Supporting Information for "Polynomial reconstruction of the magnetic field observed by multiple spacecraft with integrated velocity determination"

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Additional Supporting Information (Files uploaded separately)

1. Captions for Movies S1 to S7

Text S1 contains descriptions of the figures and movies.

1. Text S1

Figure S1 compares the reconstruction fields for case 1 to those of the simulation in the L-N plane at the M value of the centroid of the virtual spacecraft. The format is

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the same as that of Figure 6 in the paper, except for simulation reconstruction case 1.

Similarly, Figures S2–S5 compare reconstruction fields to simulation fields for simulation

reconstruction cases 7–10, respectively. Figure S3 is the same as Figure 6 in the paper,

and Figure S5 is the same as Figure 7 in the paper; these are included here for easier

comparison to the other figures.

Simulation reconstruction case 1 (Figure S1) uses the RQ-3D model with $t_{\mathrm{smooth}} =$

0.4, whereas simulation reconstruction cases 7–10 (Figures S2–S5) show results for four

different models (noted in the captions) for $t_{\mathrm{smooth}} = 0.8$. All of Figures S2–S5 show

reasonable agreement between the reconstruction and simulation fields, whereas there is

a significant disagreement in Figure S1 (especially Figure S1e). This shows that use of a

greater amount of smoothing significantly improves the reconstruction results.

Movie S1 shows the reconstruction fields for case 1 versus time in the L-N and M-

N planes for case 1. Similarly, Movies S2–S5 show reconstruction fields for simulation

reconstruction cases 7–10, respectively. The movies show the reconstruction field in the

L-N plane, similar to that shown in the top panels of Figures S1–S5. The movies also

show the reconstruction field in the M-N plane.

In principle, if the M dependence is small, there should not be any variation of the field

in the M direction. But there may appear to be significant variation in the M direction in

the movies if the B_M and B_N components of the magnetic field are small (like at $t = -0.3$;

see top and bottom right panels of Movies S1–S5 at that time). There is usually less of

this kind of problem for the magnetic field shown in the L-N plane because B_L is usually

the largest component of $\mathbf{B}$.

40   Again, Movies S2–S5, for cases 7–10, respectively, using $t_{\mathrm{smooth}} = 0.8$, show more accu-

41   rate reconstructions than Movie S1 for case 1, using $t_{\mathrm{smooth}} = 0.4$.

42   Movies S6 and S7 have the same format as Movie S1, but Movie S6 shows the recon-

43   struction fields for the 27 August 2018 MMS magnetotail reconnection event of section 4.1

44   in the paper, and Movie S7 shows the reconstruction field for the 7 December 2016 MMS

45   current sheet crossing event of section 4.2 in the paper.

Movie S1. Movie of the reconstruction fields versus time for simulation reconstruction case 1. The top panel shows the magnetic field averaged over the virtual spacecraft versus time. The current time of the movie frame is indicated by the vertical black line. The bottom left panel shows the reconstruction magnetic field in the L-N plane. The black curves are streamlines of the magnetic field in the L-N plane at the M value of the centroid of the virtual spacecraft. The color scale shows B_M, which is into the plane of the picture. The bottom right panel is similar, but showing the magnetic field in the M-N plane.

Movie S2. Movie of the reconstruction fields versus time for simulation reconstruction case 7, using the same format as Movie S1.

Movie S3. Movie of the reconstruction fields versus time for simulation reconstruction case 8 (equivalent to case 2), using the same format as Movie S1.

Movie S4. Movie of the reconstruction fields versus time for simulation reconstruction case 9, using the same format as Movie S1.

Movie S5. Movie of the reconstruction fields versus time for simulation reconstruction case 10, using the same format as Movie S1.

Movie S6. Movie of the reconstruction fields versus time for the 27 August 2018 MMS magnetotail reconnection event of section 4.1 in the paper, using the same format as Movie S1.

Movie S7. Movie of the reconstruction fields versus time for the 7 December 2016 MMS current sheet crossing event of section 4.2 in the paper, using the same format as Movie S1. Unlike Figure 13 in the paper, the color scale shows B_M (like in the other movies) rather than B_L.

68   **Figure S1. (caption not printing in Latex)** Comparison of reconstruction and sim-

69   ulation magnetic field for simulation reconstruction case 1 using the RQ-3D model with

70   $t_{\mathrm{smooth}} = 0.4$. The fields are plotted in the L-N plane at the M value of the centroid

71   of the virtual spacecraft. (a) Magnetic field averaged over the four virtual spacecraft.

72   (b–i) In each pair of vertically arranged panels, reconstructed (top, with time label) and

73   simulation (bottom, labeled "simulation") magnetic streamlines in the L-N plane (black)

74   and magnetic field into the plane of the page, B_M (color scale).

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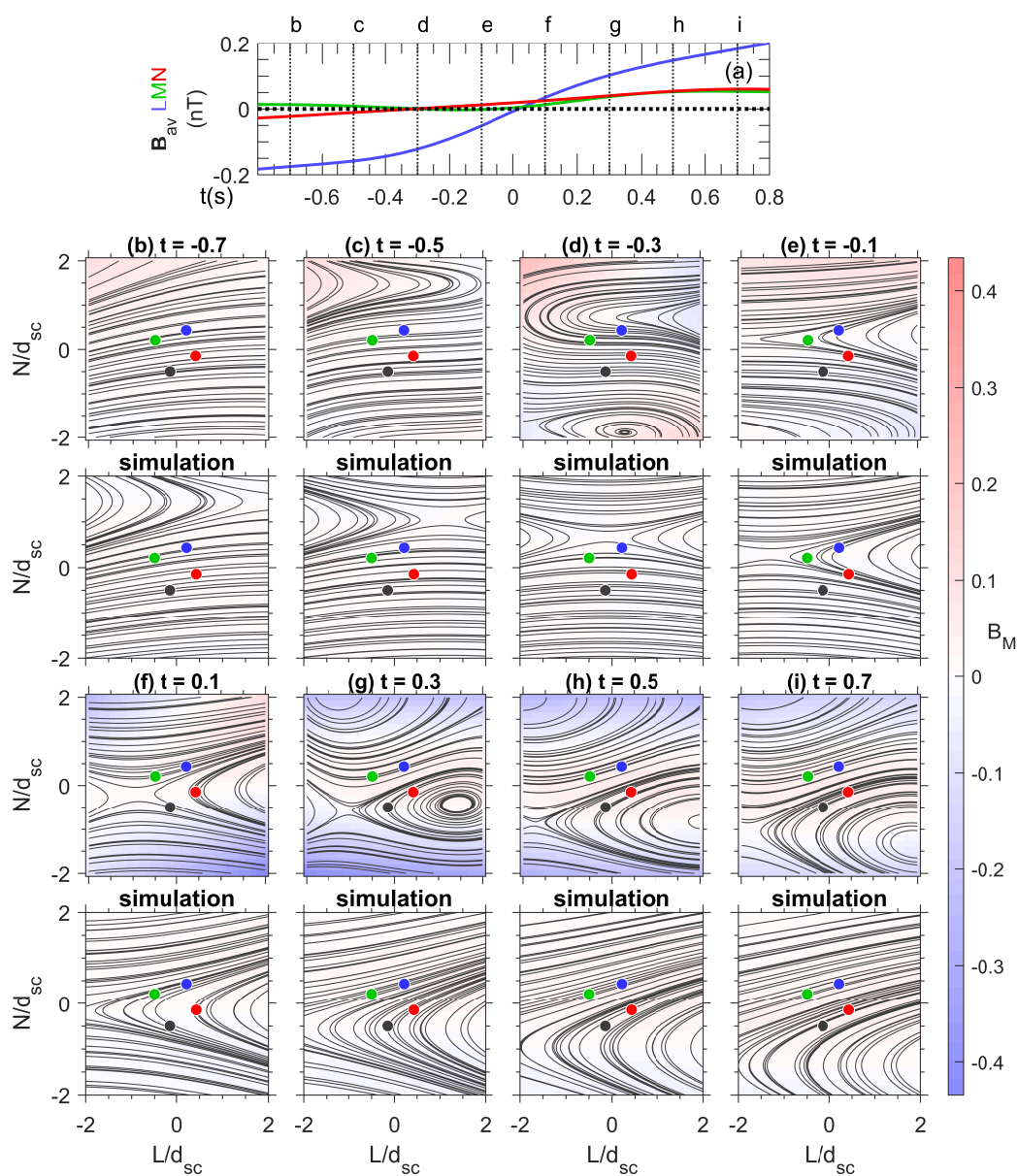
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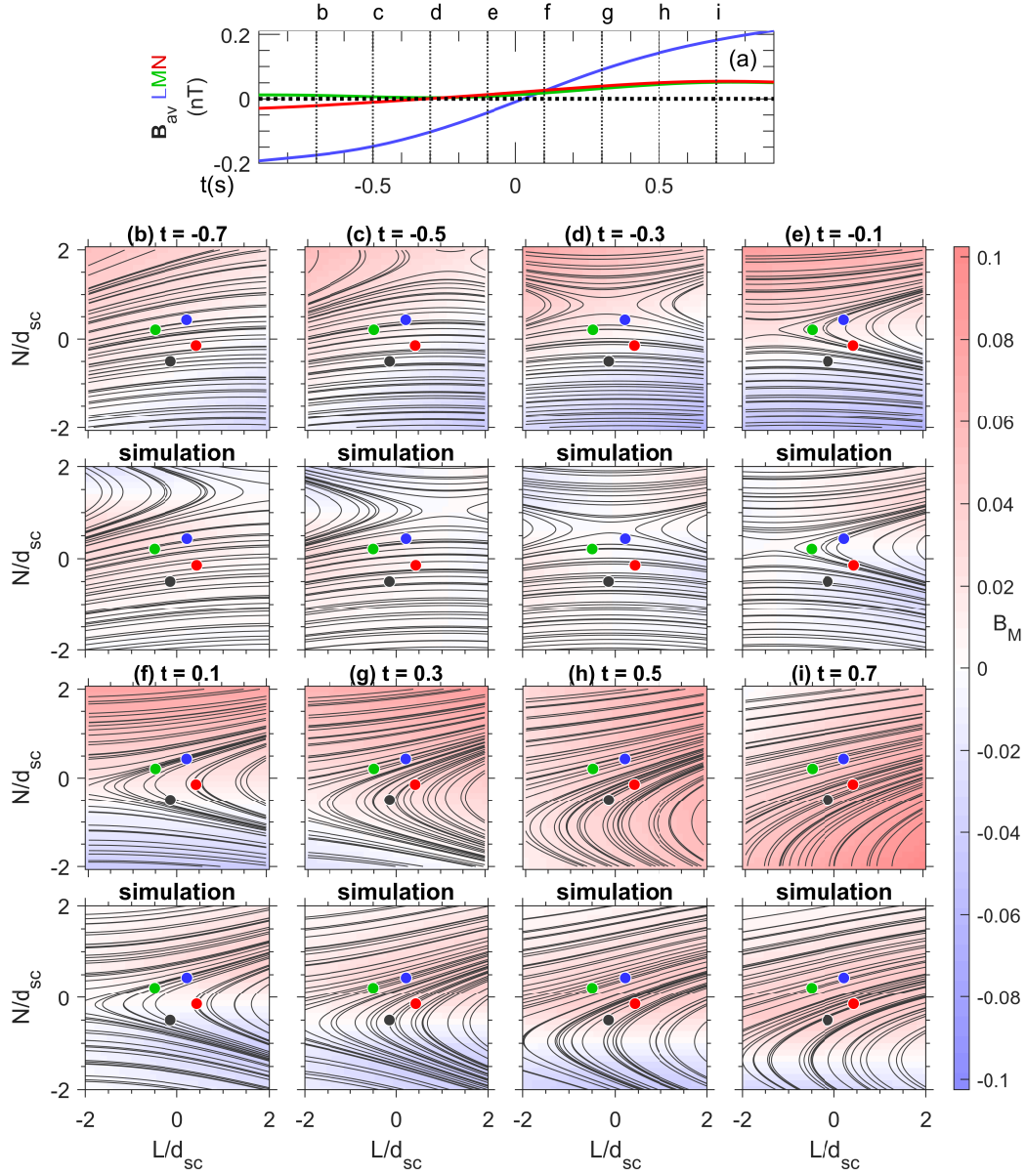
Figure S2. Comparison of reconstruction and simulation magnetic field in the *L-N* plane for reconstruction case 7 using the LB-3D model with $t_{\mathrm{smooth}} = 0.8$. The format is the same as that of Figure S1.
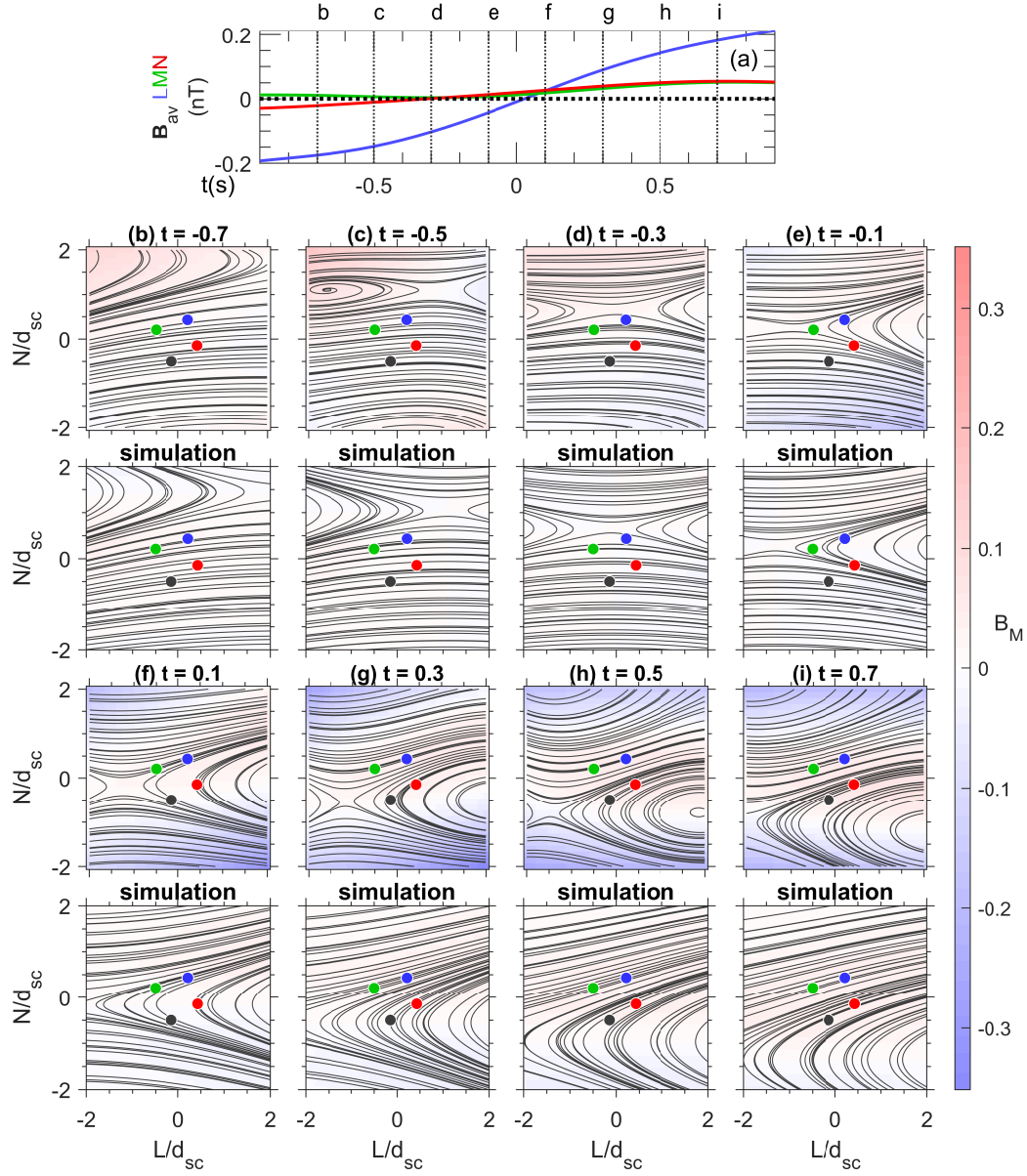
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Figure S3. Comparison of reconstruction and simulation magnetic field in the *L*-*N* plane for reconstruction case 8 (equivalent to case 2) using the RQ-3D model with $t_{\text{smooth}} = 0.8$. The format is the same as that of Figure S1.
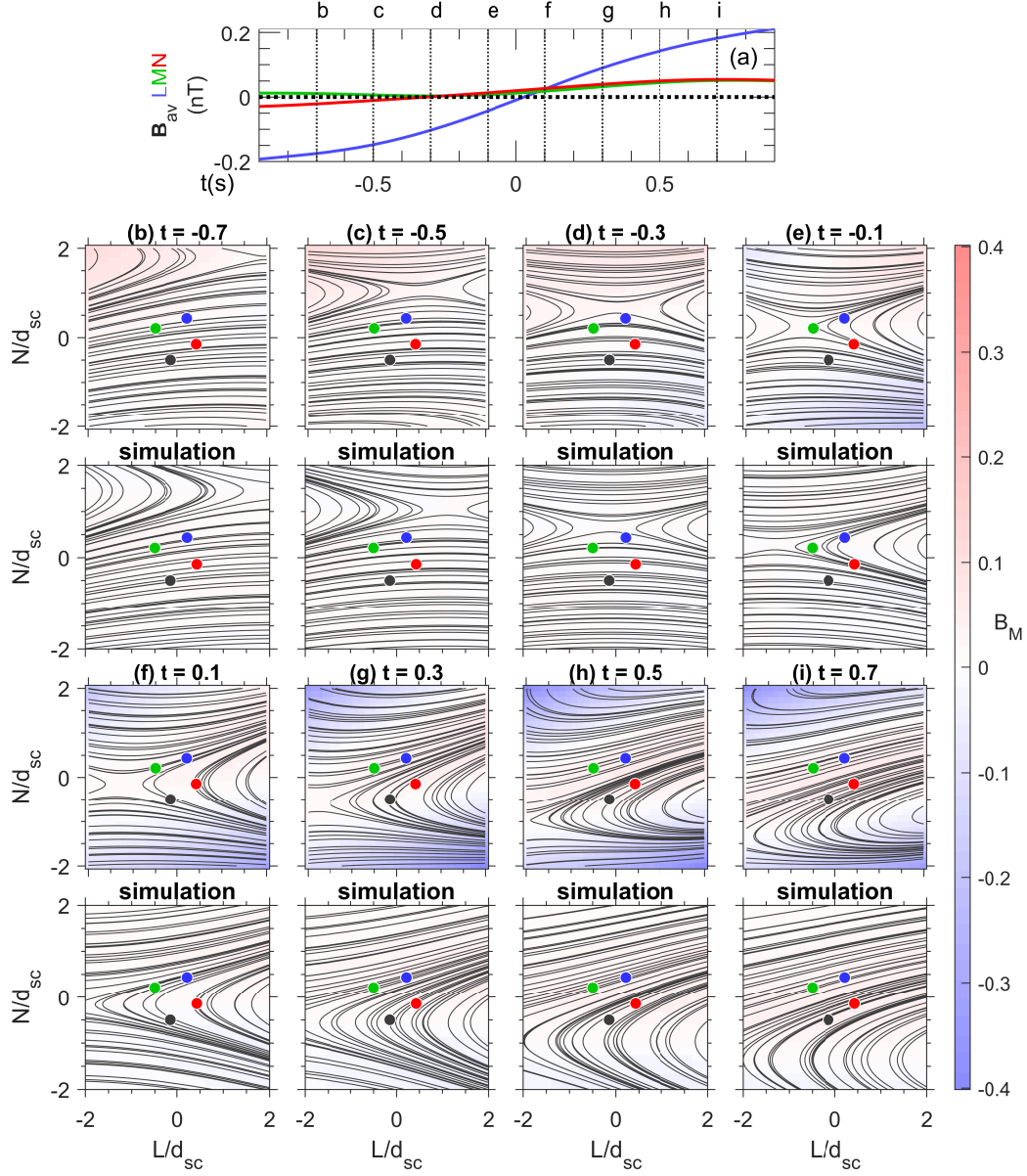
Figure S4. Comparison of reconstruction and simulation magnetic field in the *L-N* plane for reconstruction case 9 using the Q-3D model with $t_{\text{smooth}} = 0.8$. The format is the same as that of Figure S1.
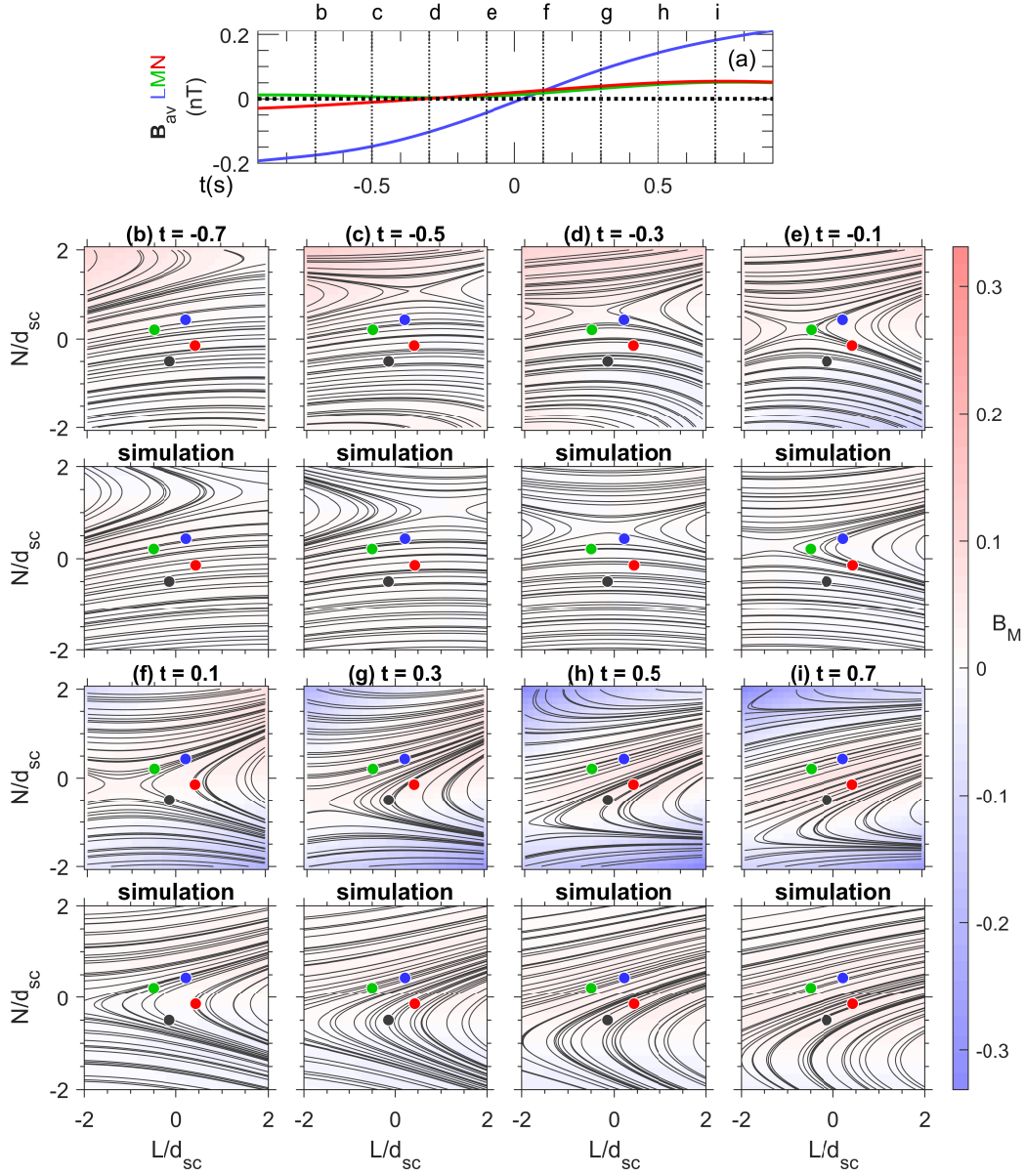
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Figure S5.   Comparison of reconstruction and simulation magnetic field in the *L-N* plane for reconstruction case 10 using the CQ-3D model with $t_{\text{smooth}} = 0.8$. The format is the same as that of Figure S1.