). \qquad (7)$$

- Flow starts from rest on a β plane. A smoothed realistic topography is used. Geostrophic contours: $f(y)/F(x,y)$.
- Aim: to investigate whether the formation of a cyclone at the Bay of Campeche is compatible with the equivalent-barotropic dynamics.

The homogeneous and quasi-homogeneous cases (Figs. 5a-c) develop a basin-scale anticyclonic circulation, with no signs of a southern cyclone. For $z_0 = 1000$ m, a cyclonic vortex is formed (d).

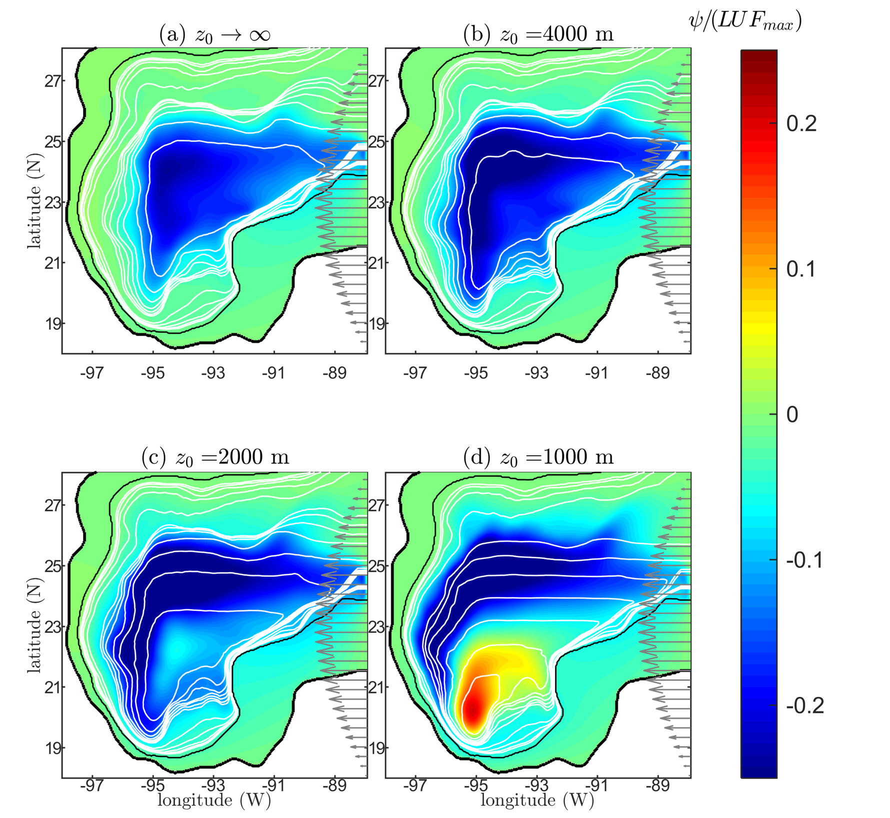


Figure 5: Transport function at day 90 for different z_0. Gray arrows at the eastern side represent the westward wind-stress. Black lines indicate the coast and the 200 m isobath. White curves are $(f_0 + \beta y)/F(x,y)$ contours

3.2 Case with $z_0 = 650$ m

According to mooring observations [6], the average vertical structure in some regions of the Campeche bay is exponential with reference depth $z_0 = 650$ m. The simulation in Fig. 6 displays a cyclonic vortex at the Campeche bay confined by the f/F contours. The maximum speed is 0.14 m/s (in the observations by [6] is 0.36 m/s).

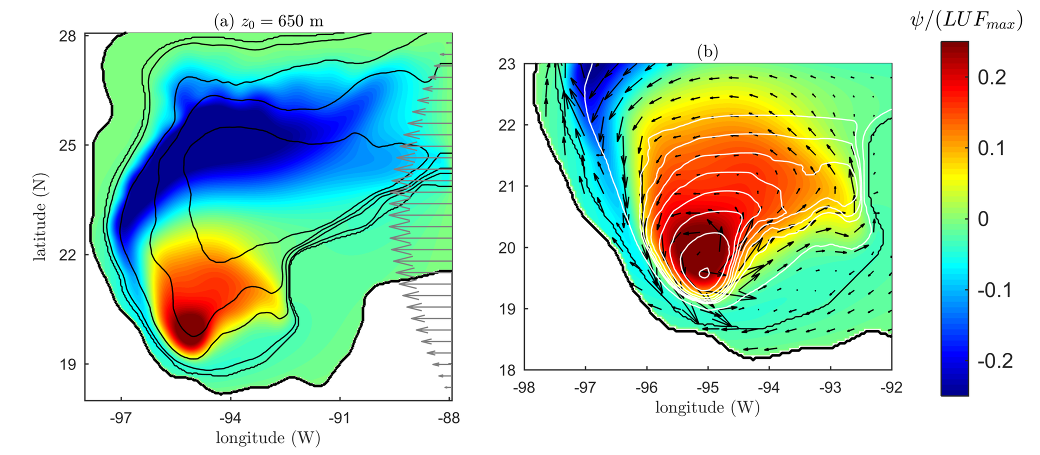


Figure 6: (a) Transport function at day 90 for $z_0 = 650$ m. Black curves are topography contours. (b) Velocity field over the Bay of Campeche. The rms speed is $U = 0.021$ m/s. White contours as in previous figure.

Conclusions 1: the model

- A time-dependent, non-linear equivalent-barotropic model is discussed for studying the effects of variable bottom topography in oceanic flows. The model simulates the vertical structure of stratified flow while maintaining a barotropic character.
- We used an exponential vertical structure $P(z)$, but the formulation admits more general cases as long as

$$\begin{aligned} P(0) &= 1 \\ P(z) &> 0 \quad -h \le z < 0 \end{aligned} \qquad (8)$$

- The error (6) grows for very abrupt $P(z)$.
- Topographic effects are decoupled from the upper region for $z_0 << h$ (equivalent depth is smoothed and bottom friction reduced).

Conclusions 2,3: the simulations

- The simulations reproduce experimental results of strongly nonlinear vortices in the extreme cases $z_0 >> h$ (Fig. 3a) and $z_0 << h$, *i.e.* nearly 2D (Fig. 3d). Thus, the intermediate cases (b-c) are "validated".
- The model is a useful tool to better understand the formation of an oceanic structure (Fig. 6) under the barotropic dynamics with a vertical structure.

Forthcoming research concerns the use of time-dependent, random forcing together with a large-scale wind in the Campeche cyclone problem. The aim is to identify the prevalence or absence of the cyclonic pattern under different physical conditions.

References

[1] Charney, J.G., 1949. J. Meteorol., 6(6), 372-385.
[2] Killworth, P. D., 1992. J. Phys. Oceanogr., 22, 1379-1387.
[3] Krupitsky *et al.* 1996. J. Phys. Oceanogr., 26(9), 1803-1824.
[4] LaCasce & Isachsen, 2010. J. Phys. Oceanogr., 84, 139-157.
[5] Zavala Sansón *et al.* 2012. J. Fluid Mech., 695, 173-198.
[6] Pérez-Brunius *et al.* 2013. Deep Sea Res. Part II, 85, 182-194.
[7] Zavala Sansón *et al.* 2017. J. Phys. Oceanogr., 47, 387-403.