Supporting Information for “Flow aware parameterizations invigorate the ocean circulation under the Pine Island ice shelf, West Antarctica”

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Text S1. Optimal interpolation methodology

The prescribed ocean state at the western boundary of the computaional domain serves as an important forcing mechanism for the ice shelf and ocean circulation. Our goal is to determine the most realistic values for the temperature, salinity, and normal velocity fields at the boundary, given the available CTD and LADCP observations during 2009 and 2014. To do this in a relatively straightforward fashion, we find the solution to the optimal interpolation (OI) problem for the generic parameter field $\mathbf{m} \coloneqq [\boldsymbol{\theta}_W^T, \mathbf{S}_W^T, \mathbf{u}_W^T]^T \in \mathbb{R}^{N_m}$:

$$\mathbf{m}_{OI} = \underset{\mathbf{m}\in\mathbb{R}^{N_m}}{\arg\min}\, \mathcal{J}(\mathbf{m}) \tag{1}$$

where

$$\mathcal{J}(\mathbf{m}) = \frac{1}{2}||f(\mathbf{m}) - \mathbf{d}||^2_{\Gamma^{-1}_{\text{Obs}}} + \frac{1}{2}||\mathbf{m} - \mathbf{m}_0||^2_{\Gamma^{-1}_{\text{prior}}}.$$

Here $f : \mathbb{R}^{N_m} \ni \mathbf{m} \to \mathbf{d} \in \mathbb{R}^{N_d}$ is simply a linear interpolation operator, mapping the parameter fields to the location of available data.

As a matter of computational convenience we make the following assumptions. First, we assume that each parameter field is independent from one another, allowing us to solve three OI problems for temperature, salinity, and velocity separately. Second, we assume that the observational and prior uncertainties can be described by Gaussian statistics. We further assume that the observations are independent, such that $\Gamma_{\text{Obs}} = \text{diag}\{\sigma_i^2\}_{i=1}^{N_{\mathbf{d}}}$. Observational uncertainties (standard deviations) are assumed to be 0.5°C for potential temperature and 0.05 g/kg for salinity as they are not provided, see Appendix A in the main text for details. The LADCP velocity data is provided with uncertainty estimates, which we use.

We specify the prior covariance as Matérn class due to the link between Matérn class Gaussian fields and the solution of the elliptic stochastic partial differential equation

(Lindgren et al., 2011):

$$\Big(\delta(\mathbf{x}) - \nabla \cdot K(\mathbf{x})\nabla\Big) m(\mathbf{x}) = \mathcal{W}(\mathbf{x}) \qquad \mathbf{x} \in \partial\Omega_{OBW} \,, \tag{2}$$

where $\mathcal{W}(\mathbf{x})$ is a standard white noise process. We employ the empirical relationship provided in Lindgren et al. (2011) and choose $\delta(\mathbf{x})$ and $K(\mathbf{x})$ such that the parameter fields exhibit a correlation of 0.1 at separation lengths: 18 km meridionally and 150 m vertically.

The last ingredient is the initial guess for the OI problem, $\mathbf{m}_0$. Simple inspection of the temperature and salinity data shows that these fields have mostly vertical structure, with slight variations in the depth of thermocline and halocline due to their horizontal location. Therefore, we specify $\boldsymbol{\theta}_0$ and $\mathbf{S}_0$ as vertical profiles based on polynomial regressions of the data. We note that using this has similar results to specifying $\boldsymbol{\theta}_0 = 0°\mathrm{C}$ and $\mathbf{S}_0 = 34.36$ g/kg, but the former provides a better fit to the observations. The spatial structure of the velocity data is less obvious *a priori* and we therefore specify $\mathbf{u}_0 = 0$ m/s.

Given these assumptions and specifications, the minimization problem in equation (1) is linear and we can write the solution to each independent OI problem as:

$$\boldsymbol{\theta}_{OI} = \boldsymbol{\theta}_0 + \Gamma_{\text{post}} F^T \Gamma_{\text{Obs}}^{-1} \Big(\mathbf{d} - F\boldsymbol{\theta}_0\Big)$$
$$\Gamma_{\text{post}} = \left(F^T \Gamma_{\text{Obs}}^{-1} F + \Gamma_{\text{prior}}^{-1}\right)^{-1} .$$

Here, potential temperature is shown as an example, and a similar solution is obtained for salinity and velocity. Before these results can be used directly as forcing for the ocean model, the spatial integral is removed from the zonal velocity:

$$u_W(\mathbf{x}) = u_{OI} - \int_{\partial\Omega_{\text{open}}} u_{OI} \, d\mathbf{x} \, .$$

Removing the spatial mean ensures that we do not add or remove mass from the domain, and there is no artificial sea level rise during the spinup to reach equilibrium. In practice, this corresponds to removing a small average velocity: 0.00943 m/s. The resulting fields are shown in comparison to the observational data in Figure S1.

References

Lindgren, F., Rue, H., & Lindström, J. (2011). An explicit link between gaussian fields and gaussian markov random fields: the stochastic partial differential equation approach. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *73*(4), 423–498. Retrieved from `http://dx.doi.org/10.1111/j.1467-9868.2011.00777.x` doi: 10.1111/j.1467-9868.2011.00777.x

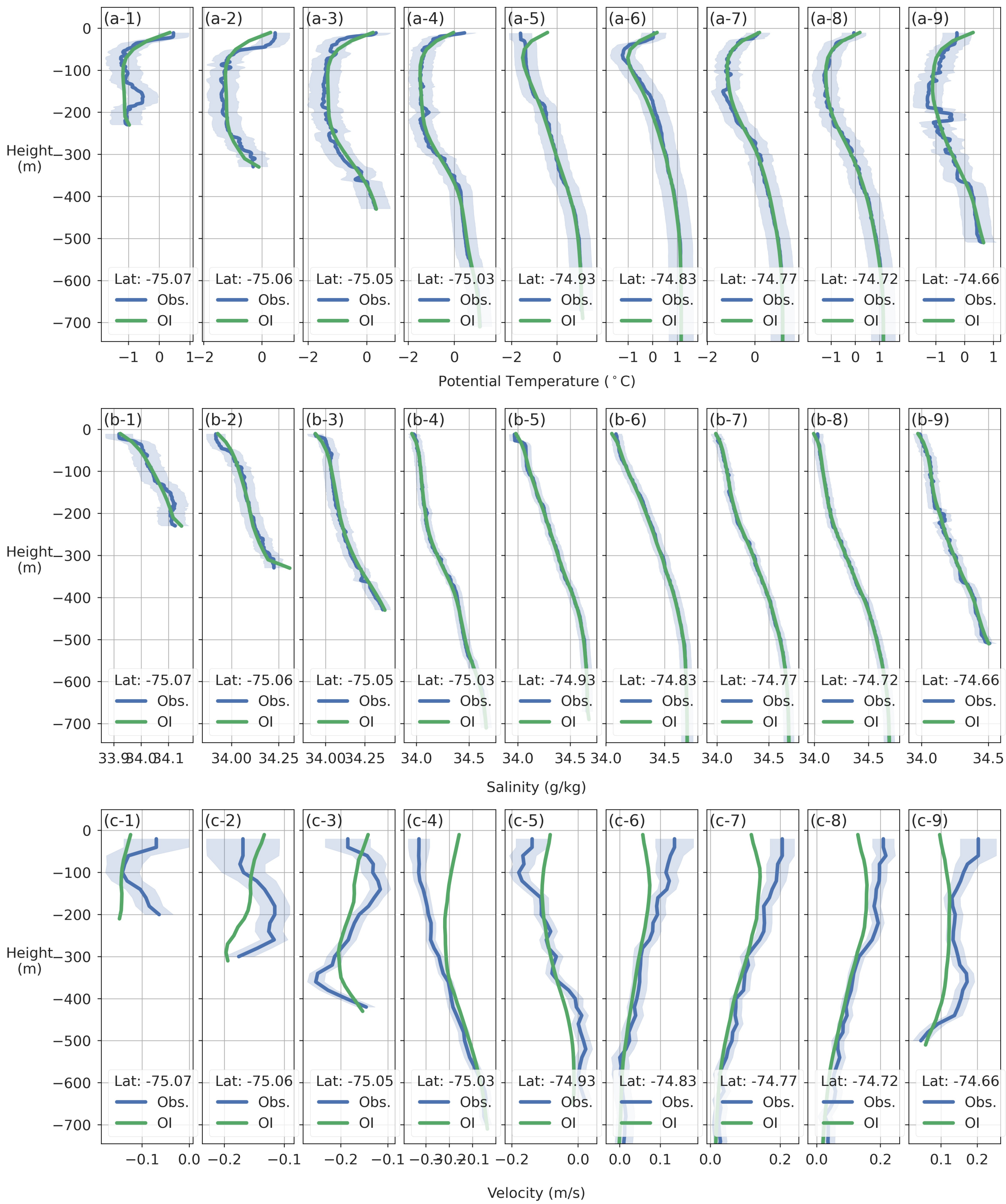

Figure S1. Optimal interpolation results (OI; green line) compared to data (Obs; blue line) at all CTD/LADCP locations used to compute open the open boundary conditions. (a-1 – a-9) potential temperature, (b-1 – b-9) salinity, (c-1 – c-9) zonal velocity.