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RESEARCH ARTICLE

Asymptotical outer synchronization control for the complex dynamical networks with unknown bounded interaction via links dynamics

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Abstract

The nodes and their connection relationships (CRs) are the two main parts for a complex dynamical network (CDN). In existing theoretical studies about the outer synchronization, the nodes are considered as the main part of synchronization phenomena mainly associated by coupling effect of CRs between nodes. However, if the CRs between nodes are time-varying, they can also be regarded as one dynamic system coupled with the nodes, and thus their state may evolve with time and maybe assist the nodes to achieve the outer synchronization. From the angle of large-scale systems, a CDN can be regarded as two interconnected subsystems, one of which is the node subsystem (NS) and the other is the link subsystem (LS). Hence, how the whole dynamic of LS contributes to the outer synchronization of NS is one of worthy research problem. In this paper, the two CDNs are considered with the unknown interaction. In each CDN, the dynamics of NS is modelled as the vector differential equation, the LS is modelled as the Riccati matrix differential equation, and the two kinds of differential equations are coupled with each other. By employing the above dynamic models of CDNs and the synthesized coupling terms in the two LSs, the adaptive controller of NS is synthesized for the response CDN. The results show that the outer synchronization happens when the two LSs tracking the synthesized auxiliary dynamic tracking targets. Finally, the numerical simulation is given to show the effectiveness of the theoretical results in this paper.

KEYWORDS:
complex dynamical network (CDN), node subsystem (NS), link subsystem (LS), auxiliary tracking targets, outer synchronization

1 | INTRODUCTION

In the past decades, synchronization, as an important collective behavior of complex dynamic network (CDN), has been widely used in various applications, such as social networks[1, 2], intelligent transportation[3, 4], power networks[5, 6], biological neural networks[7, 8]. Therefore, the research on the synchronization of CDNs attracts much attention of researchers in recent years[9–18]. In the existing literature, most studies regard the synchronization as the dynamic behavior of nodes, in which the connection relationship (CR) between nodes plays only the auxiliary role on assisting nodes to achieve synchronization. The whole dynamics of CR is ignored.

In fact, the CRs between nodes can also be regarded as one dynamic system coupled with the nodes[19]. This phenomenon also exists in some real networks. For example, Gamma oscillations in neurons (nodes) cause interneuron synapses (CR) to become facilitation in neural networks[20, 21], the tripping of transmission lines (CR) influence the power supply efficiency in some stations(nodes) of power grid[22, 23], the change of motors (nodes) speed will change the tension value(CR) between motors in industrial winding system[24, 25]. Therefore, the whole dynamics of CR should be considered and integrated into the synchronization of overall network.

From the perspective of large-scale system, a CDN can be considered as the interconnected system with the node subsystem (NS) and the link subsystem (LS), which implies that NS and LS are the two main parts to emerge the dynamical behavior of CDN[26, 27]. Therefore, the whole synchronization of network is influenced by not only the dynamics of NS but also the dynamics of LS. According to the above view, the achievements in [9–18] could be explained that the links assisted the nodes to emerge the synchronization, but the whole dynamics of LS was ignored. In [28, 29], the whole dynamics of LS is considered to investigate the state synchronization of NS by designing the controller of NS and coupling mechanism of LS, but the obtained results is only suitable to the same network. As far as we know, in the existing literature, the most studies of the synchronization of CDNs are limited to the same network (called inner synchronization), and it is regrettable that the outer synchronization of CDN is rarely considered with the whole dynamics of LS. Hence, it is very necessary to study the outer synchronization by using the whole dynamics of LS.

The outer synchronization is the phenomenon occurring in different networks, in which the state variables of corresponding nodes between different networks tend to be the same. At present, there are many research results for the outer synchronization of CDN such as the complete outer synchronization[14–16], the hybrid outer synchronization[17, 18], the projective outer synchronization [30, 31], and the generalized outer synchronization[32, 33]. However, it is noticed that the whole dynamics of links is ignored in the above-mentioned literature. On the other hand, there exists the interaction between the involved CDNs in reality, for example, the niche overlap interaction between the two biological communities[14], and the cascading interaction between the two power networks in smart grid[34]. Therefore, when investigating the outer synchronization behaviors of the two CDNs, the interaction between them should be considered. It is noticed that in [35–37], the interaction functions between CDNs are required to be known. Obviously, this requirement is too strict in the sense of application due to the interconnection between nodes interfered by the disturbances of nonlinear signals or hub nodes. To the best of our knowledge, it is seldom in the existing literature to comprehensively consider both the unknown interaction of NS and the whole dynamics of LS for investigating the outer synchronization.

Inspired by the above analysis, this paper focuses mainly on the whole dynamics of LS and the unknown interaction of NS, by which the asymptotically outer synchronization is investigated for two CDNs, one of which CDN is called as the drive network (DN), and another called as response network (RN). Compared with the existing literature, the key advantage of this paper is that **(i)** the whole dynamics of LS in each CDN is modeled by the Riccati matrix differential equation with the coupling to NS, in which the unknown interacting and inner coupling functions are mathematically represented. By employing the above model of CDN, the adaptive controller of NS in RN and the coupling terms in the two LSs are synthesized to ensure the outer synchronization happens asymptotically. **(ii)** Since it is usually difficult to measure the state of LS accurately in engineering applications, that the state of LS is unavailable is required in this paper. This requirement increases the difficulty of designing control scheme, and thus it also highlights the novelty of method in this paper. In fact, if two CDNs are considered with the unknown interaction, the advantage of this paper proposes one of solutions to the problem: how can the controlled NS and LS be coupled to facilitate the outer synchronization. In this solution, the synthesized auxiliary dynamic tracking targets of LSs play the important role.

The rest of the paper is organized as follows. In Section 2, we propose two CDN models with unknown interactions in which the NS and the LS are mutually coupled, and the corresponding mathematical assumptions are required. In Section 3, the adaptive controller is proposed for the NS in RN, and the two coupling terms in the equations of LSs are synthesized with the help of their auxiliary dynamic tracking targets, such that the asymptotic outer synchronization are achieved. In Section 4, the effectiveness of our control method is verified by the simulation example. Finally, we give conclusions and expectations in Section 5.

2 | PRELIMINARIES AND MODEL DESCRIPTION

We consider the two directed CDNs Σ and $\bar{\Sigma}$, each with N nodes. $x_{ji} = x_{ji}(t)$ denotes the connection weight of the jth node directing to the ith node in the network Σ, and $\bar{x}_{ji} = \bar{x}_{ji}(t)$ denotes the connection weight of the jth node directing to the ith node in the network $\bar{\Sigma}$. In this paper, the self-connecting weight $x_{ii}(t)$ and $\bar{x}_{ii}(t)$ are allowed, where $i, j = 1, 2, \cdots, N$, the network Σ and $\bar{\Sigma}$ are called as DN and RN, respectively.

The dynamical of NS in DN is expressed as follows

$$\dot{z}_i = f_i(z_i, t) + c_1 \sum_{j=1}^{N} x_{ji}(t) h_j(z) + \bar{c}_1 \varphi_i(\bar{z}, t) \tag{1}$$

and the dynamical of LS is expressed as

$$\dot{X} = PX + XP^T + \Psi(z, \bar{z}) \tag{2}$$

where $z_i = (z_{i1}, z_{i2}, \cdots, z_{in})^T \in R^n$ is the state vector for the ith node, $z = (z_1^T, z_2^T, \cdots, z_N^T)^T \in R^{nN}$, $c_1 > 0$ and $\bar{c}_1 > 0$ are the common coupling coefficients relative to DN and RN, respectively. $f_i(z_i, t) \in R^n$ is the nonlinear continuous vector function, $h_j(z) \in R^n$ is the internal associative function of DN, $\varphi_i(\bar{z}, t) \in R^n$ represents the state interaction function on the ith node by the internal state $\bar{z}_i \in R^n$ of RN. $X = [x_{ij}(t)]_{N\times N}$ is the CR matrix in DN, which is called to be the state matrix of LS. $\bar{z}_i \in R^n$ denotes the state of ith node in RN, $\bar{z} = (\bar{z}_1^T, \bar{z}_2^T, \cdots, \bar{z}_N^T)^T \in R^{nN}$, $i, j = 1, 2, \cdots, N$. $P \in R^{N\times N}$ is constant matrix, $\Psi(z, \bar{z}) = M(t) + \Theta(z, \bar{z})$ represents the coupling term of LS in DN, where $M = M(t) \in R^{N\times N}$ is a known bounded time-varying matrix, $\Theta(z, \bar{z}) \in R^{N\times N}$ is an unknown coupling state matrix.

The dynamical of NS in RN is expressed as follows

$$\dot{\bar{z}}_i = \bar{f}_i(\bar{z}_i, t) + \bar{c}_2 \sum_{j=1}^{N} \bar{x}_{ji}(t) \bar{h}_j(\bar{z}) + c_2 \bar{\varphi}_i(z, t) + \bar{u}_i \tag{3}$$

and the dynamical of LS is expressed as

$$\dot{\bar{X}} = \bar{P}\bar{X} + \bar{X}\bar{P}^T + \bar{\Psi}(z, \bar{z}) \tag{4}$$

where $\bar{u}_i = (\bar{u}_{i1}, \bar{u}_{i2}, \cdots, \bar{u}_{in})^T \in R^n$ is the control input for the ith node of RN, $\bar{c}_2 > 0$ and $c_2 > 0$ are the common coupling coefficients relative to RN and DN, respectively. $\bar{f}_i(\bar{z}_i, t) \in R^n$ is the nonlinear continuous vector function, $\bar{h}_j(\bar{z}) \in R^n$ is the internal associative function of RN, $\bar{\varphi}_i(z, t) \in R^n$ represents the state interaction function on the ith node by the internal state $z_i \in R^n$ of DN, $i, j = 1, 2, \cdots, N$. $\bar{X} = [\bar{x}_{ij}(t)]_{N\times N}$ is the CR matrix in RN, which is called to be the state matrix of LS. $\bar{P} \in R^{N\times N}$ is constant matrix, $\bar{\Psi}(z, \bar{z}) = \bar{M}(t) + \bar{\Theta}(z, \bar{z})$ represents the coupling state term of LS in RN, where $\bar{M} = \bar{M}(t) \in R^{N\times N}$ is a known bounded time-varying matrix, $\bar{\Theta}(z, \bar{z}) \in R^{N\times N}$ is an unknown coupling state matrix.

Remark 1. **(i)** There are two external interaction parts in the models (1) and (3), respectively ($\bar{c}_1 \varphi_i(\bar{z}, t)$ in (1), $c_2 \bar{\varphi}_i(z, t)$ in (3)), which represents the interaction between states of NS in two networks. For example, in the spread of epidemics, epidemics will spread at the infectious (one CDN) and preventive layers (another CDN) with interaction[14]. In the smart grid, the power grid (one CDN) relies on the information network (another CDN) for control, and the information network relies on the power grid for power supply [38]. This is different from the models in [14–18, 30–33]. **(ii)** The models (2) and (4) are composed of two parts. One is linear part ($PX + XP^T$ in (2), $\bar{P}\bar{X} + \bar{X}\bar{P}^T$ in (4)), which represents the dynamic state of LS. The other is coupling part with NS ($\Psi(z, \bar{z})$ in (2), $\bar{\Psi}(z, \bar{z})$ in (4)), which represents the state coupling relationship between LS and NS of the two networks. $\Psi(z, \bar{z})$ and $\bar{\Psi}(z, \bar{z})$ are composed of two parts. One is known bounded time-varying coupling part ($M(t)$ and $\bar{M}(t)$), the other is the unknown part ($\Theta(z, \bar{z})$ and $\bar{\Theta}(z, \bar{z})$) coupled with states of NS. To the best of our knowledge, structural features of the above two models have hardly been considered in the existing literature.

In order to geometrically explain the topological composition meaning of Equation (2) for LS in DN (similarly, Equation (4) for LS in RN), we can rewrite Equation (2).

$$\frac{dx_{ij}}{dt} = \sum_{k=1}^{N} p_{ik} x_{kj} + \sum_{k=1}^{N} x_{ik} p_{jk} + m_{ij}(t) + \theta_{ij}(z, \bar{z}) \tag{5}$$

where $P = (p_{kj})_{N\times N}$, $M = [m_{ij}(t)]_{N\times N}$, $\Theta(z, \bar{z}) = [\theta_{ij}(z, \bar{z})]_{N\times N}$, $\psi_{ij}(z, \bar{z}) = m_{ij}(t) + \theta_{ij}(z, \bar{z})$, $\Psi(z, \bar{z}) = [\psi_{ij}(z, \bar{z})]_{N\times N}$.

The structural composition of the dynamic Equation (5) can be illustrated in Fig.1 . Equation (5) implies that the change rate of x_{ij} is directly influenced by four parts, the first part is a linear combination about x_{kj}, the second part is linear combination about x_{ik}, the third part is a known time-varying effect, and the fourth part is the state term coupling with NSs of the two networks. From the network perspective, this means that the rate of change of the strength of CR between the ith node and the jth node in DN is affected not only by the linear superposition of the strength of CR between them both and other nodes, but also by the coupling of the two network states. The above structural features of Equation (5) also have a realistic network context. For example, in a population of robots with multiple Kilobots[39, 40], changes in the communication strength (x_{ij}) between any two of these Kilobots (the ith node and the jth node) are affected by the interaction of the other Kilobots (the kth node) with the communication between the ith node and the jth node. In winding systems with multiple motors[24, 25], the web tension (x_{ij}) change between two adjacent motors (the ith node and the jth node) are affected by the web tension of the other motors.

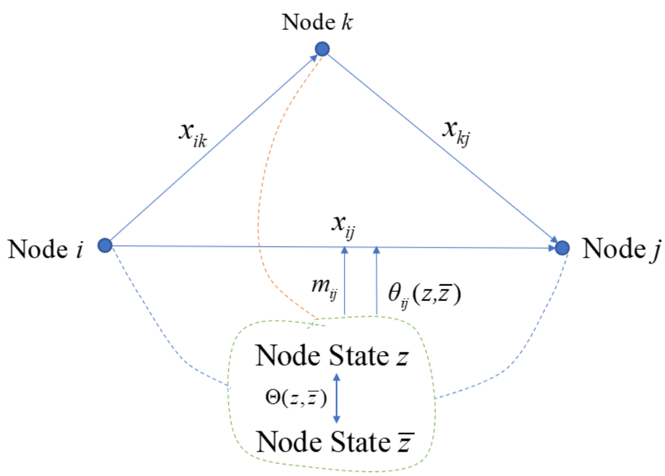

FIGURE 1 The graphical expression of the structural components of Equation (5)

Assumption 1. Consider the two CDNs with (1) and (3), where $h_j(z)$ and $\bar{h}_j(\bar{z})$ are known bounded. There exist unknown positive constant α_i and the known bounded continuous positive function $\bar{a}_i^*(\bar{z}_i,t)$ such that $\|f_i(z_i,t)\| \le \alpha_i$, $\|\bar{f}_i(\bar{z}_i,t)\| \le \bar{a}_i^*(\bar{z}_i,t)$ for $i,j = 1,2,\cdots,N$, where $\|*\|$ denotes the Euclidean norm of the vector "*" or matrix "*".

Assumption 2. Consider the two CDNs with (1) and (3), in which the state interaction functions $\varphi_i(\bar{z},t)$ and $\bar{\varphi}_i(z,t)$ are bounded. There exist the unknown positive constant θ_i and known bounded continuous positive function $\bar{b}_i(z,t)$ such that $\|\varphi_i(\bar{z},t)\| \le \theta_i$ and $\|\bar{\varphi}_i(z,t)\| \le \bar{b}_i(z,t)$ for $i,j = 1,2,\cdots,N$.

Assumption 3. Consider the two CDNs with (2) and (4), the symmetric matrices $P + P^T$ and $\bar{P} + \bar{P}^T$ are Hurwitz stable.

Remark 2. **(i)** Assumptions 1 and 2 imply that the vector function $f_i(z_i,t)$ and the state interaction function $\varphi_i(\bar{z},t)$ in Equation (1) have unknown upper bounds, while the vector function $\bar{f}_i(\bar{z}_i,t)$ and the state interaction function $\bar{\varphi}_i(z,t)$ in Equation (3) have known upper bounds. In practical engineering, since RN is constructed for DN, Assumptions 1 and 2 are consistent with realistic network requirements. **(ii)** In the existing literature, if only the case of isolated nodes is considered, there are many systems that satisfy Assumption 1, such as Lorenz chaotic systems[41], Chua circuit chaotic systems[41], Duffing chaotic systems[41], and the winding system[24, 25]. **(iii)** In this paper, the main control objective is to synchronize the node controller of RN with the states of DN, so the state coupling terms $\Theta(z,\bar{z})$ and $\bar{\Theta}(z,\bar{z})$ for the two LSs are required to be synthesized according to the control objective. This corresponds to the question "What structure of the external coupling relation consisting of states can drive the node states of two networks to be outer synchronization?" **(iv)** In particular, Assumption 3 holds if both P and $\bar{P}$ matrices are symmetric and Hurwitz stable.

Introduce $n \times N$ order state matrices $Z = Z(t) = [z_1, z_2, \cdots, z_N]$ and $\bar{Z} = \bar{Z}(t) = [\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_N]$, and $n \times N$ order control input matrix $\bar{U} = [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_N]$, and $n \times N$ order function matrices $F(z,t) = [f_1(z_1,t), f_2(z_2,t), \cdots, f_N(z_N,t)]$,

$\bar{F}(\bar{z},t) = [\bar{f}_1(\bar{z}_1,t), \bar{f}_2(\bar{z}_2,t), \cdots, \bar{f}_N(\bar{z}_N,t)]$, $H(z) = [h_1(z), h_2(z), \cdots, h_N(z)]$,$\bar{H}(\bar{z}) = [\bar{h}_1(\bar{z}), \bar{h}_2(\bar{z}), \cdots, \bar{h}_N(\bar{z})]$, $\Phi(\bar{z},t) = [\varphi_1(\bar{z},t), \varphi_2(\bar{z},t), \cdots, \varphi_N(\bar{z},t)]$, and $\bar{\Phi}(z,t) = [\bar{\varphi}_1(z,t), \bar{\varphi}_2(z,t), \cdots, \bar{\varphi}_N(z,t)]$.

Using the above symbols, Equations (1) and (3) can be expressed as follows, respectively.

$$\dot{Z} = F(z,t) + c_1 H(z) X + \bar{c}_1 \Phi(\bar{z},t) \tag{6}$$

$$\dot{\bar{Z}} = \bar{F}(\bar{z},t) + \bar{c}_2 \bar{H}(\bar{z}) \bar{X} + c_2 \bar{\Phi}(z,t) + \bar{U} \tag{7}$$

If Assumptions 1 and 2 are satisfied, it is true for the following results.

$$\|F(z,t)\| \le \alpha, \left\|\bar{F}(\bar{z},t)\right\| \le \bar{a}(\bar{z},t), \|\Phi(\bar{z},t)\| \le \theta, \left\|\bar{\Phi}(z,t)\right\| \le \bar{b}(z,t) \tag{8}$$

where $\alpha = \sqrt{\sum_{k=1}^{N} \alpha_k^2}$, $\bar{a}(\bar{z},t) = \sqrt{\sum_{k=1}^{N} [\bar{a}_k^*(\bar{z}_k,t)]^2}$, $\theta = \sqrt{\sum_{k=1}^{N} \theta_k^2}$, $\bar{b}(z,t) = \sqrt{\sum_{k=1}^{N} \bar{b}_k^2(z,t)}$.

Remark 3. **(i)** From Assumptions 1 and 2, the parameter α and θ are unknown and the bounded function $\bar{a}(\bar{z},t)$ and $\bar{b}(z,t)$ are known in Inequality (8). **(ii)** From Assumptions 1 and 2, the matrix functions $F(z,t)$, $\bar{F}(\bar{z},t)$, $H(z)$, $\bar{H}(\bar{z})$, $\Phi(\bar{z},t)$ and $\bar{\Phi}(z,t)$ are bounded.

3 | DESIGN OF OUTER SYNCHRONIZATION CONTROLLER AND COUPLING TERMS

The estimated values of the parameters α and θ in the Inequality (8) are denoted as $\hat{\alpha}$ and $\hat{\theta}$, and the estimated errors are denoted as $\bar{\alpha} = \hat{\alpha} - \alpha$ and $\bar{\theta} = \hat{\theta} - \theta$, respectively.

With reference to the literature[14, 15], we give the following definition.

Definition 1. Consider the two CDNs with (1)-(4), if $\lim_{t\to+\infty} [\bar{z}_i(t) - z_i(t)] = 0$, $i = 1, 2, \cdots, N$, the two CDNs are said as asymptotically achieving outer synchronization.

Control goal: Consider the two CDNs with (1)-(4). If Assumptions 1-3 are satisfied and suppose that the states z and $\bar{z}$ are available while $x_{ij} = x_{ij}(t)$ and $\bar{x}_{ij} = \bar{x}_{ij}(t)$ cannot available directly, design the adaptive controller $\bar{u}_i = \bar{u}_i(z_i, \bar{z}_i)$ for RN and coupling terms $\Theta(z, \bar{z})$ and $\bar{\Theta}(z, \bar{z})$ such that the two CDNs achieve asymptotical outer synchronization, that is to say that $\lim_{t\to+\infty} [\bar{z}_i(t) - z_i(t)] = 0$, $i = 1, 2, \cdots, N$, and the estimated values $\hat{\alpha}$, $\hat{\theta}$ and the matrices X and $\bar{X}$ of LS are ensured to be bounded.

To achieve the above control objective, we introduce the auxiliary dynamic tracking targets for LS with (2) and (4) as follows.

$$\dot{X}^* = PX^* + X^* P^T + M(t) \tag{9}$$

$$\dot{\bar{X}}^* = \bar{P}\bar{X}^* + \bar{X}^* \bar{P}^T + \bar{M}(t) \tag{10}$$

where $X^* = X^*(t)$ and $\bar{X}^* = \bar{X}^*(t)$ are the auxiliary dynamic tracking targets of LSs with (2) and (4), respectively.

Lemma 1[42]. Let $B \in R^{N\times N}$ and $\lambda(B)$ be an arbitrary eigenvalue of B, Let $G = \frac{1}{2}(B + B^T)$. Then $\lambda_{\min}(G) \le \mathrm{Re}\lambda(B) \le \lambda_{\max}(G)$, where $\mathrm{Re}\lambda(B)$ denotes the real part of $\lambda(B)$, $\lambda_{\min}(*)$ and $\lambda_{\max}(*)$ are the minimum and maximum eigenvalues of }} $*$ ε, respectively.

Remark 4. According to the matrix straightening operation ($vec(\cdot)$) and its properties[19], the differential Equations (9) and (10) can be rewritten as follow.

$$\frac{d[vec(X^*)]}{dt} = (I_N \otimes P + P \otimes I_N) vec(X^*) + vec[M(t)] \tag{11}$$

$$\frac{d[vec(\bar{X}^*)]}{dt} = (I_N \otimes \bar{P} + \bar{P} \otimes I_N) vec(\bar{X}^*) + vec[\bar{M}(t)] \tag{12}$$

where $\otimes$ denotes the Kronecker product. By using Assumption 3 and Lemma 1, it is seen that P is Hurwitz, and thus $\tilde{P} = I_N \otimes P + P \otimes I_N$ and $\tilde{\bar{P}} = I_N \otimes \bar{P} + \bar{P} \otimes I_N$ are also Hurwitz. Therefore, it can be obtained from (11) and (12) that $vec[X^*(t)]$ and $vec[\bar{X}^*(t)]$ are bounded due to $vec[M(t)]$ and $vec[\bar{M}(t)]$ being bounded.

The tracking errors of LSs are as follows.

$$\dot{E}_X = X - X^* = PE_X + E_X P^T + \Theta(z, \bar{z}) \tag{13}$$

$$\dot{E}_{\bar{X}} = \bar{X} - \bar{X}^* = \bar{P}E_{\bar{X}} + E_{\bar{X}}\bar{P}^T + \bar{\Theta}(z, \bar{z}) \tag{14}$$

Let the state error matrix $E = \bar{Z} - Z$. It is seen that the control goal $\lim_{t\to+\infty}[\bar{z}_i(t) - z_i(t)] = 0$ is equivalent to $\lim_{t\to+\infty}[\bar{Z}(t) - Z(t)] = 0$. Let $\overline{sign}(E) = \begin{cases} \frac{E}{\|E\|},\ E \neq O_{n\times N} \\ O_{n\times N},\ E = O_{n\times N} \end{cases}$, where $O_{n\times N}$ denotes the $n \times N$ order zero matrix. In order to achieve the above control goal, the following adaptive controller is proposed.

$$\bar{U} = -\lambda E - \bar{c}_2\bar{H}(\bar{z})\bar{X}^* + c_1 H(z)X^* + \bar{V}_1 + \bar{V}_2 \tag{15}$$

$$\bar{V}_1 = -[\bar{a}(\bar{z}, t) + c_2\bar{b}(z, t)] \cdot \overline{sign}(E) \tag{16}$$

$$\bar{V}_2 = -(\hat{\alpha} + \bar{c}_1\hat{\theta}) \cdot \overline{sign}(E) \tag{17}$$

with the following adaptive laws.

$$\dot{\hat{\alpha}} = \lambda_1 \|E\|, \dot{\hat{\theta}} = \lambda_2\bar{c}_1 \|E\| \tag{18}$$

and the composition of the coupling terms.

$$\Theta(z, \bar{z}) = c_1 H^T(z)E, \bar{\Theta}(z, \bar{z}) = -\bar{c}_2\bar{H}^T(\bar{z})E \tag{19}$$

where λ, λ_1 and λ_2 are adjustable positive constants.

Remark 5. (i) From Equations (15)-(18), the controller consists of four parts. The first is the error feedback part $-\lambda E$. The second part $-\bar{c}_2\bar{H}(\bar{z})\bar{X}^* + c_1 H(z)X^*$ is relative to the auxiliary tracking targets of LSs. The third is the robust control term $\bar{V}_1$ for the uncertainties in RN with (3), and the fourth part is the term $\bar{V}_2$ with the adaptive estimation laws in (18) for the uncertain parameters in DN with (1). **(ii)** The adaptive controller with (15)-(18) has three adjustable parameters, where λ will affect the convergence speed of outer synchronization, and thus its value should not be too small in application. λ_1 and λ_2 will affect the update speeds of the corresponding estimation laws in (19), and in general, their values should not be too large. **(iii)** From the composition of the coupling terms, $\Theta(z, \bar{z})$ and $\bar{\Theta}(z, \bar{z})$ are related to the state of the nodes in DN, the state of the nodes in RN and their node state errors. With the action of the adaptive controller and the coupling terms (influence NS by affecting LS), the node states of the two networks will achieve outer synchronization. At the same time, two LSs can track to the given reference targets, respectively.

Lemma 2[28]. Let $\Upsilon \in R^{N\times N}$ be a given negative definite matrix. It is true that $tr(\Xi^T\Upsilon\Xi) \le 0$ for the arbitrary matrix $\Xi \in R^{N\times N}$. Particularly, $tr(\Xi^T\Upsilon\Xi) = 0$ holds if only if Ξ is an $N \times N$ zero matrix.

Using Equations (6)-(7), (9)-(10) and (13)-(15), the following error equations can be obtained.

$$\dot{E} = -\lambda E + \bar{F}(\bar{z}, t) - F(z, t) + \bar{c}_2\bar{H}(\bar{z})E_{\bar{X}} - c_1 H(z)E_X + c_2\bar{\Phi}(z, t) - \bar{c}_1\Phi(\bar{z}, t) + \bar{V}_1 + \bar{V}_2 \tag{20}$$

Let $e_i = \bar{z}_i - z_i$, $i = 1, 2, \cdots, N$. It is easy to verify that $E = [e_1, e_2, \cdots, e_N]$, $tr(E^T E) = \sum_{i=1}^{N} e_i^T e_i = \sum_{i=1}^{N} \|e_i\|^2$. Consider the following positive definite function.

$$V(t) = V(E, E_X, E_{\bar{X}}, \bar{\alpha}, \bar{\theta}) = 0.5tr(E^T E) + 0.5tr(E_X^T E_X) + 0.5tr(E_{\bar{X}}^T E_{\bar{X}}) + 0.5[\lambda_1^{-1}\bar{\alpha}^2 + \lambda_2^{-1}\bar{\theta}^2] \tag{21}$$

Notice that for any matrix C and D of suitable dimensions, it has the following properties of the trace[28] $tr(CD) = tr(DC)$ and $tr(D^T) = tr(D)$. And also that $tr[E^T\overline{sign}(E)] = \begin{cases} \frac{tr(E^T E)}{\|E\|}, E \neq O_{n\times N} \\ tr(E^T O_{n\times N}), E = O_{n\times N} \end{cases} = \|E\|$, $tr(CD) \le \|D\| \cdot \|C\|$. Thus using the adaptive controller (15)-(18) and the coupling terms (19), we can obtain the orbital derivative along the error system (20) as follows.

$$
\begin{aligned}
\dot{V}(t) &= 0.5tr(\dot{E}^T E) + 0.5tr(E^T \dot{E}) + 0.5tr(\dot{E}_X^T E_X) + 0.5tr(E_X^T \dot{E}_X) + 0.5tr(\dot{E}_{\bar{X}}^T E_{\bar{X}}) + 0.5tr(E_{\bar{X}}^T \dot{E}_{\bar{X}}) \\
&\quad + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&= 0.5tr\{[-\lambda E + \bar{F}(\bar{z},t) - F(z,t) + \bar{c}_2\bar{H}(\bar{z})E_{\bar{X}} - c_1 H(z)E_X + c_2\bar{\Phi}(z,t) - \bar{c}_1\Phi(\bar{z},t) + \bar{V}_1 + \bar{V}_2]^T E\} \\
&\quad + 0.5tr\{E^T[-\lambda E + \bar{F}(\bar{z},t) - F(z,t) + \bar{c}_2\bar{H}(\bar{z})E_{\bar{X}} - c_1 H(z)E_X + c_2\bar{\Phi}(z,t) - \bar{c}_1\Phi(\bar{z},t) + \bar{V}_1 + \bar{V}_2]\} \\
&\quad + 0.5tr\{[PE_X + E_X P^T + \Theta(z,\bar{z})]^T E_X\} + 0.5tr\{E_X^T[PE_X + E_X P^T + \Theta(z,\bar{z})]\} \\
&\quad + 0.5tr\{[\bar{P}E_{\bar{X}} + E_{\bar{X}}\bar{P}^T + \bar{\Theta}(z,\bar{z})]^T E_{\bar{X}}\} + 0.5tr\{E_{\bar{X}}^T[\bar{P}E_{\bar{X}} + E_{\bar{X}}\bar{P}^T + \bar{\Theta}(z,\bar{z})]^T\} \\
&\quad + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&= -\lambda tr(E^T E) + tr[\bar{c}_2 E^T \bar{H}(\bar{z})E_{\bar{X}} - c_1 E^T H(z)E_X] \\
&\quad + tr\{E^T[\bar{F}(\bar{z},t) - F(z,t) + c_2\bar{\Phi}(z,t) - \bar{c}_1\Phi(\bar{z},t) + \bar{V}_1 + \bar{V}_2]\} \\
&\quad + tr[E_X^T(P + P^T)E_X] + tr[\Theta^T(z,\bar{z})E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}] + tr[\bar{\Theta}^T(z,\bar{z})E_{\bar{X}}] \\
&\quad + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&= -\lambda tr(E^T E) + tr[E_X^T(P + P^T)E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}] \\
&\quad + tr[\bar{c}_2 E^T \bar{H}(\bar{z})E_{\bar{X}} - c_1 E^T H(z)E_X] + tr[\Theta^T(z,\bar{z})E_X] + tr[\bar{\Theta}^T(z,\bar{z})E_{\bar{X}}] \\
&\quad + tr\{E^T[\bar{F}(\bar{z},t) + c_2\bar{\Phi}(z,t) + \bar{V}_1]\} + tr\{E^T[-F(z,t) - \bar{c}_1\Phi(\bar{z},t) + \bar{V}_2]\} + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&= -\lambda tr(E^T E) + tr[E_X^T(P + P^T)E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}] \\
&\quad + tr\{E^T[\bar{F}(\bar{z},t) + c_2\bar{\Phi}(z,t) - [\bar{a}(\bar{z},t) + c_2\bar{b}(z,t)]\cdot\overline{sign}(E)]\} \\
&\quad + tr\{E^T[-F(z,t) - \bar{c}_1\Phi(\bar{z},t) - (\hat{\alpha} + \bar{c}_1\hat{\theta})\cdot\overline{sign}(E)]\} + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&\le -\lambda tr(E^T E) + tr[E_X^T(P + P^T)E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}] \\
&\quad + \|E\|\cdot\left\|\bar{F}(\bar{z},t) + c_2\bar{\Phi}(z,t)\right\| - [\bar{a}(\bar{z},t) + c_2\bar{b}(z,t)]\cdot\|E\| \\
&\quad + \|E\|\cdot\left\|F(z,t) + \bar{c}_1\Phi(\bar{z},t)\right\| - (\hat{\alpha} + \bar{c}_1\hat{\theta})\,\|E\| + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&\le -\lambda tr(E^T E) + tr[E_X^T(P + P^T)E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}] \\
&\quad + \|E\|\,\{\left\|\bar{F}(\bar{z},t)\right\| - \bar{a}(\bar{z},t) + c_2[\left\|\bar{\Phi}(z,t)\right\| - \bar{b}(z,t)]\} \\
&\quad + \|E\|\cdot[\alpha + \bar{c}_1\theta - (\hat{\alpha} + \bar{c}_1\hat{\theta})] + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&\le -\lambda tr(E^T E) + tr[E_X^T(P + P^T)E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}] \\
&\quad + \|E\|\cdot[\alpha - \hat{\alpha} + \bar{c}_1(\theta - \hat{\theta})] + \lambda_1^{-1}\bar{\alpha}\dot{\hat{\alpha}} + \lambda_2^{-1}\bar{\theta}\dot{\hat{\theta}} \\
&= -\lambda tr(E^T E) + tr[E_X^T(P + P^T)E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}] \\
&\quad + \bar{\alpha}[-\|E\| + \lambda_1^{-1}\dot{\hat{\alpha}}] + \bar{\theta}[-\bar{c}_1\|E\| + \lambda_2^{-1}\dot{\hat{\theta}}] \\
&= -\lambda tr(E^T E) + tr[E_X^T(P + P^T)E_X] + tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}]
\end{aligned} \tag{22}
$$

By Assumption 3 and Lemma 2, we know that $tr[E_X^T(P + P^T)E_X]$ and $tr[E_{\bar{X}}^T(\bar{P} + \bar{P}^T)E_{\bar{X}}]$ are negative definite functions about E_X and $E_{\bar{X}}$, respectively. Therefore, from (22), we know that $\dot{V}(t)$ is a semi-negative definite function about E, E_X, $E_{\bar{X}}$, $\bar{\alpha}$ and $\bar{\theta}$. The error system consisting of (13), (14) and (20) with controller (16)-(17) is stable in Lyapunov sense. Therefore, the error matrices E, E_X and $E_{\bar{X}}$, the estimate values $\hat{\alpha}$ and $\hat{\theta}$ are bounded. Accordingly, by using Assumptions 1-3 and Lemmas 1-2, it is seen that $\dot{E}(t)$, $\dot{E}_X(t)$ and $\dot{E}_{\bar{X}}(t)$ are bounded, and then using Barbalat's Lemma[43], we know that $\lim_{t\to\infty} E(t) = 0$, $\lim_{t\to\infty} E_X(t) = 0$ and $\lim_{t\to\infty} E_{\bar{X}}(t) = 0$. Therefore, This completes the proof of the following theorem 1.

Theorem 1. Consider the two CDNs with Equations (1) - (4). If Assumptions 1-3 hold, the two CDNs are able to achieve asymptotical outer synchronization with the adaptive controller (15)-(18) and the coupling terms (19), and the estimate values $\hat{\alpha}$ and $\hat{\theta}$, and state matrices X and $\bar{X}$ of LS are guaranteed to be bounded.

Remark 6. (i) Theorem 1 implies that the designed node controller and coupling terms ensure that NSs of two networks achieve asymptotic outer synchronization. **(ii)** Comparing with the existing literature, if the two NSs (1) and (3) achieve the asymptotic outer synchronization, the two LSs (2) and (4) are not only guaranteed to be bounded, but also track asymptotically to our given reference targets X^* and $\bar{X}^*$, respectively. **(iii)** Since $\tilde{P}$ and $\tilde{\bar{P}}$ are Hurwitz, the given reference targets X^* and $\bar{X}^*$ are relative to the second part of Equations (11) and (12). Since the two LSs can track X^* and $\bar{X}^*$, respectively, then we have the following conclusion. (**a**) If the matrices $M(t)$ and $\bar{M}(t)$ are both zero matrices, the solutions of Equations (11) and (12) eventually converge to zero. This implies that all the connections between the nodes of each of the two networks will eventually break and both will become networks of isolated nodes. (**b**) If the matrices $M(t)$ and $\bar{M}(t)$ are both non-zero bounded time-varying matrices, the connections between all nodes of each of the two networks will eventually change with the same as the dynamics of CR, which is different from some existing results in the literature[14–18, 30–33].

Remark 7. For the applications of Theorem 1, the following steps can be proposed.
Step 1: Considering NS of DN in (1) and NS of RN in (3). Determine whether the nonlinear function $f_i(z_i,t)$, $h_j(z)$, $\bar{h}_j(\bar{z})$ and the state activation function $\bar{\varphi}_i(z,t)$ are bounded or nor. If not, Theorem 1 is invalid;
Step 2: Considering LS for DN in (2) and LS for RN in (4), respectively. Determine whether matrices $P+P^T$ and $\bar{P}+\bar{P}^T$ are Hurwitz matrices. If not, then Theorem 1 invalid;
Step 3: Considering NS of RN in (3), the norm bounds of the nonlinear functions $\bar{f}_i(\bar{z}_i,t)$ and state activation function $\bar{\varphi}_i(z,t)$ are determined, respectively;
Step 4: Design adaptive controller (15) - (18) and coupling terms of LS in (19).

4 | SIMULATION EXAMPLE

Consider two time-varying weighted CDNs with Σ and $\bar{\Sigma}$, each with 20 (N=20) nodes.

Choose the N Chua's chaotic circuit systems[29] as the isolated nodes of the network Σ, respectively, whose dynamics are given as $f_i(z_i,t) = A_i z_i + g_i(z_i,t) = \begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix} z + \begin{bmatrix} az_{i1} + b(|z_{i1}+1| - |z_{i1}-1|) \\ 0 \\ 0 \end{bmatrix}$. The corresponding internal function is $h_j(z) = [\sin(j*z_{i1}z_{i2}z_{i3}), \cos(j*z_{i1}z_{i2}z_{i3}), \arctan(j*z_{i1}z_{i2}z_{i3})]^T$, interaction function of the network $\varphi_i(\bar{z},t) = [\frac{sin(it)}{1+e^{-\bar{z}_{i1}\bar{z}_{i2}\bar{z}_{i3}}}, \frac{cos(it)}{1+(\bar{z}_{i1}\bar{z}_{i2}\bar{z}_{i3})^2}, arctan(it) * \frac{\bar{z}_{i1}\bar{z}_{i2}\bar{z}_{i3}}{1+(\bar{z}_{i1}\bar{z}_{i2}\bar{z}_{i3})^2}]^T$, $i,j = 1,2,\cdots,10$.

Choose the N Duffing chaotic systems[41] as the isolated nodes of the network $\bar{\Sigma}$, respectively, whose dynamics are given as $\bar{f}_i(\bar{z}_i,t) = \bar{A}_i\bar{z}_i + \bar{g}_i(\bar{z}_i,t) = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix} \bar{z} + \begin{bmatrix} 0 \\ -\bar{z}_{i1}\bar{z}_{i3} \\ \bar{z}_{i1}\bar{z}_{i2} \end{bmatrix}$. The corresponding internal function is $\bar{h}_j(\bar{z}) = [\cos(j*\bar{z}_{i1}\bar{z}_{i2}\bar{z}_{i3}), \arctan(j*\bar{z}_{i1}\bar{z}_{i2}\bar{z}_{i3}), \sin(j*\bar{z}_{i1}\bar{z}_{i2}\bar{z}_{i3})]^T$, interaction function of the network $\bar{\varphi}_i(z,t) = [\frac{cos(it)}{1+(z_{i1}z_{i2}z_{i3})^2}, \frac{arctan(it)}{1+e^{-z_{i1}z_{i2}z_{i3}}}, sin(it) * \frac{z_{i1}z_{i2}z_{i3}}{1+(z_{i1}z_{i2}z_{i3})^2}]^T$, $i,j = 1,2,\cdots,10$. The adaptive controller is given by Equations (15) - (18), and the coupling terms are given by Equation (19).

The matrices P and $\bar{P}$ in Equations (2) and (4) are generated by the following steps.

Step 1: By using $-abs(rand(N,1))$, N negative real numbers $\gamma_i (i = 1,2,\cdots,N)$ are generated randomly. Construct N order matrix $\Delta_1 = diag(\gamma_1,\gamma_2,\cdots,\gamma_N)$. Similarly, another group of N negative real numbers $\mu_i (i = 1,2,\cdots,N)$ is generated and thus the order matrix $\Delta_2 = diag(\mu_1,\mu_2,\cdots,\mu_N)$ is constructed.

Step 2: Generate matrices P_1 and P_2 by $P_1 = randn(N,N)$ and $P_2 = randn(N,N)$. Then we obtain two N order orthogonal matrices B_1 and B_2 by $B_1 = orth(P_1)$ and $B_2 = orth(P_2)$. Choose $P = 4(B_1\Delta_1B_1^T + P_1 - P_1^T)$ and $\bar{P} = 4(B_2\Delta_2B_2^T + P_2 - P_2^T)$.

It can be verified that the matrices P and $\bar{P}$ generated by the above two steps satisfy Assumption 3. The other parameters are chosen as follows. The adjustable parameters $\lambda = 10$, $\lambda_1 = 2.5$, and $\lambda_2 = 2.5$, the common connection strength $c_1, \bar{c}_1, c_2$ and $\bar{c}_2$ are randomly chosen in the interval $(0,10)$. The initial state $z_i(0)$, $\bar{z}_i(0)$, $i = 1,2,\cdots,20$, and $\hat{\alpha}(0)$, $\hat{\theta}(0)$ in the adaptive laws (18) are randomly chosen in the interval $(-10,10)$. Generate the random constant matrices $X(0)$ and $\bar{X}(0)$ by $X(0) = randn(N,N)$ and $\bar{X}(0) = randn(N,N)$ as the initial state of the two LSs. Similarly, the initial state tracking targets for the given two LSs are chosen as $X^*(0) = randn(N,N)$ and $\bar{X}^*(0) = randn(N,N)$. In addition, take the known coupling time-varying matrices $M(t) = (m_{ij}(t))_{N\times N}, m_{ij}(t) = 10*\sin(t)$ and $\bar{M}(t) = (\bar{m}_{ij}(t))_{N\times N}, \bar{m}_{ij}(t) = 10*\cos(t)$.The simulation results are shown in Figs.2 –4 .

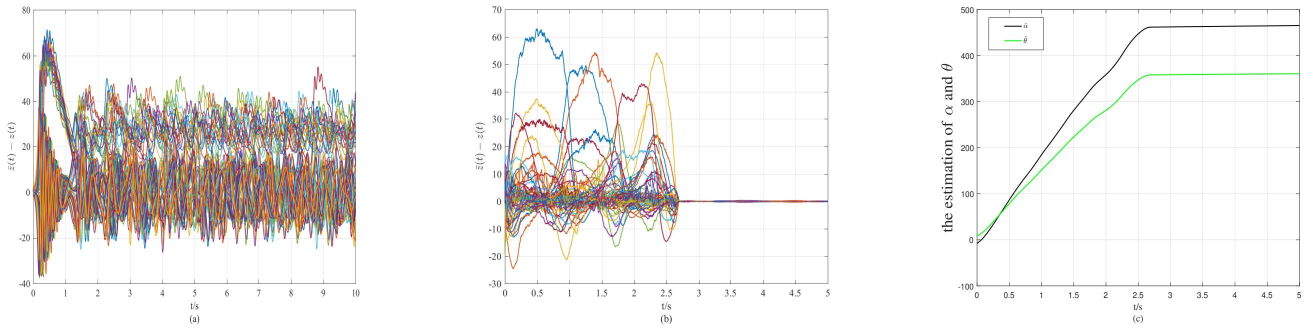

FIGURE 2 (a) The outer synchronization error curves of the nodes without controller; (b) The outer synchronization error curves of the nodes with the controller in this paper; (c) The response curves of adaptive estimated values in (18) with the controller in this paper.

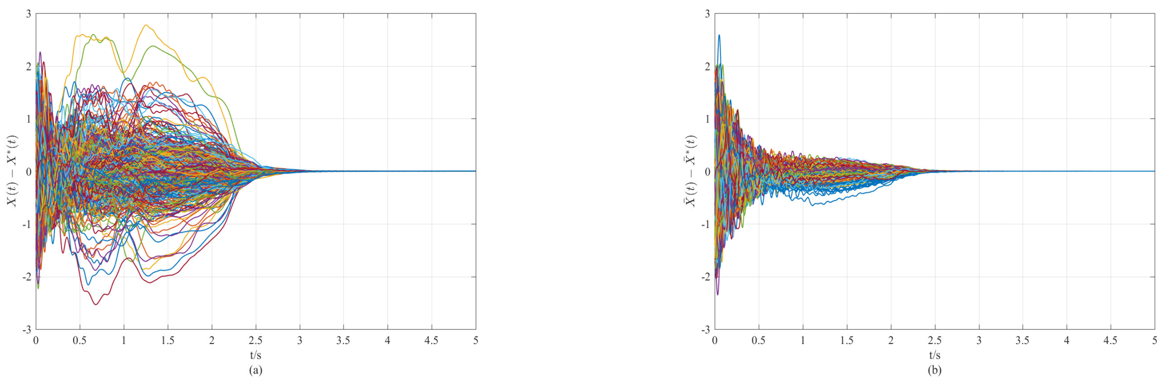

FIGURE 3 (a) The auxiliary tracking response error curves of LS with (2); (b) The auxiliary tracking response error curves of LS with (4).

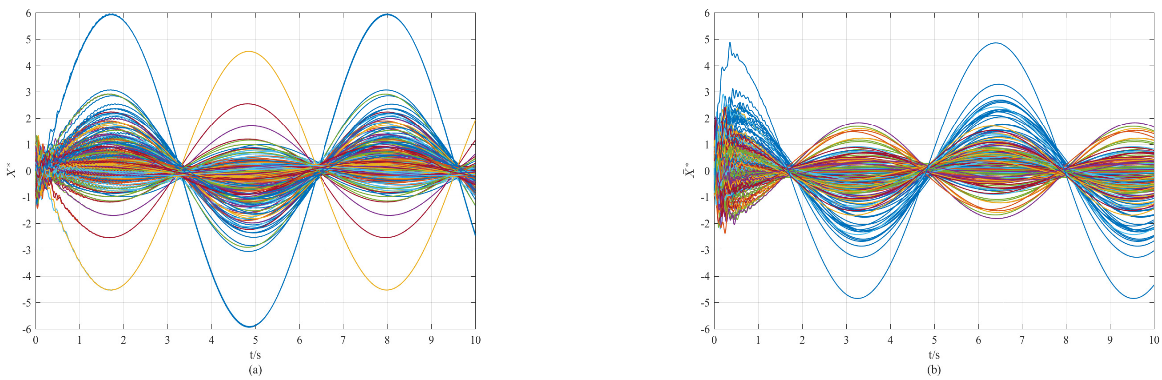

FIGURE 4 (a) The auxiliary tracking response curves of LS in (2); (b) The auxiliary tracking response curves of LS in (4).

Particularly, when both matrices $M(t)$ and $\bar{M}(t)$ are zero matrices, that is $M(t) = O$ and $\bar{M}(t) = O$, where O denotes the zero matrix of the corresponding dimension. Then the auxiliary tracking target response curves of LSs with (2) and (4) are shown in Fig.5 .

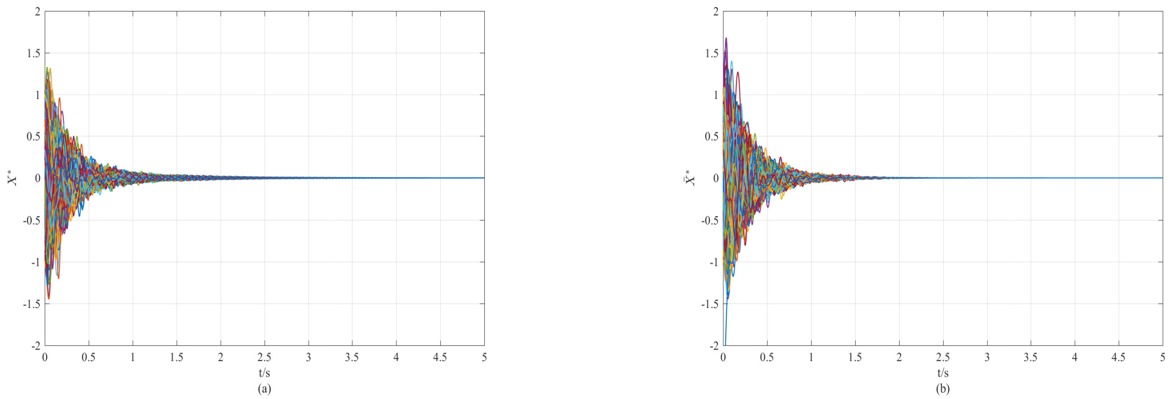

FIGURE 5 (a) The auxiliary tracking response curves of LS in (2) with $M(t) = O$; (b) The auxiliary tracking response curves of LS in (4) with $\bar{M}(t) = O$.

From the above simulation figures, we can draw the following conclusions. (**i**) From Fig.2 (a), it is known that NSs of the two networks do not achieve asymptotic outer synchronization without the application of the adaptive controller (15)-(18). However, after applying the controller designed in this paper, it can be seen from Fig.2 (b) and Fig.2 (c) that two NSs can achieve asymptotically outer synchronization and the adaptive estimation values is bounded. (**ii**) As can be seen from Fig.3 , LSs of the two networks can track to the tracking targets proposed in this paper, respectively. According to the coupling terms $M(t)$ and $\bar{M}(t)$, as can be seen from Fig.4 , the tracking targets of our given two LSs are not only bounded but also a dynamic change process, which indicates that all nodes are not isolated after achieving the outer synchronization, but are in a time-varying connected state. On the other hand, it can be seen from Fig.5 that all nodes of both networks are isolated after achieving the outer synchronization when the coupling terms $M(t)$ and $\bar{M}(t)$ vanish. This implies that the coupling terms $M(t)$ and $\bar{M}(t)$ determine whether or not all the nodes are isolated in the eventual network structure.

5 | CONCLUSIONS

Asymptotical outer synchronization has been achieved for the two CDNs by using the adaptive controller of NS in RN and the coupling terms in the two LSs in this paper. Compared with the existing results about the outer synchronization, the main advantage of this paper is that on the one hand, the whole dynamical behavior of each LS is described by employing the Riccati matrix differential equation with the coupling to the state of two NSs. On the other hand, the whole dynamical model of LS not only clearly reflects the topology of two CDNs but also better understand the topology of two CDNs due to the synthesized auxiliary dynamic tracking targets of LSs when the outer synchronization of NS is realized. Furthermore, the matrices have no more flexible mathematical operational properties than vectors, but the methods can avoid the matrix straightening operation with the methods in control theory. However, the outer synchronization of two CDNs with the discrete-time dynamics is not discussed in this paper, the research topic will be considered in the future work.

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Conflict of interest

The authors declare no potential conflict of interests.

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