AGU PUBLICATIONS

Journal of Advances in Modeling Earth Systems

Supporting Information for

Ocean Surface Wave Effects on Development of Explosive Cyclone

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Introduction

This supporting information includes an ocean model configuration in this study. It explains how the non-breaking wave-induced mixing is calculated in the ocean model.

Text S1. Ocean Model Configuration

The governing equations of the ocean model of CROCO are the same as the ones proposed by Uchiyama, McWilliams, & Shchepetkin (2010) that incorporates wave-current interactions. The modified governing equations of the ocean model with Reynolds-averaging are:

$$\frac{\partial u}{\partial t} + u_j \frac{\partial u}{\partial x_j} - fv = -\frac{\partial(\phi + \phi_B)}{\partial x} - \frac{\partial}{\partial x_j}\left(\overline{u_j' u} - \nu \frac{\partial u}{\partial x_j}\right) + V_x + F_x + F_x^w , \quad (1)$$

$$\frac{\partial v}{\partial t} + u_j \frac{\partial v}{\partial x_j} + fu = -\frac{\partial(\phi + \phi_B)}{\partial y} - \frac{\partial}{\partial x_j}\left(\overline{u_j' v} - \nu \frac{\partial v}{\partial x_j}\right) + V_y + F_y + F_y^w , \quad (2)$$

$$\frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = -u_j^{st} \frac{\partial c}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\overline{u_j' c} - \nu_c \frac{\partial c}{\partial x_j}\right) + F_c , \quad (3)$$

$$\frac{\partial \phi}{\partial z} + \frac{g\rho_w}{\rho_0} = -\frac{\partial \phi_B}{\partial z} + V_z , \quad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 , \quad (5)$$

where (u^{st}, v^{st}, w^{st}) is the Stokes drift, (V_x, V_y, V_z) is the vortex force, ϕ_B is the Bernoulli head, F is unpreserved force, F^w is an unpreserved force by the wave, ν is eddy viscosity, and ν_c is eddy diffusivity for the scalar value c (in the model, potential temperature and salinity are solved). F_c is the unpreserved force for the scalar value c. The Einstein summation convention is used in these equations. These equations are closed by parameterizing the vertical momentum flux and turbulent tracer fluxes as:

$$\overline{u'w'} = -K_M \frac{\partial u}{\partial z}, \qquad \overline{v'w'} = -K_M \frac{\partial v}{\partial z}, \qquad \overline{w'c'} = -K_C \frac{\partial c}{\partial z} , \quad (6)$$

where the eddy viscosity K_M and eddy diffusivity K_C. The Stokes velocity is defined for a monochromatic wave field by

$$\boldsymbol{u^{st}} \equiv (u^{st}, v^{st}) = \frac{A^2 \sigma}{2\, sinh\, k(h + \eta + \tilde{\eta})} cosh\, 2k(z + h)\, \boldsymbol{k} , \quad (7)$$

$$w^{st} = -\boldsymbol{\nabla}_\perp \cdot \int_{-h}^{z} (u^{st}, v^{st}) d\zeta , \quad (8)$$

where $h(x, y)$ is the ocean's depth; η is the sea level; $\tilde{\eta}$ is the quasi-static sea-level by the surface wave and air-pressure; A is the wave amplitude; $\mathbf{k}$ is its wave number vector; $\nabla_\perp$ is a horizontal differential operator. The horizontal and vertical vortex force and Bernoulli head are defined by

$$\left(V_x,\, V_y\right) = -\hat{\boldsymbol{z}} \times \boldsymbol{u^{st}}[(\hat{\boldsymbol{z}} \cdot \boldsymbol{\nabla}_\perp \times \boldsymbol{u}) + f] - w^{st} \frac{\partial \boldsymbol{u}}{\partial z}, \quad (9)$$

$$V_z = \boldsymbol{u}^{st} \cdot \frac{\partial \boldsymbol{u}}{\partial z}, \tag{10}$$

$$\phi_B = \frac{1}{4} \frac{\sigma A^2}{k sinh^2[kD]} \int_{-h}^{z} \frac{\partial (\boldsymbol{k} \cdot \boldsymbol{u})^2}{\partial \zeta^2} sinh[2k(z-\zeta)] d\zeta\,. \tag{11}$$

The quasi-static sea-level $\tilde{\eta}$ is defined by

$$\tilde{\eta} = -\frac{p_{air}}{g\rho_0} - \frac{A^2 k}{2\, sinh\, 2k(h+\eta)}. \tag{12}$$

To solve the turbulent momentum flux in the governing equations, GLS (generic length scale) approach to parameterizing them is provided in the CROCO code. The wave-current interaction that we focus on is the turbulence effects on the internal ocean by surface waves.

The generic length scale (GLS) approach is a kind of turbulence second-order closure scheme. The approach solves two quantities on turbulence: turbulence kinetic energy q^2 and generic parameter, which will be defined later. The q^2 (per unit mass) is defined as

$$q^2 = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right). \tag{13}$$

In the CROCO model, several GLS approaches are provided. In the model of this research, the Mellor-Yamada level 2.5 (MY25) scheme is assigned. In the MY25 scheme, two equations on turbulent quantities are solved explicitly. The generic parameter in this scheme is $q^2 l$, where l is the turbulent length scale. The governing two equations are:

$$\frac{\partial q^2}{\partial t} + u_j \frac{\partial q^2}{\partial x_j} = \frac{\partial}{\partial z}\left(\frac{K_M}{\sigma_q}\frac{\partial q^2}{\partial z}\right) + P + B - \varepsilon\,, \tag{14}$$

$$\frac{\partial (q^2 l)}{\partial t} + u_j \frac{\partial (q^2 l)}{\partial x_j} = \frac{\partial}{\partial z}\left[\frac{K_M}{\sigma_q}\frac{\partial (q^2 l)}{\partial z}\right] + l(c_1 P + c_3 B - c_2 \varepsilon F_{wall})\,, \tag{15}$$

where c_1, c_2, and c_3 are constant coefficients, F_{wall} is the wall proximity function, and $\sigma_q = 2.44$ is the Schmidt number for q^2. P and B represent production by shear and buoyancy as

$$P = -\overline{u'w'}\frac{\partial u}{\partial z} - \overline{v'w'}\frac{\partial v}{\partial z} = K_M\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right], \tag{16}$$

$$B = -\frac{g}{\rho_0}\overline{\rho' w'} = -K_M\left(-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}\right)\,, \tag{17}$$

And the dissipation ε is given by

$$\varepsilon = \frac{q^3}{B_1 l}\,, \tag{18}$$

where $B_1 = 16.6$ is constant. In the standard MY25 implementation, $K_Q/\sigma_q =$

$\sqrt{2q^2}lS_q$; $S_q = 0.2$; $c_1 = 0.9$, $c_3 = 0.9$, $c_2 = 0.5$ (Warner et al., 2005). This scheme's detailed settings are discussed in Umlauf and Burchard (2003) and Canuto et al. (2001).

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