Supplemental Information for Topographic Roughness as an Emergent Property of Geomorphic Process and Events

S1. Diffusion and Filling of a Channel

Here we demonstrate the role of regular channel infilling in addition to topographic diffusion. Channel infilling tends to reduce to topographic variance more rapidly than topographic diffusion alone. Numerical simulations demonstrate that the magntiude of this difference depends strongly on the value of $\lambda v/K$, where v [L T^{-1}] is the rate of infilling (Figure S1). This may be incorporated into the theory for topographic diffusion.

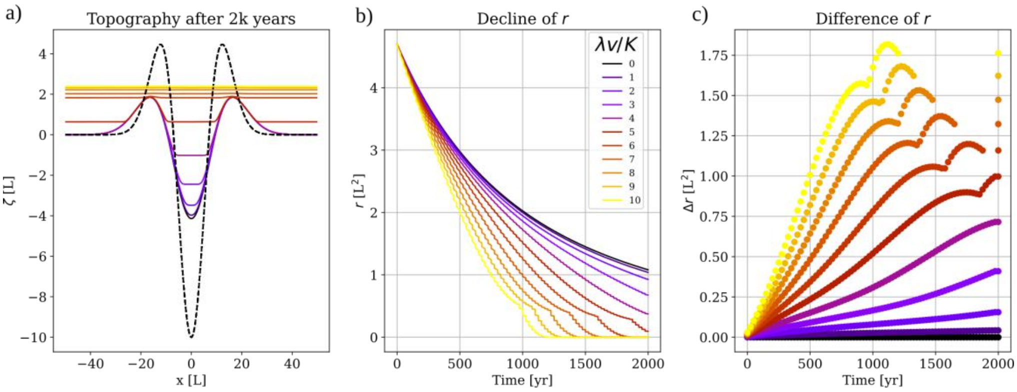

Figure S1. a) topography through time of a channel with various combinations of in-channel deposition and diffusion. b) Topographic variance through time for each combination and (c) the difference between topographic roughness with channel deposition and without it.

S2. Specifying the distribution and correlation of the roughening process

In order to specify both the correlation and the probability distribution of a time series, we develop a sampling scheme that is based on the QPPQ method (Cite Worland) commonly used in hydrology and inverse transform sampling. For this method, we first generate a time-series, γ(t), using a classic AR(1) scheme which will converge to a time series with zero mean and variance of $\sigma_\eta^2/(1 + \phi_1^2)$ and is distributed Normally. Therefore, each value of γ(t) maps to a corresponding value of the cumulative probability function, F_γ (γ) which ranges from 0 to 1. This value then maps to the cumulative probability function of the desired form which corresponds to a value that becomes *p(t)* which now is distributed by any desired probability function and shares an autocorrelation function with γ. This method preserves the autocorrelation structure for $-1 < \phi < 1$ and for probability distributions that are thin tailed (have defined variance). For heavy-tailed distributions, the method does not reproduce the exact same autocorrelation and the mismatch between autocorrelation grows with increasingly heavy tails.

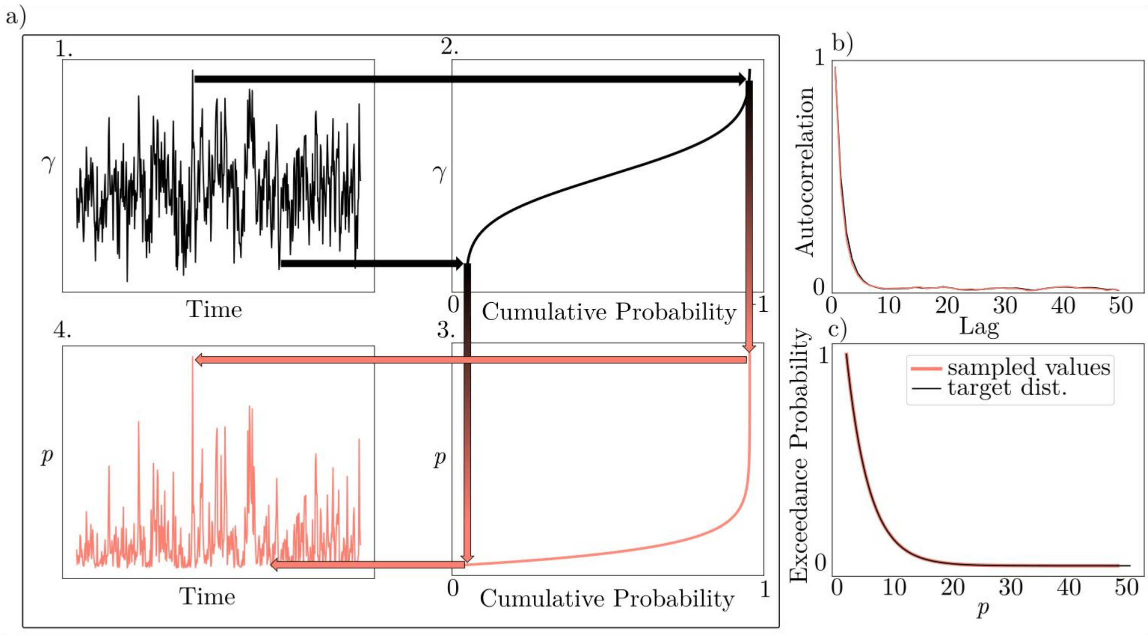

Figure S2. a) Illustration of the inverse sampling method that creates a time series with a specified probability distribution and correlation timescale. The method starts with (1) creating an AR(1) time series, (2) mapping those values to the corresponding CDF, (3) translating to the desired CDF, and (4) mapping the new values back on to a new time series. The resulting time series $p(t)$ has the same autocorrelation as the initial (γ), but is distributed (c) according to a different distribution.

References:

Worland, S. C., Steinschneider, S., Farmer, W., Asquith, W., & Knight, R. (2019). Copula theory as a generalized framework for flow-duration curve based streamflow estimates in ungaged and partially gaged catchments. *Water Resources Research*, *55*(11), 9378-9397.