True Geopotential in Meteorology

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Key Points:

- Geopotential used in meteorology is not true since it is associated with the standard/normal gravity not the true gravity $\mathbf{g} = \mathbf{i}g + \mathbf{j}g + \mathbf{k}g_z$
- A new equation of geostrophic wind is derived using the true geopotential
- With the EIGEN-6C4 and NCEP/NCAR Reanalysis data, the difference between the true and standard geopotentials is large in troposphere

Abstract

Geopotential in meteorology is the same as gravity potential in geodesy but with the opposite sign. Meteorologists don't use the true geopotential (Φ) associated with the true gravity $\mathbf{g} = \mathbf{i}g + \mathbf{j}g + \mathbf{k}g_z$, but use the normal geopotential (Φ_N) with the normal gravity [-**K**], or the standard geopotential (Φ_0) with the standard gravity (-$g_0\mathbf{k}$, g_0 = 9.81 m/s^2). Here, (, , z) are the (longitude, latitude, altitude) with (**i**, **j**, **k/K**) the corresponding unit vectors with **k/K** normal to the Earth spherical/ellipsoidal surface. In meteorology, difference between Φ_0 and Φ_N is considered minor but between Φ_0 and Φ has not been identified. This study uses two publicly available datasets: (a) ICGEM EIGEN-6C4 and (b) NCEP/NCAR Reanalysis long term mean data to obtain true geopotential Φ and standard geopotential Φ_0 for the troposphere (1000 - 100 hPa) and in turn to compute the nondimensional B and C numbers, representing the importance of the latitudinal-longitudinal gradient of disturbing geopotential (Φ-Φ_0) versus the pressure gradient force and the Coriolis force. The B number varies from 0.4176 (maximum) at 850 hPa to 0.1630 (minimum) at 200 hPa. The C number varies from 0.6168 (maximum) at 1,000 hPa to 0.1573 (minimum) at 200 hPa. These values show the importance to use the true geopotential Φ in meteorology. A new equation for the geostrophic wind is also presented.

Plain Language Summary

Meteorologists don't use the true geopotential (Φ) associated with the true gravity **g**(, , z), but use the standard geopotential (Φ_0) associated with the standard gravity (-$g_0\mathbf{k}$, g_0 = 9.81 m/s^2). Here, (, , z) are the (longitude, latitude, altitude) with (**i**, **j**, **k**) the corresponding unit vectors. Feasibility to replace the true geopotential (Φ) by the standard geopotential (Φ_0) has never been verified. This study uses the two publicly available datasets: (a) ICGEM EIGEN-6C4 and (b) NCEP/NCAR Reanalysis long term mean data to obtain true geopotential Φ and standard geopotential Φ_0 for the troposphere (1000 - 100 hPa). Evidently large disturbing geopotential (Φ-Φ_0) shows that the true geopotential

Φ cannot be replaced by the standard geopotential Φ_0 in meteorology. A new equation for the geostrophic wind is also presented.

1 Introduction

An important variable associated with the gravity has two different names with opposite sign: geopotential in meteorology and gravity potential in geodesy. Three types of geopotential (gravity potential) are used in geodesy with the true geopotential (Φ) associated with the true gravity $\mathbf{g} = \mathbf{i}g + \mathbf{j}g + \mathbf{k}g_z$, the normal geopotential (Φ_N) associated with the normal gravity [-$\mathbf{K}$], and the standard geopotential (Φ_0) associated with the standard gravity (-$g_0\mathbf{k}$, g_0 = 9.81 m/s^2) (Vaniček and Krakiwsky, 1986). Here, (, , z) are the (longitude, latitude, attitude); ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$/ $\mathbf{K}$) are the corresponding unit vectors with $\mathbf{k}$ (or $\mathbf{K}$) normal to the Earth spherical (or ellipsoidal) surface. However, only the standard and normal geopotentials are used in meteorology. The function is analytically determined. For example, Haurwitz (1941) presented the following formula in meteorology

(1)

where R = 6.3781364×10^6 m, is the Earth radius. Geodetists use more sophisticated formulas to represent g( ) such as the Somiglina equation (National Geospatial-Intelligence Agency, 1984). In addition to the (, , z) coordinates, meteorologists also use the local coordinates (x, y, z),

(2)

where the (x, y) plane is perpendicular to $\mathbf{k}$ (or $\mathbf{K}$).

Both standard and normal gravities don't have lateral (i.e., longitudinal and latitudinal) component. The standard (nomal) gravity is linked a hypothetically uniform rigid spherical (ellipsoidal) Earth with the same total mass of the true Earth and without (with) rotation. The Earth self-spinning changes the uniform rigid spherical into ellipsoidal Earth. Since the normal gravity is a single z-directional vector along $\mathbf{K}$, the normal geopotential (Φ_N) was derived by the integration of Equation 1 with respect to z from $z = 0$ (Haurwitz, 1941)

(3)

With the hydrostatic balnce in the z-direction for large scale motion,

(4)

a relationship was established between pressure (p) and the (normal) geopotential () from Equations 3 and 4

(5)

which provides the foundation in meteorology for the use of geopotential, geopotential height, and pressue coordinate system.

Since the true Earth is not a pure solid sphere/elliposod and since the mass distribution inside the true Earth is nonuniform, the true gravity linked to the true Earth is represented by a 3D vector field, $\mathbf{g} = \mathbf{g}_h + \mathbf{k}g_z$ ($\mathbf{g}_h$ $\mathbf{i}g$ +$\mathbf{j}g$). It has lateral (,) [or (x, y)] component $\mathbf{g}_h$, which is common knowledge and described in any graduate-level geodesy textbook. Interested readers are referred to Vaniček and Krakiwsky (1986). The true geopotential (Φ) assocaied with the true gravity is the sum of the normal geopotential (Φ_N) and the disturbing geopotential potential [-T(, , z)] (Hackney and Featherstone, 2003)

. (6)

where T(, , z) is the disturbing gravity potential in geodesy. Due to the spectral characteristics of T(, , z), Chu (2021b) obtained an approximated relation

(7)

for the whole troposphere with $H = 10.4$ km, being the height of the troposphere. Besides, is related to the geoid height N by the Bruns' formula (Sandwell and Smith, 1997; Chu, 2018, 2021c),

(8)

Consensus has been reached in the meteorological community that the difference between Φ_N and Φ_0 is negligible (e.g., Gill, 1982; Vallis, 2006)

(9)

However, the difference between Φ and Φ_0 has never been investigated. To fill such a gap, the disturbing geopotential

(10)

is calculated from openly available datasets to determine its importance. Here, Equations 6-9 are used. The rest of the paper is outlined as follows. Section 2 provides the dynamic equation with the true geopotential as well as the non-dimensional B and C numbers. Section 3 derives the new equation for the geostrophic wind. Section 4 intordouces the data sources. Sections 5 and 6 demonstarte the relative importance of the disturbing geopotential versus the standard/normal geopotential and the Corioilis force. Section 7 presents the conclusions.

2 Equation of Motion with the True Geopotential

With the true gravity $\mathbf{g}$, the longitudinal-latitudinal component of the dynamic equation in the pressure coordinates is given by (Chu, 2021a, b)

(11)

where $\mathbf{U} = (u, v)$, is the longitudinal-latitudinal velocity vector; $\Omega = \mathbf{\Omega} =$ 2 /(86164 s) is the Earth rotation rate; $\mathbf{F}$ is the friction force;

(12)

is the 2D vector differential operator; and

(13)

Here, is the vertical velocity in the pressure coordinates. In the pressure coordinates the standard geopotential height (Z_0) and the normal geopotential height (Z_N) are given by

(14)

Substition of (14) into (10) leads to the true geopotential,

(15)

Substitution of (15) into (11) gives

(16)

Two non-dimensional numbers B and C are defined to idedify the importance of in comparison to the pressue gradient force () and Coriolis force,

(17)

where $f = 2\Omega\sin$, is the Coriolis paramenter. If the global mean is taken to represent the order of magnitude, Equation 17 becomes

(18)

The true geopotential Φ cannot be replaced by the standard geopotential Φ_0 when the non-dimensional B and C number are not small.

3 Geostrophic Wind

Steady state flow without friction leads to the geostrophic balance

(19)

which can be rewritten by

(20)

The geostrophic wind can be decomposed into two parts, $\mathbf{U}_g = \mathbf{U}_{g0} + \mathbf{U}_g$, with

(21)

representing the classical geostrophic wind; and

(22)

denoting the geostrophic wind due to the disturbing geopotential Φ. Obviously, with Equation 17 the non-dimensional B number is also the ratio between $O(\mathbf{U}_g)$ and $O(\mathbf{U}_{g0})$,

(23)

4 Data Sources

Two datasets: (a) ICGEM global static gravity model EIGEN-6C4 (http://icgem.gfz-potsdam.de/home) (Kostelecký et al., 2015), and (b) NCEP/NCAR reanalyzed monthly long-term mean standard geopotential height (Z_0) and wind velocity (u, v) at 12 pressure levels 1,000, 925, 850, 700, 600, 500, 400, 300, 250, 200, 150, and 100 hPa (https://psl.noaa.gov/data/gridded/data.ncep.reanalysis.derived are used to calculate , and (,f**U**) at these pressure levels. In turn the B and C numbers as well as the geostrophic wind are calculated. The EIGEN-6C4 geoid data (N) is on 1°×1° grids, and the NCEP/NCAR reanalyzed data (Z_0, u, v) are on 2.5°×2.5° grids. The annual long-term mean (Z_0, u, v) are calculated from the 12 monthly long-term mean data before computing the (B, C) numbers and the geostrophic wind.

5 Disturbing Versus Standard Geopotential

The global geoid N from the static gravity model EIGEN-6C4 (Figure 1a) has the minimum of -106.20 m, the maximum of 85.83 m, and the mean of 30.57 m. The gradient and its magnitude are calculated from the global N data. The vector (Figure 1b) represents the nondimensional latitudinal-longitudinal gradient of the disturbing geopotential at $z = 0$. The histogram of (Figure 1c) shows a positively skewed diftribution with a long tail and mean of 2.36010^{-5}.

The global long-term annual mean standard geopotential height Z_0 has comparable latitudinal-longitudinal veariability at 1,000 hPa (Figure 2a) as the geoid N (Figure 1a), and becomes near-zonal at 500 hPa (Figure 2b) and 100 hPa (Figure 2c). The vector represents the nondimentsional latitudinal-longitudinal gradient of the standard geopotential at 1,000 hPa (Figure 3a), 500 hPa (Figure 3b), and 100 hPa (Figure 3c). The two vector fields (Figure 1b) and (Figure 3) are quite different. The difference becomes larger as p-level decreasing from 1,000 hPa (Figure 3a) to 100 hPa (Figure 3c). is aligned nearly latitudinal at 500 hPa (Figure 3b) and 100 hPa (Figure 3c). The histograms of show positively skewed diftributions with mean of 5.82410^{-5} at 1,000 hPa (Figure 4a), 8.26510^{-5} at 500 hPa (Figure 4b), and 1.26210^{-4} at 100 hPa (Figure 4c). Table 1 shows the global mean of , at 12 p-levels from 1,000 hPa to 100 hPa and corresponding B-numbers. The B-number has a maximum of 0.4176 at 850 hPa and a minimum of 0.1630 at 200 hPa, and is greater than 0.28 for 1,000-500 hPa. It clearly shows that the disturbing geopotential (Φ) cannot be neglected against the standard geopotential (Φ_0), and the new Equation 19 should be used for the geostrophic wind.

6 Lateral Gradient of Disturbing Geopotential Versus Coriolis Force

The global long-term annual mean wind vectors (**U**) show the general circulation at 1,000 hPa (Figure 5a), 500 hPa (Figure 5b), and 100 hPa (Figure 5c). The magnitudte of the Coriolis force is represented by . The histogram of shows a positively skewed diftribution and mean of 37.53 mGal (1 mGal = 10^{-5} m s^{-2}) at 1,000 hPa (Figure 6a), 83.19 mGal at 500 hPa (Figure 6b), and 126.0 mGal at 100 hPa (Figure 6c). The global mean of the lateral gradient of disturbing geopotential () is 23.15 mGal. Table 2 shows the global mean of , f**U** at 12 p-levels from 1,000 hPa to 100 hPa and corresponding C-numbers. The C-number has a maximum of 0.6168 at 1,000 hPa, a minimum of 0.1573 at 200 hPa, and is greater than 0.27 for 1,000-500 hPa. It clearly shows that the lateral gradient of disturbing geopotential cannot be neglected against the Coriolis force. It implies that the importance of using the true geopotential instead of the standard/normal geopotential in atmospheric dynamics.

7. Conclusions

Geopotential (i.e., gravity potential in geodesy with the opposite sign) has three forms: the standard geopotential (Φ_0) with the standard gravity (-$g_0\mathbf{k}$), the normal geopotential (Φ_N) with the normal gravity [-$\mathbf{K}$], and the true geopotential (Φ) with the true gravity $\mathbf{g} = \mathbf{i}g + \mathbf{j}g + \mathbf{k}g_z$. Meteorologists only use the normal/standard geopotential, but not the true geopotential. The disturbing geopotential, $\Phi = \Phi$- Φ_0, has never been presented and discussed in the meteorological community.

To demonstrate the importance of disturbing geopotential in atmospheric dynamics, two non-dimensional numbers are difined to identify its importance versus the pressure gradient force (B-number) and the Coriolis force (C-number). The two openly available datasets (a) ICGEM EIGEN-6C4 and (b) NCEP/NCAR Reanalysis long term mean data are used to compute the B number (0.4617 at 850 hPa to 0.1630 at 200 hPa) and C number (0.6168 at 1,000 hPa to 0.1573 at 200 hPa) for the troposphere. Both B and C numbers are greater than 0.27 for 1,000-500 hPa. These values show the importance to use the true geopotential Φ especially in lower atmosphere (1,000-500 hPa) in meteorology. A new equation for the geostrophic wind is also obtained.

Acknowledgements

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Data Availability Statement

The data used in this paper are publicly available with the geoid (N) from the ICGEM global static gravity model EIGEN-6C4 at http://icgem.gfz-potsdam.de/home, and the monthly long-term mean (normal/standard) geopotential height (Z_0) as well as the wind vector ($\mathbf{U}$) at 12 pressure levels (1,000 to 100 hPa) from the NCEP/NCAR Reanalysis at https://psl.noaa.gov/data/gridded/data.ncep.reanalysis.derived.html.

References

Chu, P.C., 2018: Two types of absolute dynamic ocean topography. *Ocean Sci.*, **14**, 947-957,

https://doi.org/10.5194/os-14-947-2018.

Chu, P.C., 2021a: Ocean dynamic equations with the real gravity. *Nature Sci. Rep.*, **11**, Article Number 3235, https://doi.org/10.1038/s41598-021-82882-1.

Chu, P.C., 2021b: True gravity in atmospheric Ekman layer dynamics. *J. Geophys. Res.*, **126**,

e2021JD035293, https://doi.org/10.1029/2021JD035293.

Chu, P. C., 2021c: World ocean annual mean absolute geostrophic velocity on marine geoid of EIGEN-6C4 from WOA13: *Geosci. Data J.*, 00: 1-9, Doi:10.1002/gdj3.124.

Chu, P.C., 2021d: True gravity in ocean dynamics Part 1 Ekman transport. *Dyn. Atmos. Oceans.*, **96**, 101268, https://doi.org/10.1016/j.dynatmoce.2021.101268.

Gill, A.E., 1982. Atmosphere-Ocean Dynamics. International Geophysics Series, Vol 30,

Academic Press, New York, 91-94.

Hackney, R.I., and W.E. Featherstone, 2003: Geodetic versus geophysical perspectives of the

'gravity anomaly'. *Geophys. J. Int.* **154**, 35-43.

Haurwitz, B., 1941: *Dynamic Meteorology*, McGraw-Hill Book Company, New York, 1-12.

Kostelecký, J., J. Klokočnìk, B. Bucha, A. Bezděk, and C. F rste, 2015: Evaluation of the

gravity model EIGEN-6C4 in comparison with EGM2008 by means of various

functions of the gravity potential and by BNSS/levelling. Geoinformatics FCE CTU,

14 (1), http://doi.org//10.14311/gi.14.1.1.

National Geospatial-Intelligence Agency, 1984: *Department of Defense World Geodetic System*

1984 (WGS84). Its definition and relationships with local geodetic systems. NIMA

TR8350.2., 3rd Edition, Equation 4-1.

Sandwell, D.T., and W.H.F. Smith, 1997: Marine gravity anomaly from Geosat and ERS 1

satellite altimetry. *J. Geophys. Res.*, **102**, 10,039-10,054.

Vallis, G.K., 2006: *Atmospheric and Oceanic Fluid Dynamics* (First Edition), Cambridge

University Press, Cambridge, 55-57.

Vaniček, P, and Krakiwsky, E., 1986: *Geodesy: the Concepts*. Part 5 Earth gravity field,

North-Holland, Amsterdam, 457-581.

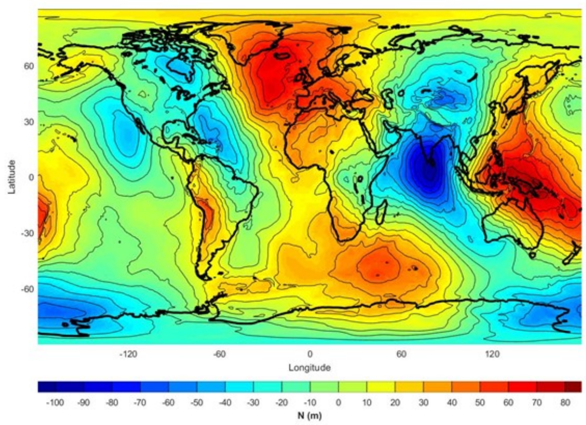
Table 1. Global means (treated as the orders of magnitudes) of and annual at 12 pressure levels in the troposphere as well as the non-dimensional *B*-number.

Pressure Level (hPa)	Mean () (10^{-5})	Mean () (10^{-5})	*B-number*
		2.360	
1,000	5.824		0.4052
925	5.686		0.4151
850	5.651		0.4176
700	6.153		0.3836
600	6.870		0.3435
500	8.265		0.2855
400	10.27		0.2298
300	12.68		0.1861
250	13.78		0.1713
200	14.48		0.1630
150	14.40		0.1639
100	12.63		0.1869

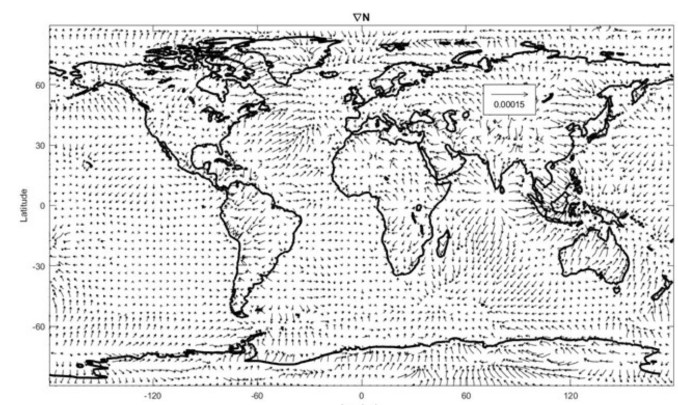
Table 2. Global means (treated as the orders of magnitudes) of and annual at 12 pressure levels in the troposphere as well as the non-dimensional *C*-number.

Pressure Level (hPa)	Mean () (mGal)	Mean (g_0) (mGal)	*C-number*
		23.15	
1,000	37.53		0.6168
925	45.52		0.5086
850	49.01		0.4724
700	60.46		0.3829
600	69.59		0.3327
500	83.19		0.2783
400	103.3		0.2241
300	128.8		0.1797
250	140.7		0.1645
200	147.2		0.1573
150	143.7		0.1611
100	126.0		0.1837

(a) N



(b) ∇N



(c) Histogram of $|\nabla N|$

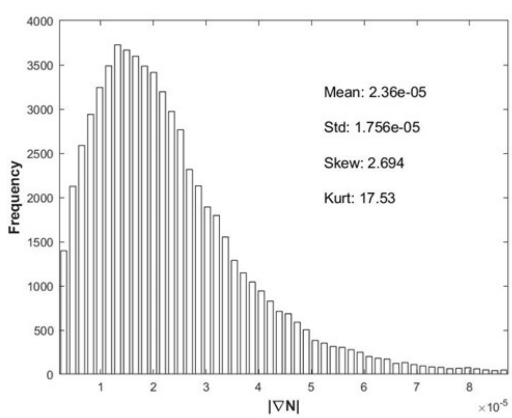
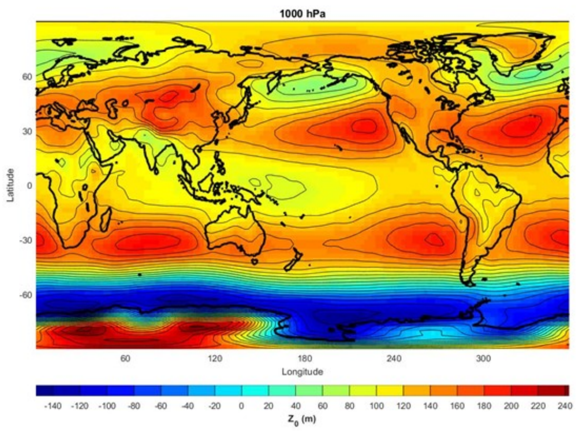
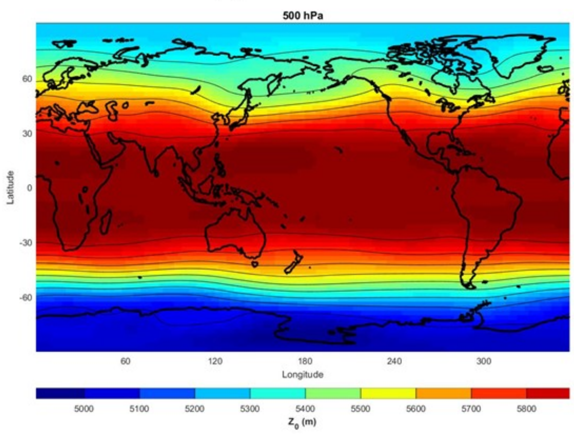


Figure 1. (a) Geoid N with $1^{\circ} \times 1^{\circ}$ resolution, obtained online at the website: http://icgem.gfz-potsdam.de/home, (b) vector plot of , and (c) histogram of with four statistical parameters (mean, standard deviation, skewness, kurtosis).

(a) 1,000 hPa



(b) 5**00 hPa**



(c) 1**00 hPa**

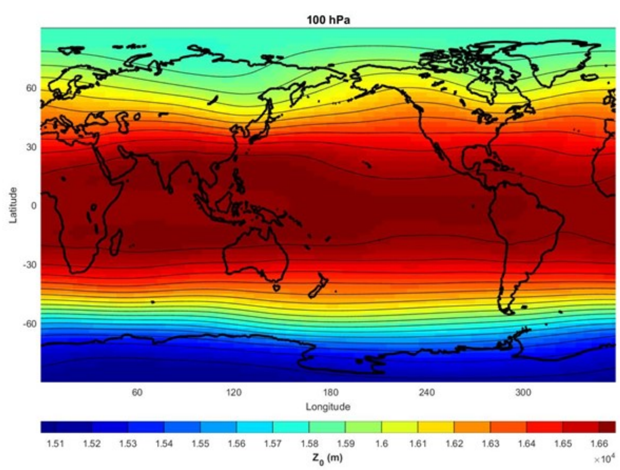
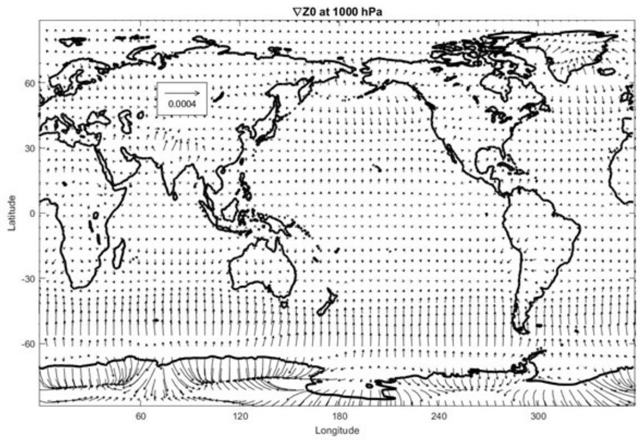
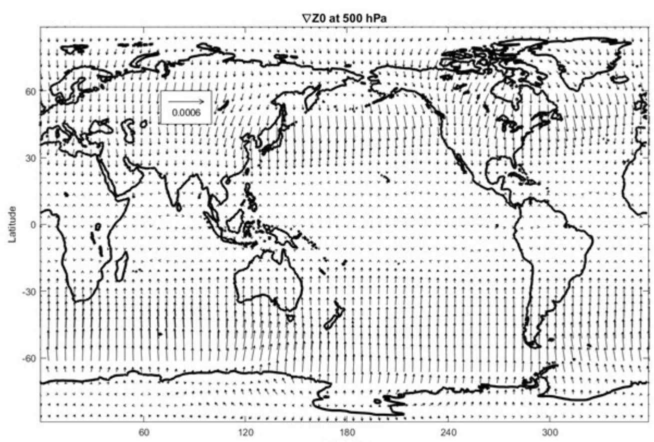


Figure 2. Long-term annual mean standard geopotential height Z_0 with $2.5^o \times 2.5^o$ resolution on the three pressure levels (a) 1,000 hPa, (b) 500 hPa, and (c) 100 hPa. The data were calculated from the long-term monthly mean standard geopotential height Z_0 obtained online at the website: https://psl.noaa.gov/data/gridded/data.ncep.reanalysis.derived.html.

(a) 1,000 hPa



(b) 500 hPa



(c) 100 hPa

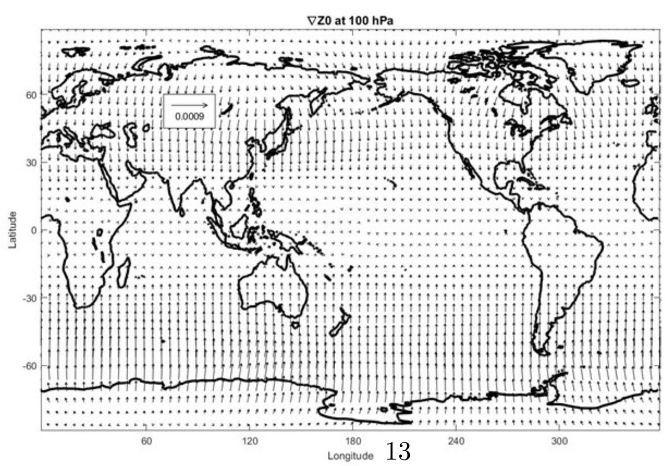
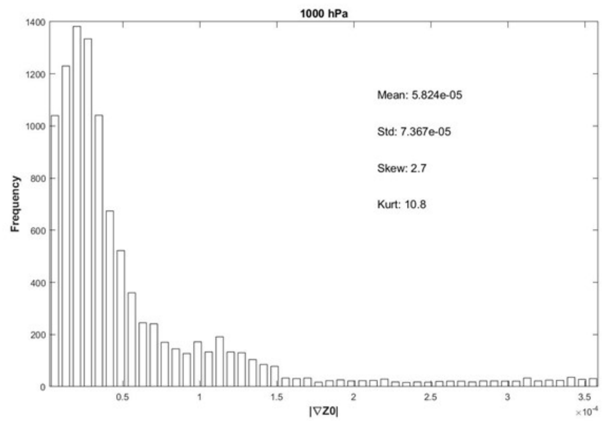
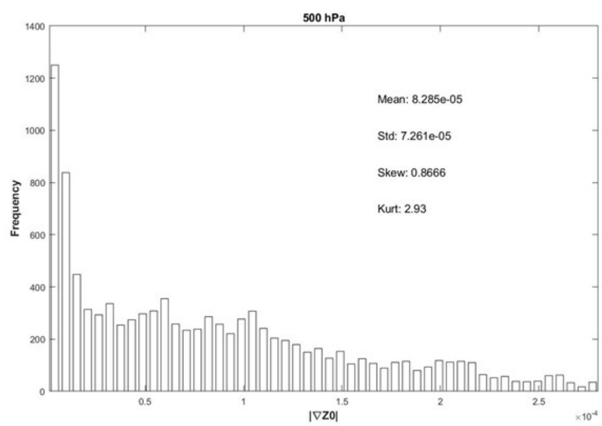


Figure 3. Vector plots of long-term annual mean standard geopotential height on (a) 1,000 hPa, (b) 500 hPa, and (c) 100 hPa.

(a) 1,000 hPa



(b) 500 hPa



(c) 100 hPa

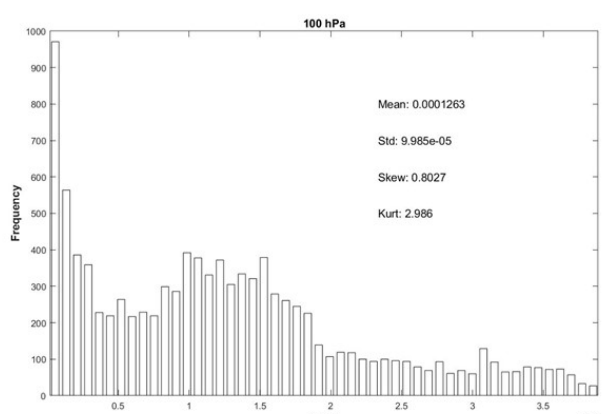
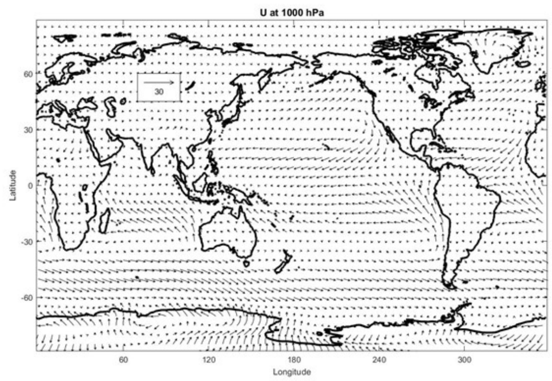
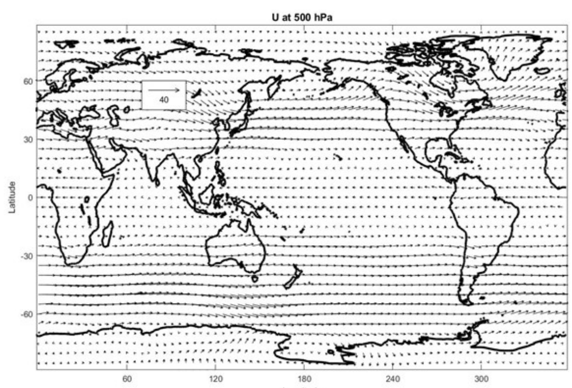


Figure 4. Histograms with four statistical parameters (mean, standard deviation, skewness, kurtosis) of long-term annual mean standard geopotential height on (a) 1,000 hPa, (b) 500 hPa, and (c) 100 hPa.

(a) **1,000 hPa**



(b) **500 hPa**



(c) **100 hPa**

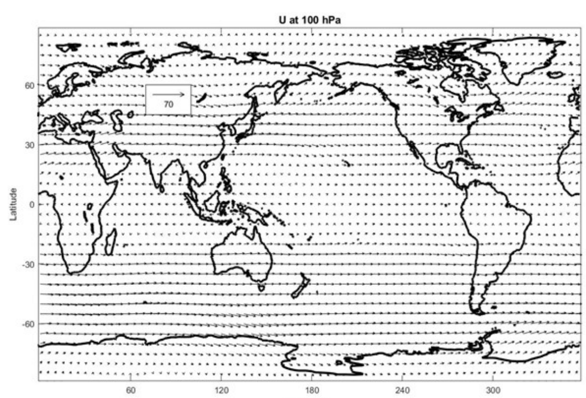
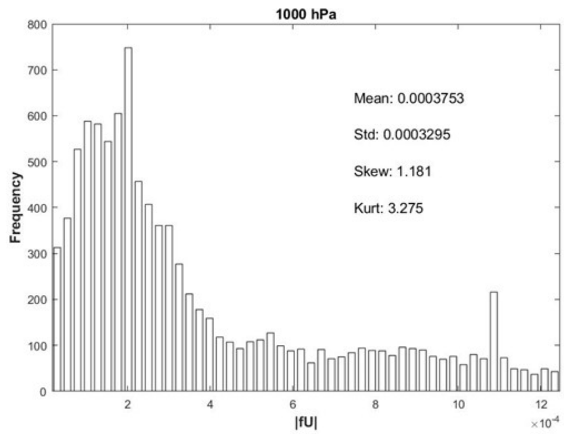
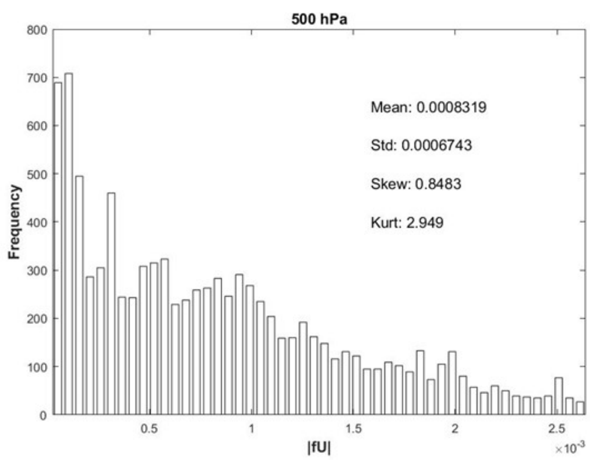


Figure 5. Plots of long-term annual mean wind vector vecotrs (**U**) on (a) 1,000 hPa, (b) 500 hPa, and (c) 100 hPa. The data were calculated from the long-term monthly mean wind vector **U** obtained online at the website: https://psl.noaa.gov/data/gridded/data.ncep.reanalysis.derived.html.

(a) 1,000 hPa



(b) 500 hPa



(c) 100 hPa

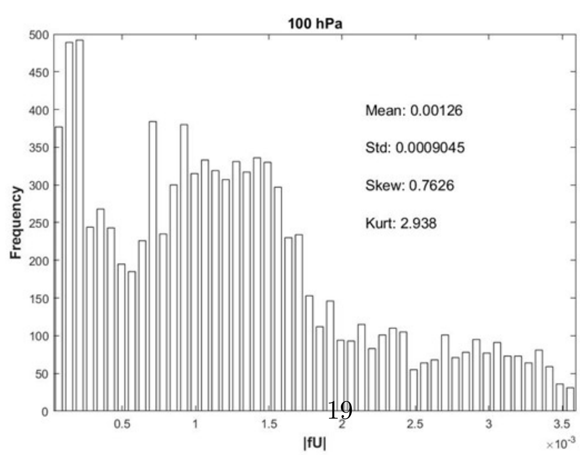


Figure 6. Histograms with four statistical parameters (mean, standard deviation, skewness, kurtosis) of long-term annual mean magnitude of the Coriolis force $f\mathbf{U}$ on the three pressure levels: (a) 1,000 hPa, (b) 500 hPa, and (c) 100 hPa.