New estimation methods of Covid-19 cases and reproduction number with using a dynamic linear model and adaptive Kalman filter for the USA: New York, Arizona, Texas and Florida utilizing data between Marc and July 28th, 2020

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Abstract

In this study, cumulative and daily cases are estimated online using a discrete-time dynamic linear model (DLM) and Adaptive Kalman Filter (AKF) based on the total COVID-19 cases between March-July 28, 2020 in USA-Florida, USA-Texas, USA-Arizona, USA-New York. Employing the data collected between Marc and July 28, 2020, it is showed that the discrete-time DLM in conjunction with AKF provides a good analysis tool for modeling the daily cases made using the in terms of mean square error (MSE) and R^2. After estimating the number of cumulative cases, the daily case number estimate was calculated. After calculating the daily case number estimate, the reproduction number estimate was obtained. The method is online. Only the data on the last day is sufficient. The AKF has never been considered for such an application. To the best of our knowledge, the estimation of COVID-19 has not been studied with this method.

KEY WORDS

COVID-19, state-space modeling, adaptive Kalman filter, reproduction number estimation.

1 INTRODUCTION

In December 2019, a new coronavirus disease emerged characterized as a viral infection with a high level of transmission in Wuhan, China. Coronavirus 19 (COVID-19) is caused by the virus known as Severe Acute Respiratory Syndrome coronavirus 2 (SARS- CoV-2) established by the Coronaviridae Study Group of the International Committee on Taxonomy of Viruses (ICTV) [1]-[2]-[3]. At the same time in the academic field, several research papers have been published focused on modeling and estimating the possible number of people infected with COVID-19 in a specific period. Applied mathematical models, such as Gompertz and Logistic, have been used successfully to predict the number of infected with COVID-19 in China, as demonstrated by. Jia et al [4] where three mathematical models were applied, including the Gompertz model and logistic model, to

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estimate the progress of COVID-19 in Wuhan, China, the results of the mathematical models predict that the COVID-19 will be over probably in late-April, 2020 in Wuhan. Cas torina et al [5] developed the mathematical Gompertz and Logistic models to evaluate the effectiveness of containment in the epidemic spread of COVID-19 in China, South Korea, Italy, and Singapore, the results of the models predict the maximum number of infected individuals for each country studied, to maintain a strong containment policy. Roosa et al [6] have used Generalized Logistic Growth Model (GLM) for the data gathered between February 5 and February 24, 2020, for China. Roosa et al [7] have used the Generalized Logistic Growth Model (GLM) and Richard model for the data gathered between February 13 and February 20, 2020 for China. Munayco et al [8] have used the Generalized Growth Model for the dates February 29 and March 30, 2020, for Peru. The mathematical models: Gompertz and Logistic, as well as the computational model: Artificial Neural Networs were applied to carry out the modeling of the number of cases of COVID-19 infection from 27 of February to 8 of May in Mexico. The results show a good fit between the observed data and those obtained by the Gompertz, Logistic, and Artificial Neural Networks models [9]. Zuzana et al [10] are to model a trajectory of the number of infections for the USA by the Gompertz curve. Cata et al [11] employed the Gompertz growing function to analyze the dynamics of the 52 spreadings of Covid-19 in several countries to make short-time predictions. Petropoulos et al [12], focused on the cumulative daily figures aggregated globally of the three main variables of interest: confirmed cases, deaths, and recoveries. To forecast confirmed cases of COVID-19, they adopted simple time series forecasting approaches.

Discrete-time linear state-space models have been employed since the 1960's, mostly in the control and signal processing areas, and Kalman filtering (KF) [13-24] has emerged as the most commonly used tool. Kalman filter solves the problem of estimating the instantaneous states of a linear dynamic system perturbed by Gaussian white noise, using measurements that are linear functions of the system state but corrupted by additive white noise. The KF has been extensively employed in many areas of estimation the extensions and applications of discrete-time linear state-space models can be found in almost all disciplines. The KF has also been utilized in electrophysiological signal analysis and compared favorably with other approaches [25-31].

The rest of this article is organized as follows: In section 2, the mathematical and computational methodologies are specified and mathematical equations of the models to be used in this study are given. In section 3 the modeling analysis and estimation results are presented. In section 4 the computation of the reproduction number with AKF is presented. Finally, the last section presents conclusions.

2 MATERIALS AND METHODS

In this work, KF[1] has been used to estimate the trend and systematic variation (SV) components from the observed COVID-19 sequences. Kalman filter is a recursive estimator for what is called the linear quadratic Gaussian problem, which is the problem of estimating the instantaneous state of a linear dynamic system perturbed by Gaussian white noise, by

[1] Kalman filter is an estimator rather than a conventional filter, however it is employed to estimate parameters from a noisy data sequence, hence the name filter.

using measurements related to the state but corrupted by Gaussian white noise. Thus, as long as a correct model that belongs to the system of interest is constructed, Kalman filtering will be the answer to the linear estimation problem in hand. The assumption that an COVID-19 sequence can be considered to comprise two parts, namely the trend and SV, which provides the means for the system to be modeled appropriately using the AKF.

In [32] a simple linear model was proposed to describe the time course of stochastic time-series. This so-called "dynamic linear model" (DLM) is defined in terms of state space representation through

$$y_t = \mu_t + e_{1t} \tag{2}$$

$$\mu_t = \mu_{t-1} + \beta_t + e_{2t} \tag{3}$$

$$\beta_t = \beta_{t-1} + e_{3t} \tag{4}$$

where, Eq. (2) is the output (observed, COVID-19 cumulative cases) where it describes the measurement at time *t*, the added term e_{1t} is the Gaussian measurement noise that corrupts the observed value of the state μ_t. The state μ_t is assumed to evolve in time as described by Eq. (3) and its value at time *t* depends on its value observed at *(t*-1*)* plus the value of the second state at time *t.* The noise term e_{2t} is the process noise component corresponding to that state, used to model unexpected changes in the state. Eq. (4) describes the evolution of the second state β_t which depends on its previous value, e_{3t} is the process noise component corresponding to the second state that is used to model the unpredicted changes in this state. Relating the COVID-19 model to Eqs. (2) - (4), μ_t represents the trend that fluctuates around a mean and is corrupted by noise where β_t is the systematic variation (SV) that is added to the trend, i.e., the SV. Thus, through this representation both the COVID-19 trend and SV can be estimated from an observed real COVID-19 sequence with an appropriate tool, (AKF). But first, it would be useful to put these equations in vector-matrix form to obtain the state-space model for an COVID-19 sequence. Re-arranging Eqs. (2) - (4) would yield

$$\begin{bmatrix} \mu_t \\ \beta_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \beta_{t-1} \end{bmatrix} + \begin{bmatrix} e_{2t} + e_{3t} \\ e_{3t} \end{bmatrix} \tag{5}$$

$$y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \mu_t \\ \beta_t \end{bmatrix} + e_{1t} \tag{6}$$

where Eq. (5) is the state equation that defines the evolution of system states and Eq. (6) is the output equation that relates system states to the observations. In above equations, process noise (e_2 and e_3) and measurement noise (e_1) sequences are assumed to be Gaussian and independent of each other. If we introduce general AKF representation[2] at this point, it will be easier to see the suitability of the (AKF) approach to this specific problem.

2.1 Modeling analysis and estimation results

[2] Please see the Appendix for the derivation of Kalman filter equations.

Lets consider a general discrete-time stochastic system represented by the state and measurement models given by

$$x_{t+1} = \Phi_t x_t + B_t u_t + w_t \tag{7}$$

$$y_t = H_t x_t + v_t \tag{8}$$

where, x_t is an $n\times1$ system vector, y_t is an $m\times1$ observation vector, Φ_t is an $n\times n$ system matrix, u_t is a $p\times1$ vector of the input forcing function, B_t is an $n\times p$ matrix, H_t is an $m\times n$ matrix, w_t an $n\times1$ vector of zero-mean white noise sequence and v_t is an $m\times1$ measurement error vector assumed to be a zero-mean white sequence uncorrelated with the w_t sequence. The matrices Φ_t, B_t, H_t, Q_t, R_t are assumed known at time t. The covariance matrices w_t and v_t are defined by $E(w_k w_k') = Q_k \delta_{kl}$, $E(v_k v_k) = R_k \delta_{kl}$, $E(w_k v_k) = 0$, where δ_{kl} is the Kronecker delta function. Please note the similarity between the Eqs. 5-7 and 8, thus, applying the state-space representation given by the Eqs. 5 and 6 to the (AKF) would result in a decomposed COVID-19 sequence and estimated values of COVID-19 trend and SV components. This is a rather simple but very effective approach and to the authors' best knowledge it has not been employed for this purpose before.

The COVID-19 sequence has been put through the AKF that has been designed to model the system described by Eqs. (5) and (6). Initial values of the COVID-19 and SV have been arbitrarily taken as $x(0) = [5\ 1]^T$. The selection of the initial values is not critical as the properly constructed model will yield these initial values to converge to the measurements. Thes standard deviation of the process noise has been chosen as 20^2 while the measurement noise standard deviation has been set at 1.7. Here the measurement noise standard deviation is determined from the recorded data and it is fixed. On the other, hand the process noise variance is a modeling parameter and it can be thought as a parameter that determines the level of variation of the states, in this case, COVID-19 trend and the IV. Forgetting factor (α) is 1.5.

The daily prevalence data of COVID-19 from January 20, 2020, to July 28, 2020, were collected from the official website of Johns Hopkins University (https://gisanddata.maps.arcgis.com/apps/opsdashboard/index.html), and Excel 2019 was used to build a time-series database [40].

Actual cumulative case estimations that have been made online using AKF. The number of daily cases can be easily calculated with $i_t = y_t - y_{t-1}$ to show the total number of cases up to y_t, t days. Since we have the estimates of y_t, we can easily find the estimations of y_t with $\hat{i}_t = \hat{y}_t - \hat{y}_{t-1}$. Daily cases and estimations are given in Fig.1-Fig.4. According to the estimation results obtained by using the daily number of cases in the DLM MSE, MAPE, and R^2, were calculated (see Table 1). These calculated values indicate that the compatibility of the model with real data is quite high. This situation tells us that estimating the daily number of cases via the DLM is a reliable method. As for AKF, utilizing only the observation in time t and the preceding estimation is the most advantageous aspect of this method.

Table 1. Calculated R^2, MSE

Region	MSE	R^2
USA-Florida	30289	0.99792
USA-Texas	40169	0.99610
USA-Arizona	19329	0.98807
USA-New York	30045	0.99661

These results have revealed that with the given system model and the assumptions, AKF could successfully be used to decompose a real COVID-19 sequence into the COVID-19 trend and SV components and estimate these components from the observed cumulative COVID-19 sequence patterns. The method estimates online. Only the data on the last day is sufficient. To the best of our knowledge, the estimation of COVID-19 has not been studied with this method.

2.2 Computation of the reproduction number with AKF

There are variants of the reproduction number, such as the basic reproduction number, the effective reproduction number, the case reproduction number, and the instantaneous reproduction number. The instantaneous reproduction number, R_t, can be estimated by the ratio of the number of new infections generated at time step t, i_t, to the total infectiousness of infected individuals at time t, given by $\sum_{s=1}^{t} i_{t-s} w_s$, the sum of infection incidence up to time step $t-1$, weighted by the infectivity function w_s. R_t is the average number of secondary cases that each infected individual would infect if the conditions remained as they were at time t. R_t is the only reproduction number easily estimated in real-time. Moreover, effective control measures undertaken at time t are expected to result in a sudden decrease in R_t. Hence, assessing the efficiency of control measures is easier by using estimates of R_t. For these reasons, we focus on estimating the instantaneous reproduction number R_t in this article. (See Anne Cori et al. [41]). Given the definition of R_t stated above, the incidence of cases at time step t is, on average, $E(i_t) = R_t \sum_{s=1}^{t} i_{t-s} w_s$, where $E(X)$ denotes the expectation of a random variable X, and i_{t-s} is the incidence at the time step $t-s$. The instantaneous reproduction number, R_t at time t can be estimated as in Eq. (8).

$$R_t = \frac{E(i_t)}{\sum_{s=1}^{t} i_{t-s} w_s} \tag{9}$$

In equation (9), i_t stands for the number of new infections generated at time step t, whereas w_s is the probability distribution of the infectivity profile which is dependent on time since the infection of the case, s, but independent of calendar time, t. Hence, an individual will be most infectious at time s when w_s is the largest. w_s is typically related to individual biological factors such as symptom severity. Effective control measures undertaken at time t are expected to result in a sudden decrease in R_t, whereas the other reproduction number variants tend to respond rather slowly. Therefore, evaluating the efficiency of control measures is more effective when estimates of R_t are used. in practice, w_s is approximated by the distribution of the serial interval. In this article, we have taken the distribution of w_s as a uniform distribution in a $f(w_s) = 1/7,\ s = 1,2,...,7$ form. Since $E(i_t) = \hat{i}_t$, Eq. (9) can be written in the form of Eq. (10).

$$R_t = \frac{\hat{i}_t}{\frac{1}{7}\sum_{s=1}^{t} i_{t-s}},\ t = 8,9,...,n-1 \tag{10}$$

The value of R_t calculated using the Eq. (10) is given in Fig.1-Fig.4. There is no need for any other model assumption in estimating R_t with this method by using the DLM. Modeling the cumulative case time-series with the DLM stochastic process and estimating the with AKF both estimate the number of daily cases and estimate the instantaneous reproduction number without any other operation. It is quite a simple method to model the daily case number time series with the DLM stochastic process and estimated the with online AKF.

3. DISCUSSION

In this study, cumulative and daily cases have been estimated online using discrete-time DLM and AKF based on the total of COVID-19 cases between February and July 28, 2020, in USA-Florida, USA-Texas, USA-Arizona, USA-New York. The cumulative case number was modeled with DLM, and the stochastic time series were estimated by online AKF. Estimation by acquired data observed between March and July 28, 2020, shows that employing the discrete-time DLM and AKF in terms of MSE, MAPE, and R^2 provides efficient analysis for modeling the total case. It is proposed that the use of discrete-time DLM and AKF will be appropriate. After estimating the number of cumulative cases, the estimation of daily cases was made. After estimating the daily case number, the estimation of reproduction number was obtained. The DLM is an appropriate estimation method for the daily cases. As for AKF, utilizing only the observation in time t and preceding the estimation is the most advantageous aspect of this method. Modeling the cumulative case time-series with the DLM stochastic process and estimating the with AKF both leads to the number of daily cases and the instantaneous reproduction number without any other operation. It is quite a simple method to model the cumulative case number time series with the DLM stochastic process and estimate the with online AKF. Among the studies made on the COVID-19 pandemic, the progress of modeling the disease is remarked primarily. The progress of modeling the disease is substantial for the precautions which will be taken by countries and interventions, and treatments to be administered. As a result of estimations by acquired data taken observed between Marc and July 28, 2020, it is proposed that the efficient analysis for modeling the total case is to be made using the discrete-time DLM and

AKF in terms of MSE, and R^2. It is thought that the method we have proposed will be suitable for the estimation of the forthcoming progress. Our suggestion is that the simplest method for the estimation of the reproduction number can be performed by modeling the daily case number time series using DLM.

Appendix: Discerete state-space model and adaptive Kalman filter

Let us consider a general linear discrete-time stochastic system represented by the state and measurement models given by

$$x_{t+1} = F_t x_t + G_t w_t$$

$$y_t = H_t x_t + v_t$$

where x_t is an n×1 system vector, y_t is an m×1 observation vector, F_t is an n×n system matrix, H_t is an m×n matrix, w_t an n×1 vector of zero mean white noise sequence and v_t is an m×1 measurement error vector assumed to be a zero mean white sequence uncorrelated with the w_t sequence. The covariance matrices w_t and w_t are defined by $w_t \sim \mathrm{N}(0,\mathrm{Q}_t)$, $v_t \sim \mathrm{N}(0,\mathrm{R}_t)$. The filtering problem is the problem of determining the best estimate of its x_t condition, given its observations $Y_t = (y_0, y_1, ..., y_t)$ [13-16]. When $Y_t = (y_0, y_1, ..., y_t)$ observations are given, the prediction of state x_t with

$$\hat{x}_t = E(x_t | y_0, y_1, ..., y_t) = E(x_t | Y_t)$$

and the covariance matrix of the error with

$$P_{t|t} = E\left[(x_t - \hat{x}_{t|t})(x_t - \hat{x}_{t|t})' | Y_t \right]$$

when $Y_{t-1} = (y_0, y_1, ..., y_{t-1})$ observations are given, the prediction of state x_t with

$$\hat{x}_{t|t-1} = E(x_t | y_0, y_1, ..., y_{t-1}) = E(x_t | Y_{t-1})$$

and the covariance matrix of the error are shown with

$$P_{t|t-1} = E\left[(x_t - \hat{x}_{t|t-1})(x_t - \hat{x}_{t|t-1})' | Y_{t-1} \right].$$

Let the initial state be assumed to have a normal distribution in the form of $x_0 \sim \mathrm{N}(\bar{x}_0, \mathrm{P}_0)$. The optimum update equations for Kalman Filter (KF) are

$$\hat{x}_{t|t-1} = F_{t-1}\hat{x}_{t-1}$$

$$P_{t|t-1} = F_{t-1}P_{t-1|t-1}F_{t-1}' + G_{t-1}Q_{t-1}G_{t-1}'$$

$$K_t = P_{t|t-1}H_t'(H_t P_{t|t-1} H_t' + R_t)^{-1}$$

$$P_{t|t} = [I - K_t H_t]P_{t|t-1}$$

$$\hat{x}_t = \hat{x}_{t|t-1} + K_t(y_t - H_t \hat{x}_{t|t-1})$$

In the above equations $\hat{x}_{t/t-1}$ is the a priori estimation and $\hat{x}_t$ is the a posteriori estimation of x_t. Also, $P_{t|t-1}$ and $P_{t|t}$ are the covariance of a priori and a posteriori estimations respectively. Asymptotic distribution theory for the Kalman filter state estimator and Central Limit Theorem was investigated in [33-34]. In order to eliminate divergence in the KF, adaptive methods are used [35-39] forgetting factor is proposed by Ozbek [36], Özbek and Aliev [37].

$$P_{t|t-1} = \alpha\left(F_{t-1}P_{t-1|t-1}F_{t-1}^{'} + G_{t-1}Q_{t-1}G_{t-1}^{'}\right)$$

Here, for $\alpha = 1$, the resulting filter is the same as the standard KF, whereas for $\alpha > 1$ the filter has an adaptive nature via exponential data weighting. The idea behind using a forgetting factor is to emphasize the effect of current data artificially by exponentially weighting the observations.

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CONFLICT OF INTEREST

The authors declared no conflict of interest.

ETHICAL APPROVAL

This article does not contain any studies involving human participants performed by any of the authors. This study was in accordance with the ethical standards of the institutional research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards.

DATA AVA I L A B I LIT Y S TATEMENT

Partial data that support the findings of this study are available in the Johns Hopkins University Center for Systems Science and Engineering site. However, partial data are available on request from the corresponding author, due to privacy restrictions.

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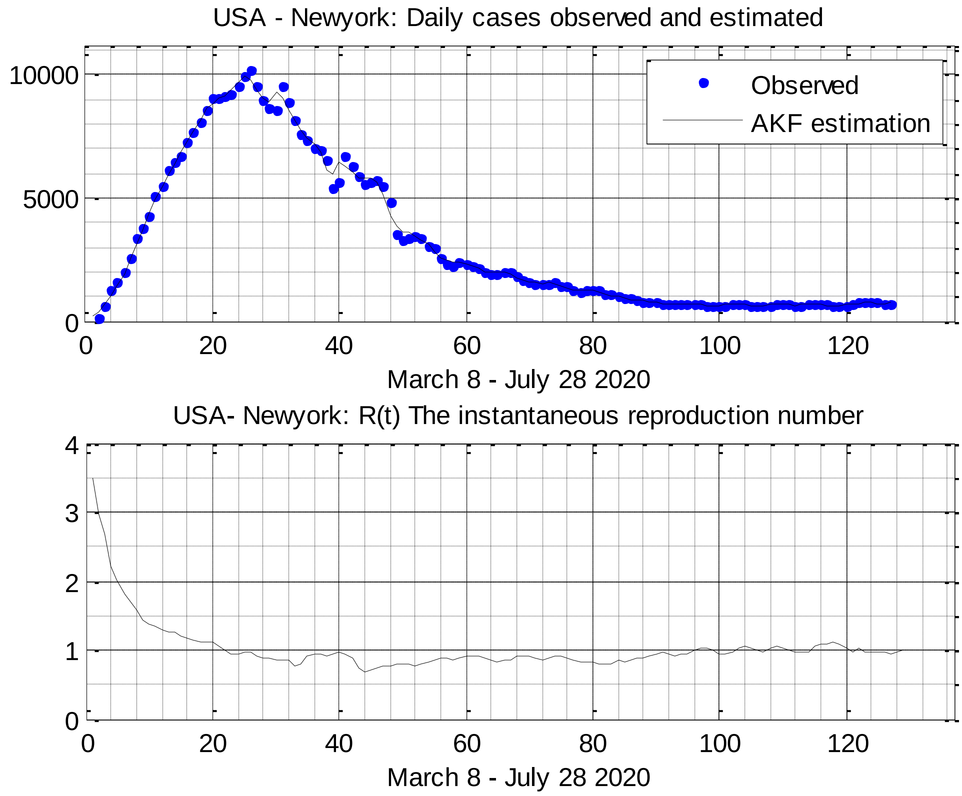

Figure-1. New York: Daily cases observed and estimated, the instantaneous reproduction number estimation.

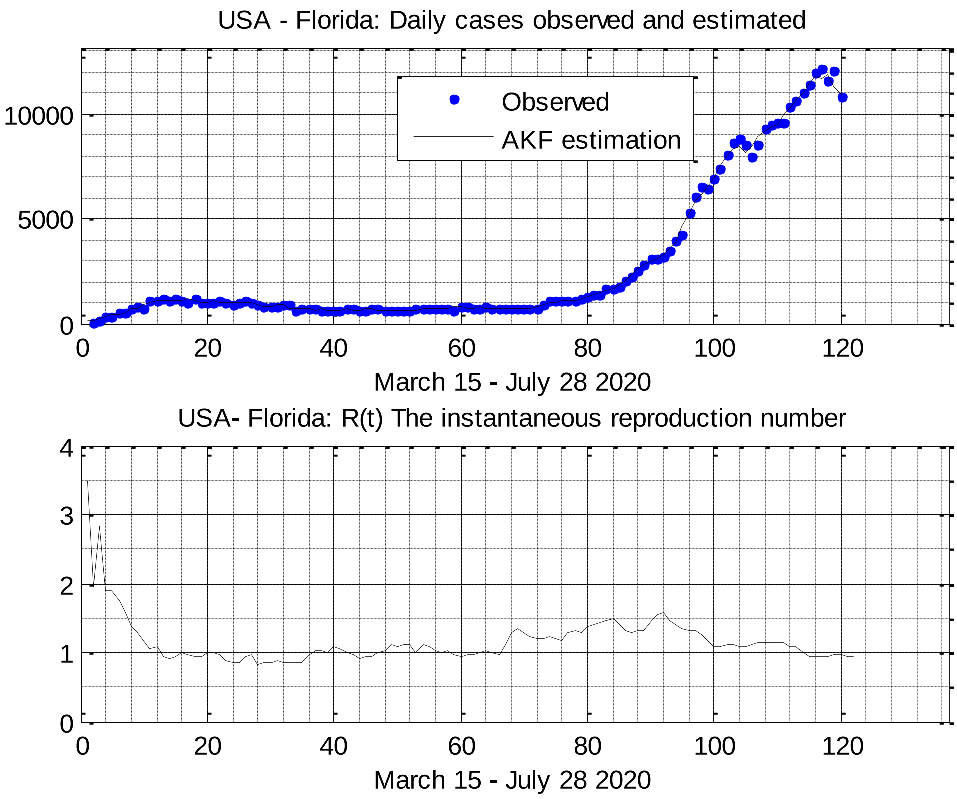

Figure-2. Florida: Daily cases observed and estimated, the instantaneous reproduction number estimation.

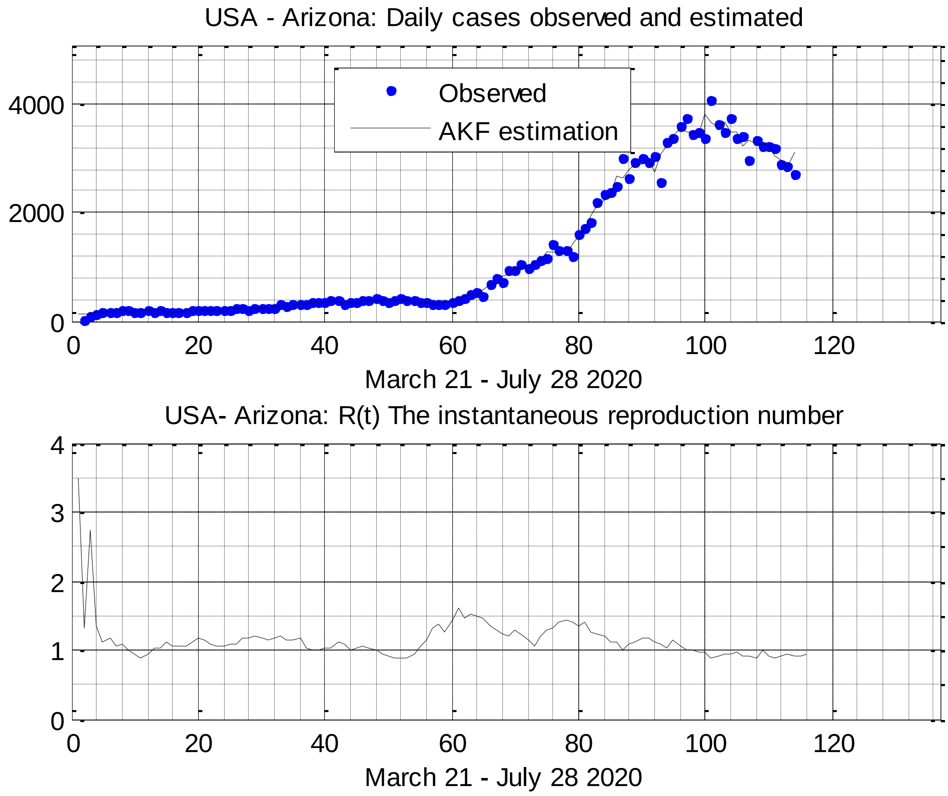

Figure-3. Arizona: Daily cases observed and estimated, the instantaneous reproduction number estimation.

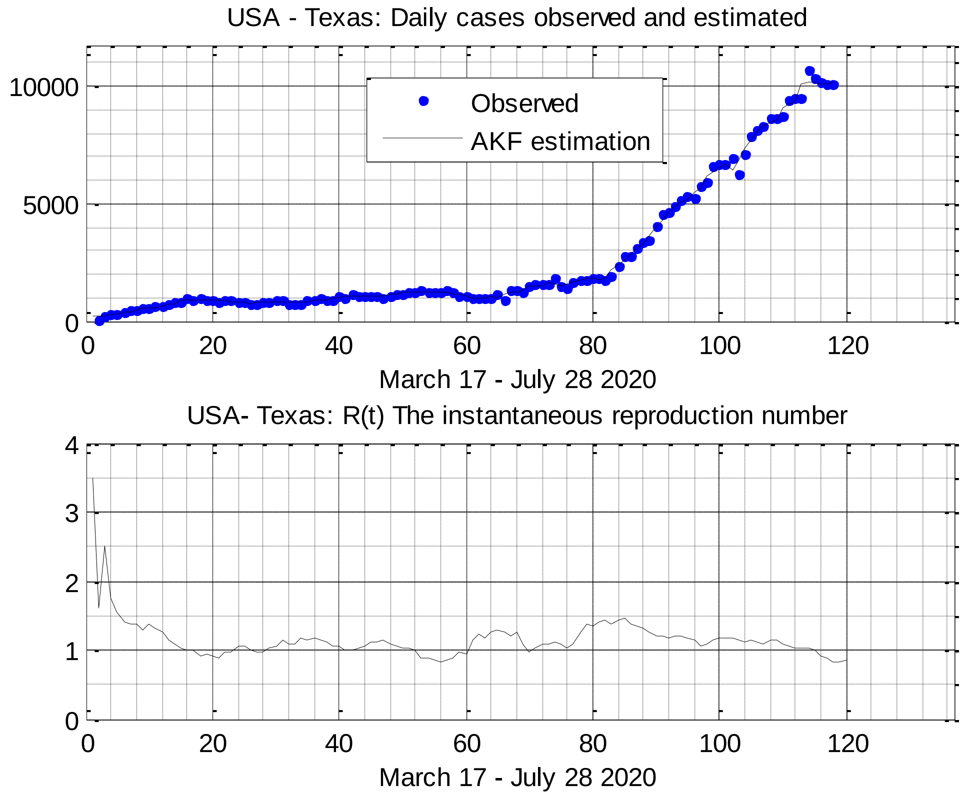

Figure-4. Texas: Daily cases observed and estimated, the instantaneous reproduction number estimation.