THE RIGOROUS DERIVATION OF UNIPOLAR EULER-MAXWELL SYSTEM FOR ELECTRONS FROM BIPOLAR EULER-MAXWELL SYSTEM BY INFINITY-ION-MASS LIMIT

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Abstract. In the paper, we consider the local-in-time and the global-in-time infinity-ion-mass convergence of bipolar Euler-Maxwell systems by setting the mass of an electron $m_e = 1$ and letting the mass of an ion $m_i \to +\infty$. We use the method of asymptotic expansions to handle the local-in-time convergence problem and find that the limiting process from bipolar models to unipolar models is actually decoupling, but not the vanishing of equations for the corresponding the other particle. Moreover, when the initial data is sufficiently close to the constant equilibrium state, we establish the global-in-time infinity-ion-mass convergence.

Keywords: Euler-Maxwell system; infinity-ion-mass limit; unipolar; bipolar; local convergence; global convergence.

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1. Introduction

In the paper, we consider the local-in-time and global-in-time infinity-ion-mass convergence of the bipolar Euler-Maxwell system, which is an important model in plasma physics. The infinity-ion-mass limit means letting the ratio of the mass of an ion to that of an electron tends to infinity. We study these problems in the case of periodic solutions. Let $\mathbb{T}^3$ be a torus in $\mathbb{R}^3$. We denote by $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ the space variable and $t > 0$ the time variable. For $\nu = i, e$, where i stands for ions and e stands for electrons, a bipolar Euler-Maxwell system in a three-dimensional torus is under the form [3, 4, 25]

$$\begin{cases} \partial_t \rho_\nu + \mathrm{div}(\rho_\nu u_\nu) = 0, \\ m_\nu \partial_t(\rho_\nu u_\nu) + m_\nu \mathrm{div}(\rho_\nu u_\nu \otimes u_\nu) + \nabla p_\nu(\rho_\nu) = q_\nu \rho_\nu (E + u_\nu \times B) - m_\nu \rho_\nu u_\nu, \\ \partial_t E - \nabla \times B = -(q_i \rho_i u_i + q_e \rho_e u_e), \quad \mathrm{div} E = q_i \rho_i + q_e \rho_e, \\ \partial_t B + \nabla \times E = 0, \quad \mathrm{div} B = 0, \quad t > 0, \quad x \in \mathbb{T}^3, \end{cases} \tag{1.1}$$

with initial conditions

$$t = 0: \quad (\rho_\nu, u_\nu, E, B) = (\rho_{\nu,0}, u_{\nu,0}, E_0, B_0), \quad x \in \mathbb{T}^3. \tag{1.2}$$

Here $\otimes$ stands for the tensor product, ρ_i and u_i (respectively, ρ_e and u_e) stand for the density and velocity vector of ions (respectively, electrons), E is the electric field and B is the magnetic field. The parameters m_i and $q_i = 1$ (respectively, m_e and $q_e = -1$) stand for the mass and the charge of an ion (respectively, an electron). The pressure functions $p_\nu(\rho)$ are supposed to be smooth and strictly increasing for all $\rho > 0$, namely,

$$p_\nu'(\rho) > 0, \quad \forall \rho > 0, \quad \nu = i, e.$$

System (1.1) admits an equilibrium state

$$(\rho_e, \rho_i, u_e, u_i, E, B) = (1, 1, 0, 0, 0, B_e),$$

where $B_e \in \mathbb{R}^3$ is an arbitrary constant vector. For smooth solutions with $\rho_\nu > 0$, the momentum equations in (1.1) are equivalent to

$$m_\nu \partial_t u_\nu + m_\nu (u_\nu \cdot \nabla) u_\nu + \nabla h_\nu(\rho_\nu) = q_\nu (E + u_\nu \times B) - m_\nu u_\nu, \tag{1.3}$$

where h is the enthalpy function, defined by

$$h_\nu'(\rho) = \frac{p_\nu'(\rho)}{\rho}.$$

Since p is strictly increasing, so is h. The terms $q_\nu(E + u_\nu \times B)$ and $m_\nu u_\nu$ on the right hand side of (1.3) represents the Lorentz force and the velocity dissipation, respectively.

The bipolar Euler-Maxwell system (1.1) is symmetrizable hyperbolic for $\rho_\nu > 0$, then the problem (1.1)-(1.2) admits a local smooth solution according to Lax [16] and Kato [14]. For smooth initial data, the global existence of smooth solutions, which are sufficiently close to the equilibrium state, to (1.1)-(1.2) was obtained by Peng [19] both in a torus and in the whole space, while Duan-Liu-Zhu [7] studied the corresponding decay rate problem in the whole space. In Xu-Xiong-Kawashima [31], the well-posedness of (1.1)-(1.2) was established in critical Besov spaces. In addition, if there is no velocity dissipation in (1.3), Guo-Ionescu-Pausader [11] proved the global existence of smooth solutions to (1.1)-(1.2) in the whole space with the extra general irrotationality condition

$$B = \nabla \times u_e = -\nabla \times u_i.$$

We refer to [8, 10, 27] and the reference therein for more related topics.

Physicians observe that in plasma physics, electrons move much more rapidly than ions. Hence, when establishing unipolar models, they often regard ions as non-moving and becoming a uniform background with a fixed unit density for simplicity. As a result, the equations for ions are neglected. In fact, if we assume

$$\rho_i = b(x), \quad u_i = 0, \tag{1.4}$$

in which $b(x)$ denotes the doping profile, then system (1.1) becomes the following unipolar Euler-Maxwell model for electrons,

$$\begin{cases} \partial_t \rho_e + \mathrm{div}(\rho_e u_e) = 0, \\ \partial_t u_e + (u_e \cdot \nabla) u_e + \nabla h_e(\rho_e) = -E - u_e \times B - u_e, \\ \partial_t E - \nabla \times B = \rho_e u_e, \quad \mathrm{div} E = b(x) - \rho_e, \\ \partial_t B + \nabla \times E = 0, \quad \mathrm{div} B = 0, \end{cases} \tag{1.5}$$

which has been widely studied. In Chen-Jerome-Wang [5], a global existence result of weak solutions in a one-dimensional space was established by using the fractional step Godunov scheme together with a compensated compactness argument. For smooth initial data, the global existence of smooth solutions, which are sufficiently close to the equilibrium state, to (1.5) was obtained in [23, 6, 30]. Moreover, the asymptotic analysis of (1.5) for smooth solutions is also a well known problem. For the local-in-time convergence of small parameters, we refer to [20, 21, 22, 6, 23, 30] and the reference therein. By establishing the uniform global estimates with respect to small parameters, the global-in-time convergence of small parameters for (1.5) are studied in [23, 24, 28].

Although the unipolar model (1.5) has been formally established, its derivation from the bipolar model (1.1) is only based on physical observations and assumptions (1.4), which lacks rigorous proof in mathematics. In other words, we need to prove correctly and explain clearly why the unipolar model is well-defined and can be regarded as the simplification of the bipolar model. However, it is not easy. One of the reasons is that there is no general method and procedure by now to reveal the relationship of the bipolar and the corresponding unipolar model which is effective for almost all fluid models.

Fortunately, in the perspective of mass, there are reasonable attempts. Based on the fact that ions are much heavier than electrons, we let the ratio $m_e/m_i \to 0$. There are two different ways to consider this limiting process. The first is setting $m_e = 1$ and letting

$m_i \to +\infty$, which is called the **infinity-ion-mass limit**, and was recently introduced in [29]. The second is setting $m_i = 1$ and letting $m_e \to 0$, which is called the **zero-electron-mass limit**. To some extent, the relationship of the bipolar model and the corresponding unipolar model can be explained well in both limits, through which the corresponding unipolar model can both be obtained from the bipolar model.

The zero-electron-mass limit in bipolar models has a long research history, and was first mathematically introduced in [13] for bipolar drift-diffusion equations. In [9], the local-in-time convergence of the zero-electron-mass limit was studied in a bounded domain in a one-dimensional space for the following bipolar Euler-Poisson system,

$$\begin{cases} \partial_t \rho_\nu + \operatorname{div}(\rho_\nu u_\nu) = 0, \\ m_\nu \partial_t(\rho_\nu u_\nu) + m_\nu \operatorname{div}(\rho_\nu u_\nu \otimes u_\nu) + \nabla p_\nu(\rho_\nu) = q_\nu \rho_\nu \nabla \phi - m_\nu \rho_\nu u_\nu, \\ \Delta \phi = \rho_i - \rho_e, \end{cases} \tag{1.6}$$

where $\rho_\nu, u_\nu, m_\nu, p_\nu$ and q_ν are defined in the same way as in (1.1), and ϕ is the scaled electric potential. See also [29] for the corresponding Cauchy problem in the whole space of any dimension. Similar to the bipolar Euler-Maxwell system, when ions are regarded as non-moving, various problems concerning the zero-electron-mass limit have been studied for the following unipolar Euler-Poisson model for electrons,

$$\begin{cases} \partial_t \rho_e + \operatorname{div}(\rho_e u_e) = 0, \\ m_e \partial_t(\rho_e u_e) + m_e \operatorname{div}(\rho_e u_e \otimes u_e) + \nabla p_e(\rho_e) = -\rho_e \nabla \phi - m_e \rho_e u_e, \\ \Delta \phi = b - \rho_e, \end{cases}$$

where b is the doping profile which is assumed to be constant. For the local-in-time convergence, we refer to [1] for the case of periodic solutions and [2] for Cauchy problem with both well- and ill-prepared initial data. For the convergence of periodic solutions in critical Besov spaces, we refer to [32, 33]. In Li-Peng-Xi [17], they studied the periodic case when the doping profile b is not a constant but a function of the space variable x. However, as pointed out in [29], when the zero-electron-mass limit is applied in bipolar models, it is not proper to simply ignore the effect of ions, and only make the asymptotic analysis to the equations for electrons. In fact, rather than staying the same, the equations for ions have a limiting process, of which the key point is actually decoupling, but not the vanishing of equations.

The zero-electron-mass limit works well in the bipolar Euler-Poisson system, but not for the bipolar Euler-Maxwell system. This is because there are strong coupling in the form of the Lorentz force in momentum equations in (1.3). Indeed, when the zero-electron-mass limit is applied to the bipolar Euler-Poisson model (1.6), formally we get the Boltzmann relation $\nabla h_e(\rho_e) = -\nabla\phi$. Together with the Poisson equation, we have

$$\rho_e - \Delta h_e(\rho_e) = \rho_i.$$

This implies the solvability of ρ_e in the expression of ρ_i, which leads to the unipolar Euler-Poisson model for ions that does not contain any information of electrons. By now the decoupling is successful (See details in [29]). Contrarily, when the zero-electron-mass limit is applied to the bipolar Euler-Maxwell system (1.1), formally the momentum equation for electrons becomes

$$\nabla h_e(\rho_e) = -E - u_e \times B.$$

Due to the Lorentz force on the right hand side, it is obvious that the decoupling is not successful. However, if setting $m_e = 1$ and letting $m_i \to +\infty$, we can avoid this strong coupling. In fact, when the infinity-ion-mass limit is applied to the bipolar Euler-Maxwell system (1.1), the formal limit of the momentum equation for ions becomes

$$\partial_t u_i + (u_i \cdot \nabla) u_i = -u_i.$$

It is a transport equation only for u_i, which results in the success in decoupling. See the details in Section 2. That is why we apply the infinity-ion-mass limit.

The aim of the present paper is to establish both the local-in-time and the global-in-time convergence of the infinity-ion-mass limit for the bipolar Euler-Maxwell system (1.1), of which the limiting system is the unipolar Euler-Maxwell model for electrons. The paper is organized as follows. In Section 2, we give some preliminaries and state our main results. In Section 3, for sufficiently smooth initial data, we establish the local-in-time convergence. Section 4 is devoted to the problem of global-in-time convergence.

2. Preliminaries and Main results

2.1. **Notations and inequalities.** In the paper, we let $m_e = 1$ and define the small parameter $\varepsilon = m_i^{-1/2}$. Thus the infinity-ion-mass limit means letting $\varepsilon \to 0$. We denote $s \geq 3$ an integer and C a generic positive constant independent of the small parameter ε. For a multi-index $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in \mathbb{N}^3$, we denote

$$\partial^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \partial x_3^{\alpha_3}} \quad \text{with} \quad |\alpha| = \alpha_1 + \alpha_2 + \alpha_3.$$

For simplicity, we denote by $\|\cdot\|$, $\|\cdot\|_\infty$ and $\|\cdot\|_l$ the usual norms of $L^2 \overset{def}{=} L^2(\mathbb{T}^3)$, $L^\infty \overset{def}{=} L^\infty(\mathbb{T}^3)$ and $H^l \overset{def}{=} H^l(\mathbb{T}^3)$ for all integers $l \geq 1$, respectively. We will repeatedly use the fact that for integers $s \geq 3$, the embedding $H^{s-1} \hookrightarrow L^\infty$ is continuous. The inner product in $L^2(\mathbb{T}^3)$ is denoted as $\langle \cdot, \cdot \rangle$. Throughout the paper, we denote $\nu = i, e$, where i

stands for ions and e stands for electrons. We first introduce the Moser-type inequalities, which we will frequently use in later proof.

Lemma 2.1. *(Moser-type calculus inequalities* [15, 18]*) . Let $l \geq 3$ be an integer. For all $\alpha \in \mathbb{N}^3$ with $1 \leq |\alpha| \leq l$, if $u \in H^l$ and $v \in H^{|\alpha|}$, then*

$$\begin{aligned} \|\partial^\alpha(uv) - u\partial^\alpha v\| &\leq C\|\nabla u\|_{l-1}\|v\|_{|\alpha|-1}, \\ \|\partial^\alpha(uv)\| &\leq C\|u\|_l\|v\|_l. \end{aligned}$$

2.2. **Results on the local-in-time convergence.** We first consider the local-in-time convergence of the infinity-ion-mass limit for the bipolar Euler-Maxwell system. When concerning the local-in-time convergence, it is not necessary to introduce the velocity dissipation term in the momentum equations (1.3). Hence, (1.1)-(1.2) becomes

$$\begin{cases} \partial_t \rho_i^\varepsilon + \mathrm{div}(\rho_i^\varepsilon u_i^\varepsilon) = 0, \\ \partial_t u_i^\varepsilon + (u_i^\varepsilon \cdot \nabla) u_i^\varepsilon + \varepsilon^2 \nabla h_i(\rho_i^\varepsilon) = \varepsilon^2 (E^\varepsilon + u_i^\varepsilon \times B^\varepsilon), \\ \partial_t \rho_e^\varepsilon + \mathrm{div}(\rho_e^\varepsilon u_e^\varepsilon) = 0, \\ \partial_t u_e^\varepsilon + (u_e^\varepsilon \cdot \nabla) u_e^\varepsilon + \nabla h_e(\rho_e^\varepsilon) = -E^\varepsilon - u_e^\varepsilon \times B^\varepsilon, \\ \partial_t E^\varepsilon - \nabla \times B^\varepsilon = \rho_e^\varepsilon u_e^\varepsilon - \rho_i^\varepsilon u_i^\varepsilon, \quad \mathrm{div} E^\varepsilon = \rho_i^\varepsilon - \rho_e^\varepsilon, \\ \partial_t B^\varepsilon + \nabla \times E^\varepsilon = 0, \quad \mathrm{div} B^\varepsilon = 0, \quad t > 0, \quad x \in \mathbb{T}^3, \\ t = 0: \quad (\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon) = (\rho_{\nu,0}^\varepsilon, u_{\nu,0}^\varepsilon, E_0^\varepsilon, B_0^\varepsilon), \quad x \in \mathbb{T}^3. \end{cases} \tag{2.1}$$

By the theory of Lax [16] and Kato [14] for the symmetrizable hyperbolic system, we have

Proposition 2.1. *(Local existence of smooth solutions) Let $s \geq 3$ be an integer and the initial data $\left(\rho_{\nu,0}^\varepsilon,\, u_{\nu,0}^\varepsilon\right) \in H^s$ with $\rho_{\nu,0}^\varepsilon \geq 2\underline{\rho}$ for some given positive constant $\underline{\rho} > 0$ independent of ε. Assume that the initial data E_0^ε and B_0^ε satisfy the following compatibility condition*

$$\mathrm{div} E_0^\varepsilon = \rho_{i,0}^\varepsilon - \rho_{e,0}^\varepsilon, \quad \mathrm{div} B_0^\varepsilon = 0,$$

then there exists $T_e^\varepsilon > 0$ such that the periodic problem (2.1) *has a unique smooth solution $(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon,\, E^\varepsilon, B^\varepsilon)$ defined on the time interval $[0, T_e^\varepsilon]$, satisfying $\rho_\nu^\varepsilon \geq \underline{\rho}$ and*

$$(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon) \quad \in \quad C\left([0,\, T_e^\varepsilon]\,;\, H^s\right) \cap C^1\left([0,\, T_e^\varepsilon]\,;\, H^{s-1}\right).$$

2.2.1. *Asymptotic expansion.* We look for an approximation of solution $(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)$ to (2.1) under the form of a power series in ε. Assume that the initial data of $(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)$ admit an asymptotic expansion with respect to ε,

$$(\rho_{\nu,0}^\varepsilon, u_{\nu,0}^\varepsilon, E_0^\varepsilon, B_0^\varepsilon)(x) = \sum_{j\geq 0} \varepsilon^{2j}\left(\bar\rho_\nu^j,\, \bar u_\nu^j,\, \bar E^j, \bar B^j\right)(x), \tag{2.2}$$

where $\left(\bar{\rho}_\nu^j,\, \bar{u}_\nu^j,\, \bar{E}^j, \bar{B}^j\right)_{j\geq 0}$ are sufficiently smooth. Then we make the following ansatz,

$$(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon,\, E^\varepsilon,\, B^\varepsilon)\,(t,\, x) = \sum_{j\geq 0} \varepsilon^{2j} \left(\rho_\nu^j,\, u_\nu^j,\, E^j,\, B^j\right)(t,\, x). \tag{2.3}$$

In what follows, we use a formal expansion defined by

$$h_\nu \left(\sum_{j\geq 0} \varepsilon^{2j} \rho_\nu^j\right) = h_\nu\left(\rho_\nu^0\right) + \varepsilon^2 h_\nu'(\rho_\nu^0)\rho_\nu^1 + \sum_{j\geq 2} \varepsilon^{2j}\left[h_\nu'(\rho_\nu^0)\rho_\nu^j + h_\nu^{j-1}\left((\rho_\nu^k)_{k\leq j-1}\right)\right],$$

where $\{h_\nu^j\}_{j\geq 1}$ are smooth functions depending only on $\left(\rho_\nu^k\right)_{k\leq j}$. Substituting the expansions (2.3) into system (2.1) and comparing the coefficients before the powers of ε, we obtain

(1) The leading profiles $(\rho_\nu^0,\, u_\nu^0,\, E^0,\, B^0)$ from the coefficients of ε^0 satisfy the following system

$$\begin{cases} \partial_t \rho_i^0 + \mathrm{div}(\rho_i^0 u_i^0) = 0, \\ \partial_t u_i^0 + (u_i^0 \cdot \nabla) u_i^0 = 0, \\ \partial_t \rho_e^0 + \mathrm{div}(\rho_e^0 u_e^0) = 0, \\ \partial_t u_e^0 + (u_e^0 \cdot \nabla) u_e^0 + \nabla h_e(\rho_e^0) = -(E^0 + u_e^0 \times B^0), \\ \partial_t E^0 - \nabla \times B^0 = \rho_e^0 u_e^0 - \rho_i^0 u_i^0, \quad \mathrm{div} E^0 = \rho_i^0 - \rho_e^0, \\ \partial_t B^0 + \nabla \times E^0 = 0, \quad \mathrm{div} B^0 = 0. \end{cases} \tag{2.4}$$

with initial conditions

$$t = 0: \quad (\rho_\nu^0, u_\nu^0, E^0, B^0) = (\bar{\rho}_\nu^0, \bar{u}_\nu^0, \bar{E}^0, \bar{B}^0). \tag{2.5}$$

The second equation in (2.4) is symmetrizable hyperbolic. Due to the theory of Lax [16] and Kato [14], a unique local smooth solution u_i^0 exists. Once u_i^0 is known, the first equation in (2.4) becomes linear. Obviously, a unique local smooth solution ρ_i^0 exists. In conclusion, a unique local smooth solution (ρ_i^0, u_i^0) exists, which satisfies the following

$$\begin{cases} \partial_t \rho_i^0 + \mathrm{div}(\rho_i^0 u_i^0) = 0, \\ \partial_t u_i^0 + (u_i^0 \cdot \nabla) u_i^0 = 0, \\ t = 0: \quad (\rho_i^0, u_i^0) = (\bar{\rho}_i^0, \bar{u}_i^0), \quad x \in \mathbb{T}^3. \end{cases} \tag{2.6}$$

Since (ρ_i^0, u_i^0) is known, the remaining equations in (2.4) become

$$\begin{cases} \partial_t \rho_e^0 + \mathrm{div}(\rho_e^0 u_e^0) = 0, \\ \partial_t u_e^0 + (u_e^0 \cdot \nabla) u_e^0 + \nabla h_e(\rho_e^0) = -(E^0 + u_e^0 \times B^0), \\ \partial_t E^0 - \nabla \times B^0 = \rho_e^0 u_e^0 - \rho_i^0(t,x) u_i^0(t,x), \quad \mathrm{div} E^0 = \rho_i^0(t,x) - \rho_e^0, \\ \partial_t B^0 + \nabla \times E^0 = 0, \quad \mathrm{div} B^0 = 0, \\ t = 0: (\rho_e^0, u_e^0, E^0, B^0) = (\bar{\rho}_e^0, \bar{u}_e^0, \bar{E}^0, \bar{B}^0), \quad x \in \mathbb{T}^3, \end{cases} \tag{2.7}$$

which we call the generalized unipolar Euler-Maxwell system for electrons. Since it is symmetrizable hyperbolic for $\rho_e > 0$, a unique local smooth solution $(\rho_e^0, u_e^0, E^0, B^0)$ to (2.7) exists due to the theory of Lax [16] and Kato [14].

(2) When $j = 1$, the profiles $(\rho_\nu^1,\ u_\nu^1,\ E^1,\ B^1)$ from the coefficients of ε^2 satisfy the following linear system

$$\begin{cases} \partial_t \rho_i^1 + \mathrm{div}(\rho_i^0 u_i^1 + \rho_i^1 u_i^0) = 0, \\ \partial_t u_i^1 + (u_i^0 \cdot \nabla) u_i^1 + (u_i^1 \cdot \nabla) u_i^0 = -\nabla h_i(\rho_i^0) + E^0 + u_i^0 \times B^0, \\ \partial_t \rho_e^1 + \mathrm{div}(\rho_e^0 u_e^1 + \rho_e^1 u_e^0) = 0, \\ \partial_t u_e^1 + (u_e^1 \cdot \nabla) u_e^0 + (u_e^0 \cdot \nabla) u_e^1 + \nabla\left(h_e'(\rho_e^0)\rho_e^1\right) + E^1 + u_e^0 \times B^1 + u_e^1 \times B^0 = 0, \\ \partial_t E^1 - \nabla \times B^1 - (\rho_e^1 u_e^0 + \rho_e^0 u_e^1) + \rho_i^0 u_i^1 + \rho_i^1 u_i^0 = 0, \quad \mathrm{div} E^1 = \rho_i^1 - \rho_e^1, \\ \partial_t B^1 + \nabla \times E^1 = 0, \quad \mathrm{div} B^1 = 0, \end{cases} \tag{2.8}$$

with initial conditions

$$t = 0: \quad (\rho_\nu^1, u_\nu^1, E^1, B^1) = (\bar\rho_\nu^1, \bar u_\nu^1, \bar E^1, \bar B^1). \tag{2.9}$$

(3) In general, for $j \geq 2$, the profiles $(\rho_\nu^j,\ u_\nu^j,\ E^j,\ B^j)$ are obtained by induction. Assume that the coefficients $\left(\rho_\nu^k,\ u_\nu^k,\ E^k, B^k\right)_{0\le k\le j-1}$ are smooth and already determined in previous steps, then the coefficients $(\rho_\nu^j,\ u_\nu^j,\ E^j, B^j)$ of order ε^{2j} satisfy the linear system

$$\begin{cases} \partial_t \rho_i^j + \mathrm{div}\left(\rho_i^0 u_i^j + \rho_i^j u_i^0\right) = -\displaystyle\sum_{k=1}^{j-1} \mathrm{div}\left(\rho_i^k u_i^{j-k}\right), \\ \partial_t u_i^j + \left(u_i^0 \cdot \nabla\right) u_i^j + \left(u_i^j \cdot \nabla\right) u_i^0 \\ \qquad = -\nabla\left(h_i'(\rho_i^0)\rho_i^{j-1} + h_i^{j-2}\left((\rho_i^k)_{k\le j-2}\right)\right) + E^{j-1} - \displaystyle\sum_{k=1}^{j-1}\left((u_i^k \cdot \nabla) u_i^{j-k}\right) + \sum_{k=0}^{j-1}\left(u_i^k \times B^{k-j}\right), \\ \partial_t \rho_e^j + \mathrm{div}\left(\rho_e^0 u_e^j + \rho_e^j u_e^0\right) = -\displaystyle\sum_{k=1}^{j-1} \mathrm{div}\left(\rho_e^k u_e^{j-k}\right), \\ \partial_t u_e^j + \left(u_e^0 \cdot \nabla\right) u_e^j + \left(u_e^j \cdot \nabla\right) u_e^0 + \nabla\left(h_e'(\rho_e^0)\rho_e^j + h_e^{j-1}\left((\rho_e^k)_{k\le j-1}\right)\right) + E^j + u_e^j \times B^0 + u_e^0 \times B^j \\ \qquad = -\displaystyle\sum_{k=1}^{j-1}\left((u_e^k \cdot \nabla) u_e^{j-k} + u_e^k \times B^{j-k}\right), \\ \partial_t E^j - \nabla \times B^j + \rho_i^j u_i^0 + \rho_i^0 u_i^j - (\rho_e^j u_e^0 + \rho_e^0 u_e^j) = -\displaystyle\sum_{k=1}^{j-1}\left(\rho_i^k u_i^{j-k} - \rho_e^k u_e^{j-k}\right), \\ \mathrm{div} E^j - \rho_i^j + \rho_e^j = 0, \quad \partial_t B^j + \nabla \times E^j = 0, \quad \mathrm{div} B^j = 0, \end{cases} \tag{2.10}$$

with the initial conditions

$$\left(\rho_\nu^j,\, u_\nu^j, E^j, B^j\right)(0,\, x) = \left(\bar\rho_\nu^j,\, \bar u_\nu^j, \bar E^j, \bar B^j\right)(x). \tag{2.11}$$

In (2.8) and (2.10), all the terms on the right hand sides are known. Generally speaking, for $j \geq 1$, we can get $\left(\rho_i^j,\ u_i^j\right)$ from the first two equations in the system (2.8) or (2.10), and then insert (ρ_i^j, u_i^j) into the remaining equations in the system to get $(\rho_e^j,\, u_e^j,\, E^j, B^j)$. Hence, linear systems (2.8)-(2.9) and (2.10)-(2.11) admit unique local smooth solutions $(\rho_\nu^j, u_\nu^j, E^j, B^j)$ for all $j \geq 1$. We then have the following proposition.

Proposition 2.2. *Let the conditions in Proposition 2.1 hold. Assume* (2.2)*, in which* $\left(\bar\rho_\nu^j,\, \bar u_\nu^j,\, \bar E^j, \bar B^j\right)_{j\geq 0}$ *are sufficiently smooth, then there exists a positive time* $T_a > 0$*, which is independent of the small parameter* ε*, such that periodic problems* (2.4)-(2.5), (2.8)-(2.9) *and* (2.10)-(2.11) *admit respectively a unique local smooth solution defined in the time interval* $[0, T_a]$*. In other words, there exists a unique asymptotic expansion of the form* (2.3) *with profiles* $(\rho_\nu^j,\, u_\nu^j,\, E^j, B^j)_{j\geq 0}$ *defined on* $[0, T_a] \times \mathbb{T}^3$ *up to any order of the small parameter* ε.

2.2.2. *Error estimates and main result.* Let $m \geq 1$ be a fixed integer and denote the approximate solution of order m by

$$\left(\rho_{\nu,\varepsilon}^m,\, u_{\nu,\varepsilon}^m,\, E_\varepsilon^m, B_\varepsilon^m\right) = \sum_{j=0}^m \varepsilon^{2j}\left(\rho_\nu^j,\, u_\nu^j,\, E^j, B^j\right),$$

where $(\rho_\nu^j,\, u_\nu^j,\, E^j, B^j)_{0\leq j\leq m}$ are constructed in the previous subsection. We define the remainders $\left(R_{\rho_\nu}^{\varepsilon,m},\, R_{u_\nu}^{\varepsilon,m}, R_E^{\varepsilon,m}\right)$ by

$$\begin{cases}
\partial_t \rho_{i,\varepsilon}^m + \operatorname{div}\left(\rho_{i,\varepsilon}^m u_{i,\varepsilon}^m\right) = R_{\rho_i}^{\varepsilon,m}, \\
\partial_t u_{i,\varepsilon}^m + \left(u_{i,\varepsilon}^m \cdot \nabla\right) u_{i,\varepsilon}^m + \varepsilon^2 \nabla h_i(\rho_{i,\varepsilon}^m) - \varepsilon^2 (E_\varepsilon^m + u_{i,\varepsilon}^m \times B_\varepsilon^m) = R_{u_i}^{\varepsilon,m}, \\
\partial_t \rho_{e,\varepsilon}^m + \operatorname{div}\left(\rho_{e,\varepsilon}^m u_{e,\varepsilon}^m\right) = R_{\rho_e}^{\varepsilon,m}, \\
\partial_t u_{e,\varepsilon}^m + \left(u_{e,\varepsilon}^m \cdot \nabla\right) u_{e,\varepsilon}^m + \nabla h_e(\rho_{e,\varepsilon}^m) + E_\varepsilon^m + u_{e,\varepsilon}^m \times B_\varepsilon^m = R_{u_e}^{\varepsilon,m}, \\
\partial_t E_\varepsilon^m - \nabla \times B_\varepsilon^m + \rho_{i,\varepsilon}^m u_{i,\varepsilon}^m - \rho_{e,\varepsilon}^m u_{e,\varepsilon}^m = R_E^{\varepsilon,m}, \\
\operatorname{div} E_\varepsilon^m - \rho_{i,\varepsilon}^m + \rho_{e,\varepsilon}^m = 0, \quad \partial_t B_\varepsilon^m + \nabla \times E_\varepsilon^m = 0, \quad \operatorname{div} B_\varepsilon^m = 0.
\end{cases}$$

It is clear that the convergence rate depends strongly on the order of the remainders with respect to ε. Since the approximate solution $\left(\rho_{\nu,\varepsilon}^m,\, u_{\nu,\varepsilon}^m,\, E_\varepsilon^m, B_\varepsilon^m\right)$ is sufficiently smooth, a straightforward computation gives

$$\sup_{0\leq t\leq T_a} \|(R_{\rho_\nu}^{\varepsilon,m}, R_{u_\nu}^{\varepsilon,m}, R_E^{\varepsilon,m})(t)\| \leq C\varepsilon^{2m+2}. \tag{2.12}$$

Let $(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon,\, E^\varepsilon, B^\varepsilon)$ be the exact local smooth solution obtained in Proposition 2.1. When the convergence holds at $t = 0$, establishing the convergence of the asymptotic expansion

(2.3) is to prove that

$$\left(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon,\, E^\varepsilon, B^\varepsilon\right) - \left(\rho_{\nu,\varepsilon}^m,\, u_{\nu,\varepsilon}^m,\, E_\varepsilon^m, B_\varepsilon^m\right) \longrightarrow 0,$$

and obtain its convergence rate as $\varepsilon \to 0$ on a time interval independent of ε. It is as follows the main result for the local-in-time convergence for the infinity-ion mass limit, of which the proof will be given in Section 3.

Theorem 2.1. *(Local convergence result) Let the conditions in Proposition 2.1 and 2.2 hold. Let $s \geq 3$ and $m \geq 1$ be integers. Assume*

$$\begin{aligned} &\left\| \left((\rho_{\nu,0}^\varepsilon - \rho_{\nu,\varepsilon}^m(0,\cdot)), \frac{1}{\varepsilon}(u_{i,0}^\varepsilon - u_{i,\varepsilon}^m(0,\cdot)), u_{e,0}^\varepsilon - u_{e,\varepsilon}^m(0,\cdot), E_0^\varepsilon - E_\varepsilon^m(0,\cdot), B_0^\varepsilon - B_\varepsilon^m(0,\cdot) \right) \right\| \\ \leq\ & C_1 \varepsilon^{2m+2}, \end{aligned} \tag{2.13}$$

where $C_1 > 0$ is a constant independent of ε, then there exists a constant $C_2 > 0$, which depends on T_a but is independent of ε, such that as $\varepsilon \to 0$, we have $T_e^\varepsilon \geq T_a$, and the local smooth solution $(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)$ to the periodic problem (2.1) *satisfies*

$$\sup_{0\leq t\leq T_a} \left\| \left(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon,\, E^\varepsilon, B^\varepsilon\right)(t) - \left(\rho_{\nu,\varepsilon}^m,\, u_{\nu,\varepsilon}^m,\, E_\varepsilon^m, B_\varepsilon^m\right)(t) \right\|_s \leq C_2 \varepsilon^{2m+1}.$$

In particular, as $\varepsilon \to 0$, we have

$$(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon) \to (\rho_\nu^0, u_\nu^0, E^0, B^0), \quad \textit{uniformly in } C([0,T_a]; H^s),$$

where (ρ_i^0, u_i^0) satisfies (2.6) *and $(\rho_e^0, u_e^0, E^0, B^0)$ satisfies the generalized unipolar Euler-Maxwell system for electrons* (2.7).

Remark 2.1. *If, as $\varepsilon \to 0$,*

$$u_{i,0}^\varepsilon \rightharpoonup 0, \quad \textit{weakly in } H^s,$$

it is obvious that $\bar{u}_i^0 = 0$. Thus the leading profile u_i^0 satisfies the following

$$\begin{cases} \partial_t u_i^0 + (u_i^0 \cdot \nabla) u_i^0 = 0, \\ t = 0 : u_i^0(0,\cdot) = \bar{u}_i^0(\cdot) = 0, \end{cases}$$

which admits a unique local solution $u_i^0 = 0$. Hence, combining the first equation in (2.6), *we have $\partial_t \rho_i^0 = 0$. As a result, $\rho_i^0 = \rho_i^0(x)$. Consequently,* (2.7) *becomes*

$$\begin{cases} \partial_t \rho_e^0 + \operatorname{div}(\rho_e^0 u_e^0) = 0, \\ \partial_t u_e^0 + (u_e^0 \cdot \nabla) u_e^0 + \nabla h_e(\rho_e^0) = -(E^0 + u_e^0 \times B^0), \\ \partial_t E^0 - \nabla \times B^0 = \rho_e^0 u_e^0, \quad \operatorname{div} E^0 = \rho_i^0(x) - \rho_e^0, \\ \partial_t B^0 + \nabla \times E^0 = 0, \quad \operatorname{div} B^0 = 0, \\ t = 0 : (\rho_e^0, u_e^0, E^0, B^0) = (\bar{\rho}_e^0, \bar{u}_e^0, \bar{E}^0, \bar{B}^0), \quad x \in \mathbb{T}^3, \end{cases}$$

which is the usual model of the unipolar Euler-Maxwell system for electrons.

2.3. **Results on the global-in-time convergence.** When considering the global-in-time convergence, we establish uniform global estimates of the solutions to (1.1)-(1.2) with respect to ε. Noticing (1.3), the periodic problem (1.1)-(1.2) is equivalent to the following

$$
\begin{cases}
\partial_t\rho_i^\varepsilon + \operatorname{div}(\rho_i^\varepsilon u_i^\varepsilon) = 0,\\
\partial_t u_i^\varepsilon + (u_i^\varepsilon\cdot\nabla)u_i^\varepsilon + \varepsilon^2\nabla h_i(\rho_i^\varepsilon) = \varepsilon^2(E^\varepsilon + u_i^\varepsilon\times B^\varepsilon) - u_i^\varepsilon,\\
\partial_t\rho_e^\varepsilon + \operatorname{div}(\rho_e^\varepsilon u_e^\varepsilon) = 0,\\
\partial_t u_e^\varepsilon + (u_e^\varepsilon\cdot\nabla)u_e^\varepsilon + \nabla h_e(\rho_e^\varepsilon) = -E^\varepsilon - u_e^\varepsilon\times B^\varepsilon - u_e^\varepsilon,\\
\partial_t E^\varepsilon - \nabla\times B^\varepsilon = \rho_e^\varepsilon u_e^\varepsilon - \rho_i^\varepsilon u_i^\varepsilon, \quad \operatorname{div}E^\varepsilon = \rho_i^\varepsilon - \rho_e^\varepsilon,\\
\partial_t B^\varepsilon + \nabla\times E^\varepsilon = 0, \quad \operatorname{div}B^\varepsilon = 0, \quad t>0, \quad x\in\mathbb{T}^3,\\
t=0: \quad (\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon) = (\rho_{\nu,0}^\varepsilon, u_{\nu,0}^\varepsilon, E_0^\varepsilon, B_0^\varepsilon), \quad x\in\mathbb{T}^3.
\end{cases}
\tag{2.14}
$$

Theorem 2.2. *(Uniform global estimates with respect to ε) Let $s\geq 3$ be an integer. There exist positive constants C_3 and δ such that for all $\varepsilon\in(0,1]$, if*

$$
\sum_{\nu=i,e}\|\rho_{\nu,0}^\varepsilon - 1\|_s + \frac{1}{\varepsilon}\|\rho_{i,0}^\varepsilon - 1\|_{s-1} + \|u_{e,0}^\varepsilon\|_s + \frac{1}{\varepsilon}\|u_{i,0}^\varepsilon\|_s + \|E_0^\varepsilon\|_s + \|B_0^\varepsilon - B_e\|_s \leq \delta,
$$

then for all $t>0$, (2.14) admits a unique global solution $(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)$ satisfying

$$
\begin{aligned}
&\sum_{\nu=i,e}\|\rho_\nu^\varepsilon(t)-1\|_s^2 + \frac{1}{\varepsilon^2}\|\rho_i^\varepsilon(t)-1\|_{s-1}^2 + \frac{1}{\varepsilon^2}\|u_i^\varepsilon(t)\|_s^2 + \|u_e^\varepsilon(t)\|_s^2 + \|E^\varepsilon(t)\|_s^2 + \|B^\varepsilon(t)-B_e\|_s^2\\
+&\int_0^t\left(\sum_{\nu=i,e}\|\nabla\rho_\nu^\varepsilon(\tau)\|_{s-1}^2 + \frac{1}{\varepsilon^2}\|u_i^\varepsilon(\tau)\|_s^2 + \|u_e^\varepsilon(\tau)\|_s^2 + \|E^\varepsilon(\tau)\|_{s-1}^2 + \|\nabla B^\varepsilon(\tau)\|_{s-2}^2\right)d\tau\\
\leq\ & C_3\left(\|\rho_{i,0}^\varepsilon - 1\|_s^2 + \|\rho_{e,0}^\varepsilon - 1\|_s^2 + \|u_{e,0}^\varepsilon\|_s^2 + \frac{1}{\varepsilon}\|u_{i,0}^\varepsilon\|_s^2 + \|E_0^\varepsilon\|_s^2 + \|B_0^\varepsilon\|_s^2\right).
\end{aligned}
$$

Theorem 2.3. *(Uniform global convergence) Let $(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)$ be the unique global smooth solution obtained in Theorem 2.2. If, as $\varepsilon\to 0$,*

$$
(\rho_{i,0}^\varepsilon, \rho_{e,0}^\varepsilon, u_{e,0}^\varepsilon, u_{i,0}^\varepsilon, E_0^\varepsilon, B_0^\varepsilon) \rightharpoonup (\bar\rho_i^0, \bar\rho_e^0, \bar u_e^0, 0, \bar E^0, \bar B^0), \quad \textit{weakly in} \quad H^s, \tag{2.15}
$$

then there exist functions $(\bar\rho_i, \bar\rho_e, u_e, \bar E, \bar B)\in L^\infty(\mathbb{R}^+; H^s)$, such that, as $\varepsilon\to 0$, up to subsequences,

$$
u_i^\varepsilon \ \to\ 0 \quad \textit{strongly in} \quad L^\infty([0,T];H^s), \quad \forall T>0, \tag{2.16}
$$

$$
(\rho_i^\varepsilon, \rho_e^\varepsilon, u_e^\varepsilon, E^\varepsilon, B^\varepsilon) \ \rightharpoonup\ (\bar\rho_i, \bar\rho_e, \bar u_e, \bar E, \bar B), \quad \textit{weakly-* in} \quad L^\infty(\mathbb{R}^+; H^s), \tag{2.17}
$$

where $\bar\rho_i = \bar\rho_i(x)$ depends only on the space variable x, and $(\bar\rho_e, \bar u_e, \bar E, \bar B)$ is the unique global smooth solution of the following unipolar Euler-Maxwell system for electrons

$$\begin{cases} \partial_t \bar\rho_e + \mathrm{div}(\bar\rho_e \bar u_e) = 0, \\ \partial_t \bar u_e + (\bar u_e \cdot \nabla)\bar u_e + \nabla h_e(\bar\rho_e) = -\bar E - \bar u_e \times \bar B - \bar u_e, \\ \partial_t \bar E - \nabla \times \bar B = \bar\rho_e \bar u_e, \quad \mathrm{div}\bar E = \bar\rho_i(x) - \bar\rho_e, \\ \partial_t \bar B + \nabla \times \bar E = 0, \quad \mathrm{div}\bar B = 0, \end{cases} \tag{2.18}$$

with initial conditions

$$t = 0 : (\bar\rho_e, \bar u_e, \bar E, \bar B) = (\bar\rho_{e,0}, \bar u_{e,0}, \bar E_0, \bar B_0). \tag{2.19}$$

3. Local convergence

3.1. **Energy estimates.** In this section, we prove Theorem 2.1. Let $(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon,\, E^\varepsilon, B^\varepsilon)$ be the exact local smooth solution obtained in Proposition 2.1, which is defined on the time interval $[0,\, T_e^\varepsilon]$. Since the approximate solution $\left(\rho_{\nu,\varepsilon}^m,\, u_{\nu,\varepsilon}^m,\, E_\varepsilon^m, B_\varepsilon^m\right)$ is defined on the time interval $[0,\, T_a]$, we set

$$T_b^\varepsilon = \min\left(T_e^\varepsilon,\, T_a\right),$$

then the exact solution and the approximate solution are both defined on the time interval $[0,\, T_b^\varepsilon]$, on which we denote

$$(N_\nu^\varepsilon,\, w_\nu^\varepsilon, \chi^\varepsilon, G^\varepsilon) \triangleq \left(\rho_\nu^\varepsilon - \rho_{\nu,\varepsilon}^m, u_\nu^\varepsilon - u_{\nu,\varepsilon}^m, E^\varepsilon - E_\varepsilon^m, B^\varepsilon - B_\varepsilon^m\right).$$

It is easy to check that $(N_\nu^\varepsilon,\, w_\nu^\varepsilon, \chi^\varepsilon, G^\varepsilon)$ satisfies

$$\begin{cases} \partial_t N_i^\varepsilon + u_i^\varepsilon \cdot \nabla N_i^\varepsilon + \rho_i^\varepsilon \mathrm{div} w_i^\varepsilon = -(N_i^\varepsilon \mathrm{div} u_{i,\varepsilon}^m + \nabla \rho_{i,\varepsilon}^m \cdot w_i^\varepsilon) - R_{\rho_i}^{\varepsilon,m}, \\ \partial_t w_i^\varepsilon + (u_i^\varepsilon \cdot \nabla) w_i^\varepsilon + \varepsilon^2 h_i'(\rho_i^\varepsilon)\nabla N_i^\varepsilon \\ \qquad = -(w_i^\varepsilon \cdot \nabla) u_{i,\varepsilon}^m - \varepsilon^2 \left(h_i'(N_i^\varepsilon + \rho_{i,\varepsilon}^m) - h_i'(\rho_{i,\varepsilon}^m)\right) \nabla \rho_{i,\varepsilon}^m \\ \qquad\quad + \varepsilon^2(\chi^\varepsilon + w_i^\varepsilon \times B_\varepsilon^m + u_i^\varepsilon \times G^\varepsilon) - R_{u_i}^{\varepsilon,m}, \\ \partial_t N_e^\varepsilon + u_e^\varepsilon \cdot \nabla N_e^\varepsilon + \rho_e^\varepsilon \mathrm{div} w_e^\varepsilon = -(N_e^\varepsilon \mathrm{div} u_{e,\varepsilon}^m + \nabla \rho_{e,\varepsilon}^m \cdot w_e^\varepsilon) - R_{\rho_e}^{\varepsilon,m}, \\ \partial_t w_e^\varepsilon + (u_e^\varepsilon \cdot \nabla) w_e^\varepsilon + h_e'(\rho_e^\varepsilon)\nabla N_e^\varepsilon \\ \qquad = -(w_e^\varepsilon \cdot \nabla) u_{e,\varepsilon}^m - \left(h_e'(N_e^\varepsilon + \rho_{e,\varepsilon}^m) - h_e'(\rho_{e,\varepsilon}^m)\right) \nabla \rho_{e,\varepsilon}^m \\ \qquad\quad - (\chi^\varepsilon + w_e^\varepsilon \times B_\varepsilon^m + u_e^\varepsilon \times G^\varepsilon) - R_{u_e}^{\varepsilon,m}, \\ \partial_t \chi^\varepsilon - \nabla \times G^\varepsilon = N_e^\varepsilon u_e^\varepsilon + w_e^\varepsilon \rho_{e,\varepsilon}^m - (N_i^\varepsilon u_i^\varepsilon + w_i^\varepsilon \rho_{i,\varepsilon}^m) - R_E^{\varepsilon,m}, \\ \mathrm{div}\chi^\varepsilon = N_i^\varepsilon - N_e^\varepsilon, \quad \partial_t G^\varepsilon + \nabla \times \chi^\varepsilon = 0, \quad \mathrm{div} G^\varepsilon = 0, \end{cases} \tag{3.1}$$

with initial conditions

$$
\begin{aligned}
& \left(N_\nu^\varepsilon,\, w_\nu^\varepsilon, \chi^\varepsilon, G^\varepsilon\right)\big|_{t=0} \\
= \ & \left(\rho_{\nu,0}^\varepsilon - \rho_{\nu,\varepsilon}^m(0,\cdot), u_{\nu,0}^\varepsilon - u_{\nu,\varepsilon}^m(0,\cdot), E_0^\varepsilon - E_\varepsilon^m(0,\cdot), B_0^\varepsilon - B_\varepsilon^m(0,\cdot)\right).
\end{aligned} \tag{3.2}
$$

For $\nu = i, e$, we set

$$
W_i^\varepsilon = \begin{pmatrix} N_i^\varepsilon \\ w_i^\varepsilon \end{pmatrix}, \quad W_e^\varepsilon = \begin{pmatrix} N_e^\varepsilon \\ w_e^\varepsilon \end{pmatrix}, \quad W^\varepsilon = \begin{pmatrix} W_i^\varepsilon \\ W_e^\varepsilon \\ \chi^\varepsilon \\ G^\varepsilon \end{pmatrix}.
$$

We further denote

$$
\begin{aligned}
H_{i,\varepsilon}^1 &= \begin{pmatrix} N_i^\varepsilon \mathrm{div} u_{i,\varepsilon}^m + w_i^\varepsilon \cdot \nabla \rho_{i,\varepsilon}^m \\ \left(w_i^\varepsilon \cdot \nabla\right) u_{i,\varepsilon}^m + \varepsilon^2 \left(h_i'(N_i^\varepsilon + \rho_{i,\varepsilon}^m) - h_i'(\rho_{i,\varepsilon}^m)\right) \nabla \rho_{i,\varepsilon}^m \end{pmatrix}, \\
H_{e,\varepsilon}^1 &= \begin{pmatrix} N_e^\varepsilon \mathrm{div} u_{e,\varepsilon}^m + w_e^\varepsilon \cdot \nabla \rho_{e,\varepsilon}^m \\ \left(w_e^\varepsilon \cdot \nabla\right) u_{e,\varepsilon}^m + \left(h_e'(N_e^\varepsilon + \rho_{e,\varepsilon}^m) - h_e'(\rho_{e,\varepsilon}^m)\right) \nabla \rho_{e,\varepsilon}^m \end{pmatrix},
\end{aligned}
$$

$$
\begin{aligned}
H_{i,\varepsilon}^2 &= \begin{pmatrix} 0 \\ \varepsilon^2(\chi^\varepsilon + w_i^\varepsilon \times B_\varepsilon^m + u_i^\varepsilon \times G^\varepsilon) \end{pmatrix}, \\
H_{e,\varepsilon}^2 &= \begin{pmatrix} 0 \\ -\chi^\varepsilon - w_e^\varepsilon \times B_\varepsilon^m - u_e^\varepsilon \times G^\varepsilon \end{pmatrix}.
\end{aligned}
$$

The remaining terms are defined as

$$
R_i^\varepsilon = \begin{pmatrix} R_{\rho_i}^{\varepsilon,m} \\ R_{u_i}^{\varepsilon,m} \end{pmatrix}, \quad R_e^\varepsilon = \begin{pmatrix} R_{\rho_e}^{\varepsilon,m} \\ R_{u_e}^{\varepsilon,m} \end{pmatrix}.
$$

For $j = 1, 2, 3$, we set $u_\nu^\varepsilon = (u_{\nu,1}^\varepsilon, u_{\nu,2}^\varepsilon, u_{\nu,3}^\varepsilon)$, and

$$
\begin{aligned}
A_i^j\left(\rho_i^\varepsilon,\, u_i^\varepsilon\right) &= \begin{pmatrix} u_{i,j}^\varepsilon & \rho_i^\varepsilon \xi_j^\top \\ \varepsilon^2 h_i'\left(\rho_i^\varepsilon\right)\xi_j & u_{i,j}^\varepsilon \mathbf{I}_3 \end{pmatrix}, \\
A_e^j\left(\rho_e^\varepsilon,\, u_e^\varepsilon\right) &= \begin{pmatrix} u_{e,j}^\varepsilon & \rho_e^\varepsilon \xi_j^\top \\ h_e'\left(\rho_e^\varepsilon\right)\xi_j & u_{e,j}^\varepsilon \mathbf{I}_3 \end{pmatrix},
\end{aligned}
$$

where $\mathbf{I}_3$ is the 3×3 unit matrix, $\{\xi_k\}_{k=1}^3$ is the canonical basis of $\mathbb{R}^3$ and the superscript $\top$ denotes the transpose of a vector or a matrix. Thus, the first four equations in (3.1) can be written as

$$
\partial_t W_\nu^\varepsilon + \sum_{j=1}^3 A_\nu^j\left(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon\right)\partial_{x_j} W_\nu^\varepsilon = -H_{\nu,\varepsilon}^1 + H_{\nu,\varepsilon}^2 - R_\nu^\varepsilon, \tag{3.3}
$$

with initial conditions

$$
t = 0: \quad W_\nu^\varepsilon = W_{\nu,0}^\varepsilon, \tag{3.4}
$$

where

$$
\begin{aligned}
W_{i,0}^{\varepsilon} &= \begin{pmatrix} N_i^{\varepsilon}(0,\,\cdot) \\ w_i^{\varepsilon}(0,\,\cdot) \end{pmatrix} = \begin{pmatrix} \rho_{i,0}^{\varepsilon} - \rho_{i,\varepsilon}^{m}(0,\,\cdot) \\ u_{i,0}^{\varepsilon} - u_{i,\varepsilon}^{m}(0,\,\cdot) \end{pmatrix}, \\
W_{e,0}^{\varepsilon} &= \begin{pmatrix} N_e^{\varepsilon}(0,\,\cdot) \\ w_e^{\varepsilon}(0,\,\cdot) \end{pmatrix} = \begin{pmatrix} \rho_{e,0}^{\varepsilon} - \rho_{e,\varepsilon}^{m}(0,\,\cdot) \\ u_{e,0}^{\varepsilon} - u_{e,\varepsilon}^{m}(0,\,\cdot) \end{pmatrix}.
\end{aligned}
$$

Since $\rho_\nu^\varepsilon \geq \underline{\rho} > 0$, system (3.3)-(3.4) is indeed symmetrizable hyperbolic. In fact, if we introduce the symmetrizers $A_\nu^0(\rho_\nu^\varepsilon)$ as

$$
A_i^0(\rho_i^\varepsilon) = \begin{pmatrix} h_i'(\rho_i^\varepsilon) & 0 \\ 0 & \varepsilon^{-2}\rho_i^\varepsilon \mathbf{I}_3 \end{pmatrix}, \quad A_e^0(\rho_e^\varepsilon) = \begin{pmatrix} h_e'(\rho_e^\varepsilon) & 0 \\ 0 & \rho_e^\varepsilon \mathbf{I}_3 \end{pmatrix},
$$

which for $j = 1, 2, 3$, results in

$$
\begin{aligned}
\tilde{A}_i^j(\rho_i^\varepsilon,\, u_i^\varepsilon) &= A_i^0(\rho_i^\varepsilon)\, A_i^j(\rho_i^\varepsilon,\, u_i^\varepsilon) = \begin{pmatrix} h_i'(\rho_i^\varepsilon)\, u_{i,j}^\varepsilon & p_i'(\rho_i^\varepsilon)\, \xi_j^\top \\ p_i'(\rho_i^\varepsilon)\, \xi_j & \varepsilon^{-2}\rho_i^\varepsilon u_{i,j}^\varepsilon \mathbf{I}_3 \end{pmatrix}, \\
\tilde{A}_e^j(\rho_e^\varepsilon,\, u_e^\varepsilon) &= A_e^0(\rho_e^\varepsilon)\, A_e^j(\rho_e^\varepsilon,\, u_e^\varepsilon) = \begin{pmatrix} h_e'(\rho_e^\varepsilon)\, u_{e,j}^\varepsilon & p_e'(\rho_e^\varepsilon)\, \xi_j^\top \\ p_e'(\rho_e^\varepsilon)\, \xi_j & \rho_e^\varepsilon u_{e,j}^\varepsilon \mathbf{I}_3 \end{pmatrix},
\end{aligned}
$$

then for $\rho_\nu^\varepsilon > 0$, A_ν^0 is positively definite and $\tilde{A}_\nu^j$ is symmetric for all $1 \leq j \leq 3$. Thus, the theorem of Lax [16] and Kato [14] for the local existence of smooth solutions can also be applied to (3.3)-(3.4).

By standard arguments, to prove Theorem 2.1, it suffices to establish uniform estimates of W^ε with respect to ε. We denote

$$
W_{i,*}^\varepsilon = \begin{pmatrix} N_i^\varepsilon \\ \varepsilon^{-1} w_i^\varepsilon \end{pmatrix}, \quad W_{e,*}^\varepsilon = \begin{pmatrix} N_e^\varepsilon \\ w_e^\varepsilon \end{pmatrix}, \quad W_*^\varepsilon = \begin{pmatrix} W_{i,*}^\varepsilon \\ W_{e,*}^\varepsilon \\ \chi^\varepsilon \\ G^\varepsilon \end{pmatrix}.
$$

The theorem of Lax and Kato for the local existence of smooth solutions implies $W_*^\varepsilon \in C([0, T_b^\varepsilon]; H^s)$ and the function $t \to \|W_*^\varepsilon\|_s$ is continuous on $[0, T_b^\varepsilon]$. From the assumption (2.13), there exists a $T^\varepsilon \in [0, T_b^\varepsilon]$ such that

$$
\|W_*^\varepsilon(t)\|_s \leq C, \quad \forall t \in [0, T^\varepsilon], \tag{3.5}
$$

provided that $\varepsilon < 1$. Actually, we may assume that T^ε is the maximum time such that W_*^ε exists and satisfies (3.5). For $\varepsilon < 1$, the approximate solution established in Proposition 2.2 is sufficiently smooth. This, together with (3.5), implies the exact solution to (2.1) satisfies

$$
\|(\rho_\nu^\varepsilon, u_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)(t)\|_s \leq C, \quad \forall t \in [0, T^\varepsilon]. \tag{3.6}
$$

In order to prove that T^ε is indeed independent of ε, first we need to show that there exists a constant $\mu > 0$, such that

$$\sup_{0\le t\le T^\varepsilon} \|W_*^\varepsilon(t)\|_s \le C\varepsilon^\mu.$$

In what follows, we always assume that the conditions in Theorem 2.1 hold.

Lemma 3.1. *It holds*

$$\frac{d}{dt}\big\langle A_\nu^0(\rho_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\big\rangle \le C\|W_*^\varepsilon\|_s^2 + C\varepsilon^{4m+2}. \tag{3.7}$$

Proof. For a multi-index $\alpha\in\mathbb{N}^3$ with $|\alpha|\le s$, applying $A_\nu^0(\rho_\nu^\varepsilon)\partial^\alpha$ to (3.3), making the inner product of the resulting equations with $\partial^\alpha W^\varepsilon$ in $L^2(\mathbb{T}^3)$, we obtain the following energy equality for W_ν^ε,

$$\begin{aligned}\frac{d}{dt}\big\langle A_\nu^0(\rho_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\big\rangle =& \langle \mathrm{div}A_\nu(\rho_\nu^\varepsilon,u_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\rangle - 2\big\langle A_\nu^0\left(\rho_\nu^\varepsilon\right)\partial^\alpha W_\nu^\varepsilon,\, \partial^\alpha H_{\nu,\varepsilon}^1\big\rangle\\ &+ 2\big\langle A_\nu^0\left(\rho_\nu^\varepsilon\right)\partial^\alpha W_\nu^\varepsilon,\, \partial^\alpha H_{\nu,\varepsilon}^2\big\rangle - 2\big\langle A_\nu^0\left(\rho_\nu^\varepsilon\right)\partial^\alpha W_\nu^\varepsilon,\, \partial^\alpha R_\nu^\varepsilon\big\rangle,\\ &+ 2\left\langle A_\nu^0(\rho_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, J_{\nu,\varepsilon}^\alpha\right\rangle,\end{aligned} \tag{3.8}$$

where

$$\begin{aligned}\mathrm{div}A_\nu\left(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon\right) &= \partial_t A_\nu^0\left(\rho_\nu^\varepsilon\right) + \sum_{j=1}^3 \partial_{x_j}\tilde{A}_\nu^j\left(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon\right),\\ J_{\nu,\varepsilon}^\alpha &= -\partial^\alpha\left(\sum_{j=1}^3 A_\nu^j\left(\rho_\nu^\varepsilon,\, u_\nu^\varepsilon\right)\partial_{x_j}W_\nu^\varepsilon\right) + \sum_{j=1}^3 A_\nu^j(\rho_\nu^\varepsilon, u_\nu^\varepsilon)\partial^\alpha\partial_{x_j}W_\nu^\varepsilon.\end{aligned}$$

Now we deal with each term on the right hand side of (3.8). First, since $\partial_t\rho_\nu^\varepsilon = -\mathrm{div}(\rho_\nu^\varepsilon u_\nu^\varepsilon)$, we have

$$\|\partial_t\rho_\nu^\varepsilon\|_\infty \le \|u_\nu^\varepsilon\|_s.$$

Since $\varepsilon<1$, in view of the expressions of A_ν^0, using (3.6) and the Cauchy-Schwarz inequality, we have

$$\begin{aligned}|\left\langle\partial_t\left(A_i(\rho_i^\varepsilon,u_i^\varepsilon)H_{i,\varepsilon}^0\right)\partial^\alpha W_i^\varepsilon, \partial^\alpha W_i^\varepsilon\right\rangle| &\le C\left(\|N_i^\varepsilon\|^2 + \left\|\frac{w_i^\varepsilon}{\varepsilon}\right\|^2\right) \le C\|W_{i,*}^\varepsilon\|_{|\alpha|}^2,\\ |\left\langle\partial_t\left(A_e(\rho_e^\varepsilon,u_e^\varepsilon)H_{e,\varepsilon}^0\right)\partial^\alpha W_e^\varepsilon, \partial^\alpha W_e^\varepsilon\right\rangle| &\le C\left(\|N_e^\varepsilon\|^2 + \|w_e^\varepsilon\|^2\right) \le C\|W_{e,*}^\varepsilon\|_{|\alpha|}^2,\end{aligned}$$

Hence,

$$|\left\langle\partial_t\left(A_\nu(\rho_\nu^\varepsilon,u_\nu^\varepsilon)H_{\nu,\varepsilon}^0\right)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\right\rangle| \le C\|W_*^\varepsilon\|_{|\alpha|}^2. \tag{3.9}$$

Similarly, in view of the expression $\tilde{A}_\nu^j$, we have

$$\left|\left\langle\partial_{x_j}\tilde{A}_\nu^j(\rho_\nu^\varepsilon,u_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\right\rangle\right| \le C\|W_*^\varepsilon\|_{|\alpha|}^2,$$

Thus, combining (3.9), we have

$$|\langle\mathrm{div}A_\nu(\rho_\nu^\varepsilon,u_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\rangle| \le C\|W_*^\varepsilon\|_{|\alpha|}^2. \tag{3.10}$$

Since the approximate solution $(\rho_{\nu,\varepsilon}^m, u_{\nu,\varepsilon}^m, E_\varepsilon^m, B_\varepsilon^m)$ is sufficiently smooth, noticing (3.5)-(3.6) and the expressions of $H_{\nu,\varepsilon}^1$, using the Cauchy-Schwarz inequalities and the Moser-type inequality in Lemma 2.1, we have

$$\begin{aligned}
&\left|\left\langle A_i^0\left(\rho_i^\varepsilon\right)\partial^\alpha W_i^\varepsilon,\, \partial^\alpha H_{i,\varepsilon}^1\right\rangle\right| \\
\leq\ & \left|\left\langle h_i'(\rho_i^\varepsilon)\partial^\alpha N_i^\varepsilon, \partial^\alpha\left(N_i^\varepsilon \mathrm{div} u_{i,\varepsilon}^m + w_i^\varepsilon\cdot\nabla\rho_{i,\varepsilon}^m\right)\right\rangle\right| \\
&+\left|\left\langle \rho_i^\varepsilon\partial^\alpha(\varepsilon^{-1}w_i^\varepsilon), \partial^\alpha(\varepsilon^{-1}\left(w_i^\varepsilon\cdot\nabla\right)u_{i,\varepsilon}^m) + \partial^\alpha(\left(h_i'(N_i^\varepsilon+\rho_{i,\varepsilon}^m) - h_i'(\rho_{i,\varepsilon}^m)\right)\nabla\rho_{i,\varepsilon}^m)\right\rangle\right| \\
\leq\ & C\left(\|N_i^\varepsilon\|^2 + \left\|\frac{w_i^\varepsilon}{\varepsilon}\right\|^2\right) + \left|\left\langle \rho_i\partial^\alpha(\varepsilon^{-1}w_i^\varepsilon), \partial^\alpha(h_i''(\rho_{i,*}^\varepsilon)N_i^\varepsilon\nabla\rho_{i,\varepsilon}^m)\right\rangle\right| \leq C\|W_{i,*}^\varepsilon\|_{|\alpha|}^2,
\end{aligned}$$

in which we have used the Taylor formula and $\rho_{i,*}^\varepsilon$ is between $N_i^\varepsilon + \rho_{i,\varepsilon}^m$ and $\rho_{i,\varepsilon}^m$. Similar estimates can also be established for the term containing $H_{e,\varepsilon}^1$ and $H_{\nu,\varepsilon}^2$. Thus, we have

$$\left|\left\langle A_\nu^0\left(\rho_\nu^\varepsilon\right)\partial^\alpha W_\nu^\varepsilon,\, \partial^\alpha H_{\nu,\varepsilon}^1\right\rangle\right| + \left|\left\langle A_\nu^0\left(\rho_\nu^\varepsilon\right)\partial^\alpha W_\nu^\varepsilon,\, \partial^\alpha H_{\nu,\varepsilon}^2\right\rangle\right| \leq C\|W_*^\varepsilon\|_{|\alpha|}^2. \tag{3.11}$$

For the term containing R_ν^ε in (3.8), noticing (2.12) and applying the Cauchy-Schwarz inequality, we have

$$\begin{aligned}
\left|\left\langle A_i^0\left(\rho_i^\varepsilon\right)\partial^\alpha W_i^\varepsilon,\, \partial^\alpha R_i^\varepsilon\right\rangle\right| &\leq C\|W_{i,*}^\varepsilon\|_{|\alpha|}^2 + C\|\varepsilon^{-1}R_i^\varepsilon\|_{|\alpha|}^2 \leq C\|W_*^\varepsilon\|_{|\alpha|}^2 + C\varepsilon^{4m+2}, &(3.12)\\
\left|\left\langle A_\nu^0\left(\rho_\nu^\varepsilon\right)\partial^\alpha W_e^\varepsilon,\, \partial^\alpha R_e^\varepsilon\right\rangle\right| &\leq C\|W_{e,*}^\varepsilon\|_{|\alpha|}^2 + C\|R_e^\varepsilon\|_{|\alpha|}^2 \leq C\|W_*^\varepsilon\|_{|\alpha|}^2 + C\varepsilon^{4(m+1)}, &(3.13)
\end{aligned}$$

If $|\alpha| = 0$, the term containing $J_{\nu,\varepsilon}^\alpha$ in (3.8) vanishes. For $1 \leq |\alpha| \leq s$, by using the Moser-type inequalities in Lemma 2.1 , we have

$$\|J_{\nu,\varepsilon}^\alpha\| \leq C\|\nabla\rho_\nu^\varepsilon\|_{s-1}\|\nabla u_\nu^\varepsilon\|_{s-1} \leq C\|\nabla u_\nu^\varepsilon\|_{s-1},$$

which implies

$$\left|\left\langle A_\nu^0(\rho_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, J_{\nu,\varepsilon}^\alpha\right\rangle\right| \leq C\|W_*^\varepsilon\|_s^2. \tag{3.14}$$

Combining (3.8) and the estimates (3.10)-(3.14) yields (3.7). □

Lemma 3.2. *It holds*

$$\sup_{0\leq t\leq T^\varepsilon}\|W_*^\varepsilon(t)\|_s^2 \leq C\varepsilon^{4m+2}. \tag{3.15}$$

Proof. For a multi-index $\alpha\in\mathbb{N}^3$ with $|\alpha|\leq s$, applying ∂^α to the Maxwell equations in (3.1), we have

$$\begin{cases}
\partial_t\partial^\alpha\chi^\varepsilon - \nabla\times\partial^\alpha G^\varepsilon = \partial^\alpha(N_e^\varepsilon u_e^\varepsilon + w_e^\varepsilon\rho_{e,\varepsilon}^m) - \partial^\alpha(N_i^\varepsilon u_i^\varepsilon + w_i^\varepsilon\rho_{i,\varepsilon}^m) - \partial^\alpha R_E^{\varepsilon,m},\\
\partial_t\partial^\alpha G^\varepsilon + \nabla\times\partial^\alpha\chi^\varepsilon = 0,\\
\mathrm{div}\partial^\alpha\chi^\varepsilon = \partial^\alpha N_i^\varepsilon - \partial^\alpha N_e^\varepsilon, \quad \mathrm{div}\partial^\alpha G^\varepsilon = 0.
\end{cases} \tag{3.16}$$

Making the inner product in $L^2(\mathbb{T}^3)$ of the first equation in (3.16) with $\partial^\alpha \chi^\varepsilon$ as well as of the second equation with $\partial^\alpha G^\varepsilon$, adding the resulting two equations and noting the vector analysis formula

$$\operatorname{div}(f \times g) = (\nabla \times f) \cdot g - (\nabla \times g) \cdot f, \quad \forall f, g \in \mathbb{R}^3,$$

we have

$$\frac{d}{dt}\left(\|\partial^\alpha \chi^\varepsilon\|^2 + \|\partial^\alpha G^\varepsilon\|^2\right) = 2\left\langle \partial^\alpha \chi^\varepsilon, \partial^\alpha (N_e^\varepsilon u_e^\varepsilon + w_e^\varepsilon \rho_{e,\varepsilon}^m) - \partial^\alpha (N_i^\varepsilon u_i^\varepsilon + w_i^\varepsilon \rho_{i,\varepsilon}^m) - \partial^\alpha R_E^{\varepsilon,m}\right\rangle.$$

Noticing (2.12), applying the Cauchy-Schwarz inequality and the Moser-type inequalities in Lemma 2.1 to the right hand side of the above energy equality, we have

$$\frac{d}{dt}\left(\|\partial^\alpha \chi^\varepsilon\|^2 + \|\partial^\alpha G^\varepsilon\|^2\right) \le C\|W_*^\varepsilon\|_s^2 + C\varepsilon^{4(m+1)}.$$

Thus, combining the above estimate and (3.7), we have

$$\frac{d}{dt}\left(\sum_{\nu=i,e}\left\langle A_\nu^0(\rho_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\right\rangle + \|\partial^\alpha \chi^\varepsilon\|^2 + \|\partial^\alpha G^\varepsilon\|^2\right) \le C\|W_*^\varepsilon\|_s^2 + C\varepsilon^{4m+2}. \tag{3.17}$$

Since $A_\nu^0(\rho_\nu^\varepsilon)$ is positive definite, we obtain that there exists a constant $c_1 > 0$, such that

$$\left\langle A_\nu^0(\rho_\nu^\varepsilon)\partial^\alpha W_\nu^\varepsilon, \partial^\alpha W_\nu^\varepsilon\right\rangle \ge c_1\|W_{\nu,*}^\varepsilon\|^2.$$

Consequently, summing (3.17) for all $0 \le |\alpha| \le s$, integrating the resulting equation over $[0,t]$ for any $t \in (0, T^\varepsilon]$ and noticing (2.13), we have

$$\|W_*^\varepsilon(t)\|_s^2 \le C\varepsilon^{4m+2} + \int_0^t \|W_*^\varepsilon(s)\|_s^2 ds, \quad \forall t \in [0, T^\varepsilon].$$

Applying the Gronwall inequality to the above estimate yields (3.15). □

3.2. **Proof of Theorem 2.1.** It suffices to prove $T_e^\varepsilon \ge T_a$, i.e., $T_b^\varepsilon = T_a$. Recall that T^ε is the maximum time interval on which W_*^ε exists and satisfies

$$\|W_*^\varepsilon(t)\|_s \le C, \quad \forall t \in [0, T^\varepsilon].$$

By the definition of T_b^ε, T_a and T^ε, we have $T^\varepsilon \le T_b^\varepsilon \le T_a$. We want to prove $T^\varepsilon = T_a$. If $T^\varepsilon < T_a$, we apply the theorem of Kato for the local existence of smooth solutions with initial data $W_*^\varepsilon(T^\varepsilon)$. Consequently, there exists $T_\varepsilon > T^\varepsilon$ and a smooth solution $W_*^\varepsilon \in C([0, T_\varepsilon]; H^s)$. Since the function $t \to \|W_*^\varepsilon(t)\|_s$ is continuous on $[T^\varepsilon, T_\varepsilon]$, there exists $T'_\varepsilon \in (T^\varepsilon, T_\varepsilon]$, such that

$$\|W_*^\varepsilon(t)\|_s \le C, \quad \forall t \in [0, T'_\varepsilon].$$

This is contradictory to the maximality of T^ε. Thus, we have proved $T^\varepsilon = T_b^\varepsilon = T_a$. □

4. Uniform Global existence and convergence

In this section, we denote by $s \geq 3$ an integer and C a generic positive constant independent of ε and any time. We first prove Theorem 2.2. In what follows, we will drop the superscript ε for simplicity and assume that the conditions in Theorem 2.2 hold. Let

$$n_\nu = \rho_\nu - 1, \quad F = B - B_e, \quad U_\nu = \begin{pmatrix} n_\nu \\ u_\nu \end{pmatrix}, \quad U = \begin{pmatrix} U_i \\ U_e \\ E \\ F \end{pmatrix}.$$

For $j = 1, 2, 3$ and $u_\nu = (u_\nu^1, u_\nu^2, u_\nu^3)$, we denote

$$\begin{aligned} D_i^j(\rho_i, u_i) &= \begin{pmatrix} u_i^j & \rho_i \xi_j^T \\ \varepsilon^2 h_i'(\rho_i)\xi_j & u_i \mathbf{I}_3 \end{pmatrix}, \\ D_e^j(\rho_e, u_e) &= \begin{pmatrix} u_e^j & \rho_e \xi_j^T \\ h_e'(\rho_e)\xi_j & u_e \mathbf{I}_3 \end{pmatrix}, \end{aligned}$$

and

$$\begin{aligned} Q_i(u_i, E, B) &= \begin{pmatrix} 0 \\ \varepsilon^2(E + u_i \times B) - u_i \end{pmatrix}, \\ Q_e(u_e, E, B) &= \begin{pmatrix} 0 \\ -E - u_e \times B - u_e \end{pmatrix}, \end{aligned}$$

where $\mathbf{I_3}$ is the unit matrix, $\{\xi_k\}_{k=1}^3$ is the canonical basis of $\mathbb{R}^3$ and $\xi_j^\top$ is the transpose of ξ_j. Then (2.14) becomes

$$\partial_t U_\nu + \sum_{j=1}^3 D_\nu^j(\rho_\nu, u_\nu)\partial_{x_j} U_\nu = Q_\nu(u_\nu, E, B), \tag{4.1}$$

with initial conditions

$$t = 0: \quad (\rho_\nu, u_\nu, E, B) = (\rho_{\nu,0}^\varepsilon, u_{\nu,0}^\varepsilon, E_0^\varepsilon, B_0^\varepsilon), \quad x \in \mathbb{T}^3. \tag{4.2}$$

System (4.1) is symmetrizable hyperbolic when $\rho_\nu > 0$. Indeed, if we define the symmetrizers as

$$\begin{aligned} D_i^0(\rho_i) &= \begin{pmatrix} h_i'(\rho_i) & 0 \\ 0 & \frac{1}{\varepsilon^2}\rho_i \mathbf{I}_3 \end{pmatrix}, \\ D_e^0(\rho_e) &= \begin{pmatrix} h_e'(\rho_e) & 0 \\ 0 & \rho_e \mathbf{I}_3 \end{pmatrix}, \end{aligned}$$

which are symmetric and positive definite for $\rho_\nu > 0$. The matrix $\tilde{D}_\nu^j$ are defined by

$$\begin{aligned}\tilde{D}_i^j(\rho_i, u_i) &= D_i^0(\rho_i) D_i^j(\rho_i, u_i) = \begin{pmatrix} h_i'(\rho_i) u_i^j & p_i'(\rho_i)\xi_j^T \\ p_i'(\rho_i)\xi_j & \frac{1}{\varepsilon^2}\rho_i u_i^j \mathbf{I}_3 \end{pmatrix}, \\ \tilde{D}_e^j(\rho_e, u_e) &= D_i^0(\rho_e) D_e^j(\rho_e, u_e) = \begin{pmatrix} h_e'(\rho_e) u_e^j & p_e'(\rho_e)\xi_j^T \\ p_e'(\rho_e)\xi_j & \rho_e u_e^j \mathbf{I}_3 \end{pmatrix},\end{aligned}$$

which are symmetric.

Without loss of generality, let $T > 0$ and U be the local smooth solution of (4.1)-(4.2) defined on the interval $[0, T]$. We introduce the total energy and the dissipative energy as follows

$$\begin{aligned}\mathscr{E}(t) &= \sum_{\nu=i,e} \|n_\nu\|_s^2 + \frac{1}{\varepsilon^2}\|n_i(t)\|_{s-1}^2 + \frac{1}{\varepsilon^2}\|u_i(t)\|_s^2 + \|u_e(t)\|_s^2 + \|E(t)\|_s^2 + \|F(t)\|_s^2, \\ \mathscr{D}(t) &= \sum_{\nu=i,e} \|\nabla n_\nu(t)\|_{s-1}^2 + \frac{1}{\varepsilon^2}\|u_i(t)\|_s^2 + \|u_e(t)\|_s^2 + \|\mathrm{div} E\|_{s-1}^2.\end{aligned}$$

Moreover, we set

$$\mathscr{E}_T = \sup_{0 \le t \le T} \mathscr{E}(t),$$

which we assume to be uniformly sufficiently small with respect to T and ε. Besides, because of the smallness of $\mathscr{E}_T$, it is reasonable to assume

$$\frac{1}{2} \le \rho_\nu \le \frac{3}{2}, \quad \text{and} \quad h_\nu'(\rho_\nu) \ge h_0, \tag{4.3}$$

where h_0 is a positive constant independent of the small parameter ε and any time. We first have

Lemma 4.1. *(L^2-estimate) It holds*

$$\frac{d}{dt}\int_{\mathbb{T}^3} \rho_e|u_e|^2 + h_e'(\rho_e^*) n_e^2 + \frac{1}{\varepsilon^2}\rho_i|u_i|^2 + h_i'(\rho_i^*) n_i^2 + |E|^2 + |F|^2 dx + \|u_e\|^2 + \frac{1}{\varepsilon^2}\|u_i\|^2 \le 0, \tag{4.4}$$

in which ρ_ν^* is between 1 and ρ_ν.

Proof. The entropy and the corresponding entropy flux for the bipolar Euler system in (2.14) are

$$\begin{cases} \eta_0(\rho_\nu, u_\nu) = \frac{1}{2}\rho_e|u_e|^2 + H_e(\rho_e) + \frac{1}{2\varepsilon^2}\rho_i|u_i|^2 + H_i(\rho_i), \\ \psi_0(\rho_\nu, u_\nu) = \frac{1}{2}\rho_e|u_e|^2 u_e + \rho_e h_e(\rho_e) u_e + \frac{1}{2\varepsilon^2}\rho_i|u_i|^2 u_i + \rho_i h_i(\rho_i) u_i, \end{cases}$$

in which $H_\nu'(\rho) = h_\nu(\rho)$. Then the entropy identity for the bipolar Euler system is

$$\partial_t \eta_0(\rho_\nu, u_\nu) + \mathrm{div}\psi_0(\rho_\nu, u_\nu) + \rho_e|u_e|^2 + \frac{1}{\varepsilon^2}\rho_i|u_i|^2 = -E(\rho_e u_e - \rho_i u_i).$$

The energy conservation of Maxwell equations in (2.14) is

$$\frac{1}{2}\partial_t\left(|E|^2+|F|^2\right)+\mathrm{div}(E\times F)=E(\rho_e u_e-\rho_i u_i).$$

Thus, combining the above two equations, we have

$$\partial_t\eta(\rho_\nu,u_\nu,E,F)+\mathrm{div}\psi(\rho_\nu,u_\nu,E,F)+\rho_e|u_e|^2+\frac{1}{\varepsilon^2}\rho_i|u_i|^2=0, \tag{4.5}$$

in which

$$\begin{cases}\eta(\rho_\nu,u_\nu,E,F)=\eta_0+\frac{1}{2}|E|^2+\frac{1}{2}|F|^2,\\ \psi(\rho_\nu,u_\nu,E,F)=\psi_0+E\times F.\end{cases}$$

By the Taylor formula, we obtain

$$H_\nu(\rho_\nu)=H_\nu(1)+h_\nu(1)n_\nu+\frac{1}{2}h'(\rho_\nu^*)n_\nu^2,$$

where ρ_ν^* is between 1 and ρ_ν. Since $\partial_t n_\nu=-\mathrm{div}(\rho_\nu u_\nu)$, we have

$$\partial_t H_\nu(\rho_\nu)=-h_\nu(1)\mathrm{div}(\rho_\nu u_\nu)+\frac{1}{2}\partial_t\left(h'(\rho_\nu^*)n_\nu^2\right).$$

Thus, substituting the above into (4.5), integrating the resulting equation over $\mathbb{T}^3$ and noticing (4.3) yield (4.4). □

Lemma 4.2. *(Higher order estimates) It holds*

$$\begin{aligned}&\frac{d}{dt}\sum_{\nu=i,e}\sum_{1\le|\alpha|\le s}\left\langle\partial^\alpha U_\nu,D_\nu^0\partial^\alpha U_\nu\right\rangle\\ +\ &\frac{d}{dt}\left(\int_{\mathbb{T}^3}\rho_e|u_e|^2+\sum_{\nu=i,e}h_\nu'(\rho_\nu^*)n_\nu^2+\frac{1}{\varepsilon^2}\rho_i|u_i|^2dx+\|E\|_s^2+\|F\|_s^2\right)+\|u_e\|_s^2+\frac{1}{\varepsilon^2}\|u_i\|_s^2\\ \le\ &C\mathscr{E}_T^{1/2}\mathscr{D}(t).\end{aligned}\tag{4.6}$$

Proof. For a multi-index $\alpha\in\mathbb{N}^3$ with $1\le|\alpha|\le s$, applying ∂^α to both sides of (4.1), we obtain

$$\partial_t\partial^\alpha U_\nu+\sum_{j=1}^3 D_\nu^j(\rho_\nu,u_\nu)\partial^\alpha\partial_{x_j}U_\nu=\partial^\alpha Q(u_\nu,E,B)+J_\nu^\alpha, \tag{4.7}$$

in which

$$J_\nu^\alpha=\sum_{j=1}^3(D_\nu^j(\rho_\nu,u_\nu)\partial^\alpha\partial_{x_j}U_\nu-\partial^\alpha(D_\nu^j(\rho_\nu,u_\nu)\partial_{x_j}U_\nu)).$$

Since D_ν^0 and $\tilde{D}_\nu^j$ are symmetric, taking the inner product of (4.7) with $D_\nu^0(\rho_\nu)\partial^\alpha U_\nu$ in $L^2(\mathbb{T}^3)$, we have

$$\begin{aligned}\frac{d}{dt}\left\langle\partial^\alpha U_\nu,D_\nu^0\partial^\alpha U_\nu\right\rangle &= \left\langle\mathrm{div}D_\nu\partial^\alpha U_\nu,\partial^\alpha U_\nu\right\rangle+2\left\langle J_\nu^\alpha,D_\nu^0\partial^\alpha U_\nu\right\rangle+2\left\langle\partial^\alpha Q_\nu,D_\nu^0\partial^\alpha U_\nu\right\rangle\\ &= I_\nu^1+I_\nu^2+I_\nu^3,\end{aligned}\tag{4.8}$$

with the natural correspondence of I_ν^1, I_ν^2 and I_ν^3, and

$$\mathrm{div}D_\nu = \partial_t D_\nu^0(\rho_\nu) + \sum_{j=1}^3 \partial_{x_j} \tilde{D}_\nu^j(\rho_\nu, u_\nu).$$

In what follows, we will estimate I_ν^1, I_ν^2 and I_ν^3 term by term. First, since $\partial_t n_\nu = -\mathrm{div}(\rho_\nu u_\nu)$, we have

$$\|\partial_t n_\nu\|_\infty \le \|u_\nu\|_s.$$

Since $\varepsilon < 1$, it follows that

$$\begin{aligned}\left\langle \partial_t D_i^0 \partial^\alpha U_i, \partial^\alpha U_i \right\rangle &\le C\left(\|\nabla n_i\|_{s-1}^2 + \left\|\frac{u_i}{\varepsilon}\right\|_s^2\right)\left\|\frac{u_i}{\varepsilon}\right\|_s, \\ \left\langle \partial_t D_e^0 \partial^\alpha U_e, \partial^\alpha U_e \right\rangle &\le C\left(\|\nabla n_e\|_{s-1}^2 + \|u_e\|_s^2\right)\|u_e\|_s.\end{aligned}$$

Similarly, for $1 \le j \le 3$,

$$\begin{aligned}\left\langle \partial_{x_j} \tilde{D}_i^j(\rho_i, u_i) \partial^\alpha U_i, \partial^\alpha U_i \right\rangle &\le C\left(\|\nabla n_i\|_{s-1}^2 + \left\|\frac{u_i}{\varepsilon}\right\|_s^2\right)\left\|\frac{u_i}{\varepsilon}\right\|_s, \\ \left\langle \partial_{x_j} \tilde{D}_e^j(\rho_e, u_e) \partial^\alpha U_e, \partial^\alpha U_e \right\rangle &\le C\left(\|\nabla n_e\|_{s-1}^2 + \|u_e\|_s^2\right)\|u_e\|_s.\end{aligned}$$

Therefore

$$|I_\nu^1| \le C\left(\|\nabla n_i\|_{s-1}^2 + \left\|\frac{u_i}{\varepsilon}\right\|_s^2\right)\left\|\frac{u_i}{\varepsilon}\right\|_s + C\left(\|\nabla n_e\|_{s-1}^2 + \|u_e\|_s^2\right)\|u_e\|_s \le C\mathscr{E}_T^{1/2}\mathscr{D}(t). \quad (4.9)$$

For I_ν^2, since $I_\nu^2 = 2\sum_{j=1}^3 I_{\nu,j}^2$, with

$$\begin{aligned} I_{i,j}^2 = & \left\langle h_i'(\rho_i)\left(\partial^\alpha(u_i^j \partial_{x_j} n_i) - u_i^j \partial^\alpha \partial_{x_j} n_i\right), \partial^\alpha n_i \right\rangle \\ & + \left\langle h_i'(\rho_i)\left(\partial^\alpha(\rho_i \partial_{x_j} u_i^j) - \rho_i \partial^\alpha \partial_{x_j} u_i^j\right), \partial^\alpha n_i \right\rangle \\ & + \left\langle \rho_i \left(\partial^\alpha(h_i'(\rho_i)\partial_{x_j} n_i) - h_i'(\rho_i)\partial^\alpha \partial_{x_j} n_i\right), \partial^\alpha u_i^j \right\rangle \\ & + \varepsilon^{-2}\left\langle \rho_i \left(\partial^\alpha(u_i \partial_{x_j} u_i) - u_i \partial^\alpha \partial_{x_j} u_i\right), \partial^\alpha u_i \right\rangle, \end{aligned}$$

and

$$\begin{aligned} I_{e,j}^2 = & \left\langle h_e'(\rho_e)\left(\partial^\alpha(u_e^j \partial_{x_j} n_e) - u_e^j \partial^\alpha \partial_{x_j} n_e\right), \partial^\alpha n_e \right\rangle \\ & + \left\langle h_e'(\rho_e)\left(\partial^\alpha(\rho_e \partial_{x_j} u_e^j) - \rho_e \partial^\alpha \partial_{x_j} u_e^j\right), \partial^\alpha n_e \right\rangle \\ & + \left\langle \rho_e \left(\partial^\alpha(h_e'(\rho_e)\partial_{x_j} n_e) - h_e'(\rho_e)\partial^\alpha \partial_{x_j} n_e\right), \partial^\alpha u_e^j \right\rangle \\ & + \left\langle \rho_e \left(\partial^\alpha(u_e \partial_{x_j} u_e) - u_e \partial^\alpha \partial_{x_j} u_e\right), \partial^\alpha u_e \right\rangle. \end{aligned}$$

Noticing $\nabla h_\nu'(\rho_\nu) = h_\nu''(\rho_\nu)\nabla n_\nu$, by the Moser-type inequalities in Lemma 2.1, we have

$$\begin{aligned}|I_{i,j}^2| &\le C\left(\|\nabla n_i\|_{s-1}^2 + \left\|\frac{u_i}{\varepsilon}\right\|_s^2\right)\left\|\frac{u_i}{\varepsilon}\right\|_s, \\ |I_{e,j}^2| &\le C\left(\|\nabla n_e\|_{s-1}^2 + \|u_e\|_s^2\right)\|u_e\|_s,\end{aligned}$$

which similarly implies

$$|I_\nu^2| \le C\mathscr{E}_T^{1/2}\mathscr{D}(t). \tag{4.10}$$

For I_ν^3, by using the Moser-type inequalities in Lemma 2.1, we obtain

$$\begin{aligned} I_i^3 &= 2\langle \partial^\alpha E, \rho_i \partial^\alpha u_i\rangle + 2\langle \partial^\alpha (u_i \times B), \rho_i\partial^\alpha u_i\rangle - \frac{2}{\varepsilon^2}\langle \rho_i \partial^\alpha u_i, \partial^\alpha u_i\rangle \\ &= 2\langle \partial^\alpha E, \rho_i \partial^\alpha u_i\rangle - \frac{2}{\varepsilon^2}\langle \rho_i \partial^\alpha u_i, \partial^\alpha u_i\rangle \\ &\quad +2\langle \partial^\alpha u_i \times B, \rho_i \partial^\alpha u_i\rangle + 2\langle \partial^\alpha (u_i\times B) - \partial^\alpha u_i \times B, \rho_i \partial^\alpha u_i\rangle \\ &\le 2\langle \partial^\alpha E, \rho_i \partial^\alpha u_i\rangle - \frac{1}{\varepsilon^2}\|\partial^\alpha u_i\|^2 + C\|u_i\|_{s-1}\|\nabla F\|_{s-1}\|\partial^\alpha u_i\| \\ &\le 2\langle \partial^\alpha E, \rho_i \partial^\alpha u_i\rangle - \frac{1}{\varepsilon^2}\|\partial^\alpha u_i\|^2 + C\mathscr{E}_T^{1/2}\mathscr{D}(t), \end{aligned}$$

similarly

$$I_e^3 \le -2\langle \partial^\alpha E, \rho_e \partial^\alpha u_e\rangle - \|\partial^\alpha u_e\|^2 + C\mathscr{E}_T^{1/2}\mathscr{D}(t).$$

Substituting (4.9), (4.10) and the above two estimates into (4.8), and adding the resulting equation for $\nu = i, e$ yield

$$\frac{d}{dt}\sum_{\nu=i,e}\left\langle \partial^\alpha U_\nu, D_\nu^0 \partial^\alpha U_\nu\right\rangle + \frac{1}{\varepsilon^2}\|\partial^\alpha u_i\|^2 + \|\partial^\alpha u_e\|^2 - 2\langle \partial^\alpha E, \rho_i\partial^\alpha u_i - \rho_e \partial^\alpha u_e\rangle \le C\mathscr{E}_T^{1/2}\mathscr{D}(t). \tag{4.11}$$

Now we estimate $2\langle \partial^\alpha E, \rho_i\partial^\alpha u_i - \rho_e\partial^\alpha u_e\rangle$. Applying ∂^α to the Maxwell equations in (2.14), we have

$$\begin{cases} \partial_t \partial^\alpha E - \nabla \times \partial^\alpha F = -\partial^\alpha(\rho_i u_i - \rho_e u_e), \\ \partial_t \partial^\alpha F + \nabla\times \partial^\alpha E = 0. \end{cases}$$

Taking the inner product in $L^2(\mathbb{T}^3)$ of the first equation in the above system with $2\partial^\alpha E$ and of the second equation with $2\partial^\alpha F$, and adding the two resulting equations, we have

$$\begin{aligned} \frac{d}{dt}\left(\|\partial^\alpha E\|^2 + \|\partial^\alpha G\|^2\right) &= -2\langle \partial^\alpha E, \rho_i \partial^\alpha u_i - \rho_e\partial^\alpha u_e\rangle \\ &\quad -2\langle \partial^\alpha E, \partial^\alpha(\rho_i u_i) - \rho_i\partial^\alpha u_i\rangle + 2\langle \partial^\alpha E, \partial^\alpha(\rho_e u_e) - \rho_e \partial^\alpha u_e\rangle, \end{aligned}$$

in which by the Moser-type inequalities, we obtain

$$\begin{aligned} |\langle \partial^\alpha E, \partial^\alpha(\rho_i u_i) - \rho_i \partial^\alpha u_i\rangle| &\le C\|\partial^\alpha E\|\|\nabla \rho_i\|_{s-1}\left\|\frac{u_i}{\varepsilon}\right\|_{s-1} \le C\mathscr{E}_T^{1/2}\mathscr{D}(t), \\ |\langle \partial^\alpha E, \partial^\alpha(\rho_e u_e) - \rho_e \partial^\alpha u_e\rangle| &\le C\|\partial^\alpha E\|\|\nabla\rho_e\|_{s-1}\|u_e\|_{s-1} \le C\mathscr{E}_T^{1/2}\mathscr{D}(t). \end{aligned}$$

Hence,

$$-2\langle \partial^\alpha E, \rho_i\partial^\alpha u_i - \rho_e\partial^\alpha u_e\rangle \ge \frac{d}{dt}\left(\|\partial^\alpha E\|^2 + \|\partial^\alpha G\|^2\right) - C\mathscr{E}_T^{1/2}\mathscr{D}(t).$$

Substituting the above into (4.11), adding the resulting equation for all $1 \le |\alpha| \le s$ and combining (4.4) yield (4.6). $\square$

Lemma 4.3. *(Dissipation of ∇n_ν) It holds*

$$\begin{aligned} &\sum_{|\beta|\le s-1}\frac{d}{dt}\left(\frac{1}{\varepsilon^2}\left(\|\partial^\beta n_i\|^2+2\left\langle\partial^\beta u_i,\partial^\beta\nabla n_i\right\rangle\right)+\|\partial^\beta n_e\|^2+2\left\langle\partial^\beta u_e,\partial^\beta\nabla n_e\right\rangle\right)\\ &+\frac{h_0}{2}\|\nabla n_i\|_{s-1}^2+\frac{h_0}{2}\|\nabla n_e\|_{s-1}^2+\|\mathrm{div}E\|_{s-1}^2\\ \le\ & C\left\|\frac{u_e}{\varepsilon}\right\|_s^2+C\|u_e\|_s^2+C\mathscr{E}_T^{1/2}\mathscr{D}(t). \end{aligned}\tag{4.12}$$

Proof. For a multi-index $\beta\in\mathbb{N}^3$ with $|\beta|\le s-1$, multiplying $\varepsilon^{-2}\rho_i$ to both sides of the momentum equation of ions in (2.14), and applying ∂^β to the resulting equation, we have

$$\begin{aligned} p_i'(\rho_i)\partial^\beta\nabla n_i-\partial^\beta(\rho_iE) &= -\varepsilon^{-2}\partial_t\partial^\beta(\rho_iu_i)-\varepsilon^{-2}\partial^\beta(\rho_i(u_i\cdot\nabla)u_i)-\varepsilon^{-2}\partial^\beta(\rho_iu_i)\\ &\quad+\partial^\beta(\rho_iu_i\times B)+\left(p_i'(\rho_i)\partial^\beta\nabla n_i-\partial^\beta(p_i'(\rho_i)\nabla n_i)\right). \end{aligned}$$

Taking the inner product of the above equation with $\partial^\beta\nabla n_i$ in $L^2(\mathbb{T}^3)$ yields

$$\begin{aligned} &\left\langle p_i'(\rho_i)\partial^\beta\nabla n_i,\partial^\beta\nabla n_i\right\rangle-\left\langle\partial^\beta(\rho_iE),\partial^\beta\nabla n_i\right\rangle\\ =\ &-\left\langle\varepsilon^{-2}\partial_t\partial^\beta(\rho_iu_i),\partial^\beta\nabla n_i\right\rangle-\left\langle\varepsilon^{-2}\partial^\beta(\rho_i(u_i\cdot\nabla)u_i),\partial^\beta\nabla n_i\right\rangle-\left\langle\varepsilon^{-2}\partial^\beta(\rho_iu_i),\partial^\beta\nabla n_i\right\rangle\\ &+\left\langle\partial^\beta(\rho_iu_i\times B),\partial^\beta\nabla n_i\right\rangle+\left\langle\left(p_i'(\rho_i)\partial^\beta\nabla n_i-\partial^\beta(p_i'(\rho_i)\nabla n_i)\right),\partial^\beta\nabla n_i\right\rangle. \end{aligned}\tag{4.13}$$

Now we treat each terms in the above equation. First, the estimate for the term containing $\partial^\beta(\rho_iE)$ on the left hand side requires a little more calculations. Let us first remark that

$$\left\langle\partial^\beta(\rho_iE),\partial^\beta\nabla n_i\right\rangle = \left\langle\rho_i\partial^\beta E,\partial^\beta\nabla n_i\right\rangle+\left\langle\left(\partial^\beta(\rho_iE)-\rho_i\partial^\beta E\right),\partial^\beta\nabla n_i\right\rangle,$$

in which by the Moser-type inequalities, we have

$$\left|\left\langle\left(\partial^\beta(\rho_iE)-\rho_i\partial^\beta E\right),\partial^\beta\nabla n_i\right\rangle\right|\le C\|E\|_{s-1}\|\nabla n_i\|_{s-1}^2\le C\mathscr{E}_T^{1/2}\mathscr{D}(t),$$

and

$$\begin{aligned} \left\langle\rho_i\partial^\beta E,\partial^\beta\nabla n_i\right\rangle &= \frac12\left\langle\partial^\beta E,\partial^\beta(\nabla(\rho_i)^2)\right\rangle-\left\langle\partial^\beta E,\partial^\beta(\rho_i\nabla\rho_i)-\rho_i\partial^\beta\nabla\rho_i\right\rangle\\ &\le \frac12\left\langle\partial^\beta E,\partial^\beta(\nabla(\rho_i)^2)\right\rangle-C\mathscr{E}_T^{1/2}\mathscr{D}(t). \end{aligned}$$

Hence,

$$\left\langle\partial^\beta(\rho_iE),\partial^\beta\nabla n_i\right\rangle\le\frac12\left\langle\partial^\beta E,\partial^\beta(\nabla(\rho_i)^2)\right\rangle-C\mathscr{E}_T^{1/2}\mathscr{D}(t).$$

Moreover, noticing (4.3), we have

$$\left\langle p_i'(\rho_i)\partial^\beta\nabla n_i,\partial^\beta\nabla n_i\right\rangle\ge\frac12h_0\|\partial^\beta\nabla n_i\|^2.$$

Let us move on to the right hand side of (4.13). An integration by parts gives

$$\begin{aligned} \varepsilon^{-2}\left\langle\partial_t\partial^\beta(\rho_iu_i),\partial^\beta\nabla n_i\right\rangle &= \varepsilon^{-2}\frac{d}{dt}\left\langle\partial^\beta(\rho_iu_i),\partial^\beta\nabla n_i\right\rangle+\varepsilon^{-2}\left\langle\partial^\beta\mathrm{div}(\rho_iu_i),\partial^\beta\mathrm{div}(\rho_iu_i)\right\rangle\\ &\le \varepsilon^{-2}\frac{d}{dt}\left\langle\partial^\beta(\rho_iu_i),\partial^\beta\nabla n_i\right\rangle+C\left\|\frac{u_i}{\varepsilon}\right\|_s^2. \end{aligned}$$

By applying the Young inequality, the Cauchy-Schwarz inequality and the Moser-type inequalities, it is obvious that

$$
\begin{aligned}
\varepsilon^{-2}|\left\langle\partial^\beta(\rho_i(u_i\cdot\nabla)u_i),\partial^\beta\nabla n_i\right\rangle| &\leq C\|\nabla n_i\|_{s-1}\left\|\frac{u_i}{\varepsilon}\right\|_s^2\leq C\mathscr{E}_T^{1/2}\mathscr{D}(t),\\
\varepsilon^{2}|\left\langle\partial^\beta(\rho_i u_i\times B),\partial^\beta\nabla n_i\right\rangle| &\leq C\left\|\frac{u_i}{\varepsilon}\right\|_s^2+\frac{h_0}{4}\|\partial^\beta\nabla n_i\|^2,\\
|\left\langle p_i'(\rho_i)\partial^\beta\nabla n_i-\partial^\beta(p_i'(\rho_i)\nabla n_i),\partial^\beta\nabla n_i\right\rangle| &\leq C\|\nabla n_i\|_{s-1}^3\leq C\mathscr{E}_T^{1/2}\mathscr{D}(t).
\end{aligned}
$$

Next, using the mass equation of ions in (2.14), we have

$$
\varepsilon^{-2}\left\langle\partial^\beta(\rho_i u_i),\partial^\beta\nabla n_i\right\rangle=-\varepsilon^{-2}\left\langle\partial^\beta\mathrm{div}(\rho_i u_i),\partial^\beta n_i\right\rangle=\varepsilon^{-2}\left\langle\partial_t\partial^\beta n_i,\partial^\beta\rho_i\right\rangle=\frac{1}{2\varepsilon^2}\frac{d}{dt}\|\partial^\beta n_i\|^2.
$$

Thus, combining all these estimates, we have

$$
\begin{aligned}
&\frac{1}{\varepsilon^2}\frac{d}{dt}\left(\|\partial^\beta n_i\|^2+2\left\langle\partial^\beta u_i,\partial^\beta\nabla n_i\right\rangle\right)+\frac{h_0}{2}\|\partial^\beta\nabla n_i\|^2-\left\langle\partial^\beta E,\partial^\beta(\nabla(\rho_i)^2)\right\rangle\\
\leq\ & C\left\|\frac{u_i}{\varepsilon}\right\|_s^2+C\mathscr{E}_T^{1/2}\mathscr{D}(t). \qquad (4.14)
\end{aligned}
$$

Similarly, applying the same procedure as above to the momentum equation for electrons, we obtain the similar estimate for electrons.

$$
\begin{aligned}
&\frac{d}{dt}\left(\|\partial^\beta n_e\|^2+2\left\langle\partial^\beta u_e,\partial^\beta\nabla n_e\right\rangle\right)+\frac{h_0}{2}\|\partial^\beta\nabla n_e\|^2+\left\langle\partial^\beta E,\partial^\beta(\nabla(\rho_e)^2)\right\rangle\\
\leq\ & C\left\|u_e\right\|_s^2+C\mathscr{E}_T^{1/2}\mathscr{D}(t). \qquad (4.15)
\end{aligned}
$$

Now it remains to estimate the term $\left\langle\partial^\beta E,\partial^\beta\nabla((\rho_i)^2-(\rho_e)^2)\right\rangle$. Since

$$
\partial^\beta\mathrm{div}E=\partial^\beta\rho_i-\partial^\beta\rho_e,
$$

then

$$
\begin{aligned}
\left\langle\partial^\beta E,\partial^\beta\nabla((\rho_i)^2-(\rho_e)^2)\right\rangle &= -\left\langle\partial^\beta\mathrm{div}E,\partial^\beta((\rho_i-\rho_e)(\rho_i+\rho_e))\right\rangle\\
&= -\left\langle(\rho_i+\rho_e)\partial^\beta\mathrm{div}E,\partial^\beta\mathrm{div}E\right\rangle\\
&\quad -\left\langle\partial^\beta\mathrm{div}E,\partial^\beta((\rho_i-\rho_e)(\rho_i+\rho_e))-(\rho_i+\rho_e)\partial^\beta(\rho_i-\rho_e)\right\rangle,
\end{aligned}
$$

in which noticing (4.3), we have

$$
\left\langle(\rho_i+\rho_e)\partial^\beta\mathrm{div}E,\partial^\beta\mathrm{div}E\right\rangle\geq\|\partial^\beta\mathrm{div}E\|^2,
$$

and by the Cauchy-Schwarz inequality and the Moser-type inequality, we have

$$
\begin{aligned}
&|\left\langle\partial^\beta\mathrm{div}E,\partial^\beta((\rho_i-\rho_e)(\rho_i+\rho_e))-(\rho_i+\rho_e)\partial^\beta(\rho_i-\rho_e)\right\rangle|\\
\leq\ & C\|\partial^\beta\mathrm{div}E\|\|\nabla\rho_i+\nabla\rho_e\|_{s-1}\|\rho_i-\rho_e\|_{s-1}\\
\leq\ & C\mathscr{D}(t)(\|\rho_i-1\|_{s-1}+\|\rho_e-1\|_{s-1})\\
\leq\ & C\mathscr{E}_T^{1/2}\mathscr{D}(t).
\end{aligned}
$$

Hence, combining these estimates, we have

$$\left\langle \partial^\beta E, \partial^\beta \nabla((\rho_i)^2-(\rho_e)^2)\right\rangle \le -\|\partial^\beta \mathrm{div} E\|^2 + C\mathscr{E}_T^{1/2}\mathscr{D}(t).$$

Adding (4.14) and (4.15), and combining the above estimate, we have

$$\begin{aligned}
&\frac{1}{\varepsilon^2}\frac{d}{dt}\left(\|\partial^\beta n_i\|^2 + 2\left\langle \partial^\beta u_i, \partial^\beta \nabla n_i\right\rangle\right) + \frac{d}{dt}\left(\|\partial^\beta n_e\|^2 + 2\left\langle \partial^\beta u_e, \partial^\beta \nabla n_e\right\rangle\right)\\
&+\frac{h_0}{2}\|\partial^\beta \nabla n_i\|^2 + \frac{h_0}{2}\|\partial^\beta \nabla n_e\|^2 + \|\mathrm{div}\partial^\beta E\|^2\\
\le\ & C\left\|\frac{u_e}{\varepsilon}\right\|_s^2 + C\|u_e\|_s^2 + C\mathscr{E}_T^{1/2}\mathscr{D}(t).
\end{aligned}$$

Adding the above for all $|\beta|\le s-1$ yields (4.12). □

Lemma 4.4. *For* $\forall\, t>0$, *it holds*

$$\begin{aligned}
&\sum_{\nu=i,e}\|\rho_\nu(t)-1\|_s^2 + \|u_e(t)\|_s^2 + \frac{1}{\varepsilon^2}\|u_i(t)\|_s^2 + \|E(t)\|_s^2 + \|B(t)-B_e\|_s^2\\
&+\int_0^t \sum_{\nu=i,e}\|\nabla\rho_\nu(\tau)\|_{s-1}^2 + \frac{1}{\varepsilon^2}\|u_i(\tau)\|_s^2 + \|u_e(\tau)\|_s^2 d\tau\\
\le\ & C(\|\rho_{\nu,0}^\varepsilon - 1\|_s^2 + \frac{1}{\varepsilon^2}\|\nabla \rho_{i,0}^\varepsilon\|_{s-1}^2 + \|u_{e,0}^\varepsilon\|_s^2 + \frac{1}{\varepsilon}\|u_{i,0}^\varepsilon\|_s^2 + |E_0^\varepsilon\|_s^2 + \|B_0^\varepsilon\|_s^2).
\end{aligned}\tag{4.16}$$

Proof. Now let us define the following

$$\begin{aligned}
\mathbb{E}(t) =\ & \kappa\varepsilon^{-2}\|n_i\|_{s-1}^2 + \kappa\|n_e\|_{s-1}^2 + \|E\|_s^2 + \|F\|_s^2 + \int_{\mathbb{T}^3}\rho_e|u_e|^2 + \sum_{\nu=i,e}h'_\nu(\rho_\nu^*)n_\nu^2 + \frac{1}{\varepsilon^2}\rho_i|u_i|^2 dx\\
&+\sum_{\nu=i,e}\sum_{1\le|\alpha|\le s}\left\langle \partial^\alpha U_\nu, D_\nu^0\partial^\alpha U_\nu\right\rangle + \sum_{|\beta|\le s-1}2\kappa\left(\varepsilon^{-2}\left\langle \partial^\beta u_i, \partial^\beta\nabla n_i\right\rangle + \left\langle \partial^\beta u_e, \partial^\beta \nabla n_e\right\rangle\right),
\end{aligned}$$

and

$$\mathbb{D}(t) = \|u_e\|_s^2 + \frac{1}{\varepsilon^2}\|u_i\|_s^2 + \frac{\kappa h_0}{2}\|\nabla n_i\|_{s-1}^2 + \frac{\kappa h_0}{2}\|\nabla n_e\|_{s-1}^2 + \kappa\|\mathrm{div}E\|_{s-1}^2,$$

where $\kappa>0$ is a small constant to be determined later. Using Lemma 4.2-4.3, adding the two resulting estimates in the way (4.6)+κ(4.12), we have

$$\frac{d}{dt}\mathbb{E}(t) + \mathbb{D}(t) \le C\kappa\left(\|u_e\|_s^2 + \frac{1}{\varepsilon^2}\|u_i\|_s^2\right) + C\mathscr{E}_T^{1/2}\mathscr{D}(t).$$

Since $\mathbb{D}(t)$ is equivalent to $\mathscr{D}(t)$, and $\mathscr{E}_T$ is sufficiently small, a straightforward calculation implies that there exists a positive constant $\kappa_1>0$, such that

$$\frac{d}{dt}\mathbb{E}(t) + \kappa_1\mathscr{D}(t) \le 0,$$

provided that κ is chosen to be sufficiently small. Integrating the above inequality over $[0,t]$ for any $t\in(0,T]$ yields

$$\mathbb{E}(t) + \kappa_1\int_0^t \mathscr{D}(\tau)d\tau \le \mathbb{E}(0). \tag{4.17}$$

Noticing (4.3) and the fact that D_ν^0 is positive definite, it is obvious that $\mathbb{E}(t)$ is equivalent to $\mathscr{E}(t)$. Thus, from (4.17), we have

$$\mathscr{E}(t)+\int_0^t \mathscr{D}(\tau)d\tau \leq C\mathscr{E}(0),$$

This gives (4.16), which implies the uniform global existence of solution to (2.14) by a bootstrap argument. □

The proof of Theorem 2.2 follows from the following lemma.

Lemma 4.5. *It holds*

$$\int_0^t \left(\|E(\tau)\|_{s-1}^2+\|\nabla F(\tau)\|_{s-2}^2\right) d\tau \leq C\mathscr{E}(0), \quad \forall t\geq 0. \tag{4.18}$$

Proof. Let $t>0$ and

$$E=-\partial_t u_e - f, \tag{4.19}$$

in which

$$f=(u_e\cdot\nabla)u_e+\nabla h(\rho_e)+u_e+u_e\times B.$$

Since $\varepsilon\in(0,1]$, from (4.16), we obtain

$$\int_0^t \|f(\tau)\|_{s-1}^2 d\tau \leq C\mathscr{E}(0). \tag{4.20}$$

For a multi-index $\beta\in\mathbb{N}^3$ with $|\beta|\leq s-1$, applying ∂^β to both sides of (4.19), we obtain

$$\partial^\beta E=-\partial_t\partial^\beta u_e-\partial^\beta f.$$

Taking the inner product of the above with $\partial^\beta E$ in $L^2(\mathbb{T}^3)$ yields

$$\|\partial^\beta E\|^2=-\left\langle \partial^\beta E,\partial^\beta f\right\rangle-\left\langle \partial_t\partial^\beta u_e,\partial^\beta E\right\rangle,$$

in which by the Young inequality,

$$\left|\left\langle \partial^\beta E,\partial^\beta f\right\rangle\right|\leq \frac{1}{2}\|\partial^\beta E\|^2+C\|\partial^\beta f\|^2,$$

and by (2.14) and the Moser-type inequalities,

$$\begin{aligned}
\left\langle \partial_t\partial^\beta u_e,\partial^\beta E\right\rangle &= \frac{d}{dt}\left\langle \partial^\beta u_e,\partial^\beta E\right\rangle-\left\langle \partial^\beta u_e,\partial_t\partial^\beta E\right\rangle\\
&= \frac{d}{dt}\left\langle \partial^\beta u_e,\partial^\beta E\right\rangle-\left\langle \partial^\beta u_e,\nabla\times\partial^\beta F+\partial^\beta(\rho_e u_e-\rho_i u_i)\right\rangle\\
&\geq \frac{d}{dt}\left\langle \partial^\beta u_e,\partial^\beta E\right\rangle-C\mathscr{D}(t)-\mu\|\nabla F\|_{s-2}^2,
\end{aligned}$$

where $\mu>0$ is a sufficiently small constant to be determined later. Hence,

$$2\frac{d}{dt}\left\langle \partial^\beta u_e,\partial^\beta E\right\rangle+\|\partial^\beta E^\varepsilon\|^2\leq C\mu\|\nabla F\|_{s-2}^2+C\|\partial^\beta f\|^2+C\mathscr{D}(t).$$

Summing for all $|\beta| \le s-1$, integrating the resulting equation over $[0,t]$ and noticing (4.16) and (4.20), we have

$$\begin{aligned}\int_0^t \|E(\tau)\|_{s-1}^2 d\tau &\le -2\sum_{|\beta|\le s-1}\left\langle \partial^\beta u_e(t), \partial^\beta E(t)\right\rangle + C\mathscr{E}(0) + C\mu\int_0^t \|\nabla F(\tau)\|_{s-2}^2 d\tau \\ &\le C\mathscr{E}(0) + C\mu\int_0^t \|\nabla F(\tau)\|_{s-2}^2 d\tau. \end{aligned} \tag{4.21}$$

For a multi-index $\gamma \in \mathbb{N}^3$ with $|\gamma| \le s-2$, applying ∂^γ to the equation for $\partial_t E$ in (2.14), we obtain

$$\nabla \times \partial^\gamma F = \partial_t \partial^\gamma E - \partial^\gamma(\rho_e u_e - \rho_i u_i).$$

Taking the inner product of the above with $\nabla \times \partial^\gamma F$ in $L^2(\mathbb{T}^3)$, and using the Maxwell equations for E and B in (2.14), the Cauchy-Schwarz inequality and the Moser-type inequalities, we have

$$\begin{aligned}\|\nabla\times\partial^\gamma F\|^2 &= \frac{d}{dt}\langle \partial^\gamma E, \nabla\times\partial^\gamma F\rangle - \langle \partial^\gamma E, \nabla\times\partial^\gamma F\rangle - \langle \partial^\gamma(\rho_e u_e - \rho_i u_i), \nabla\times\partial^\gamma F\rangle \\ &\le \frac{d}{dt}\langle \partial^\gamma E, \nabla\times\partial^\gamma F\rangle - \langle \nabla\times\partial^\gamma E, \partial_t\partial^\gamma F\rangle + \frac{1}{2}\|\nabla\times\partial^\gamma F\|^2 + C\mathscr{D}(t) \\ &\le \frac{d}{dt}\langle \partial^\gamma E, \nabla\times\partial^\gamma F\rangle + \|\nabla\times\partial^\gamma E\|^2 + \frac{1}{2}\|\nabla\times\partial^\gamma F\|^2 + C\mathscr{D}(t),\end{aligned}$$

which implies

$$\|\nabla\times\partial^\gamma F\|^2 \le 2\frac{d}{dt}\langle \partial^\gamma E, \nabla\times\partial^\gamma F\rangle + 2\|\nabla\times\partial^\gamma E\|^2 + C\mathscr{D}(t).$$

Summing for all $|\gamma| \le s-2$, integrating the resulting equation over $[0,t]$ and using (4.16), we have

$$\begin{aligned}\int_0^t \|\nabla F(\tau)\|_{s-2}^2 d\tau &\le 2\sum_{|\gamma|\le s-2}\langle \partial^\gamma E(t), \nabla\times\partial^\gamma F(t)\rangle + C\int_0^t \|E(\tau)\|_{s-1}^2 d\tau + C\mathscr{E}(0) \\ &\le C\int_0^t \|E(\tau)\|_{s-1}^2 d\tau + C\mathscr{E}(0).\end{aligned}$$

Substituting the above into (4.21), we have

$$\int_0^t \left(\|E(\tau)\|_{s-1}^2 + \|\nabla F(\tau)\|_{s-2}^2\right) d\tau \le C\mathscr{E}(0),$$

provided that μ is sufficiently small. This proves (4.18). □

Proof of Theorem 2.3. The uniform estimate (2.15) implies that

$$u_i^\varepsilon \longrightarrow 0, \text{ strongly in } C([0,T]; H^s), \quad \forall T > 0.$$

Besides, we obtain that the sequences $(\rho_\nu^\varepsilon - 1)_{\varepsilon>0}$, $(u_e^\varepsilon)_{\varepsilon>0}$, $(E^\varepsilon)_{\varepsilon>0}$ and $(B^\varepsilon - B_e)_{\varepsilon>0}$ are uniformly bounded in $L^\infty(\mathbb{R}^+; H^s)$. It follows that there exist functions $\bar{\rho_\nu}, \bar{u_e}, \bar{E}$ and $\bar{B}$, such that as $\varepsilon \to 0$, (2.17) holds. This allows us to pass the limit in the mass and momentum equations for ions in the sense of distributions. In particular, we have

$$\begin{aligned} \partial_t \rho_i &\rightharpoonup \partial_t \bar{\rho}_i, \\ \mathrm{div}(\rho_i^\varepsilon u_i^\varepsilon) &\rightharpoonup 0, \end{aligned}$$

which implies $\partial_t \bar{\rho}_i = 0$. Thus, $\bar{\rho}_i$ is a function that depends only on the space variable x.

Moreover, noticing $\varepsilon < 1$, $(\rho_e^\varepsilon)_{\varepsilon>0}$ and $(u_e^\varepsilon)_{\varepsilon>0}$ are uniformly bounded in $L^\infty(\mathbb{R}^+; H^{s-1})$, by a classical compactness theorem [26], $(\rho_e^\varepsilon)_{\varepsilon>0}$ and $(u_e^\varepsilon)_{\varepsilon>0}$ are relatively compact in $C([0,T]; H_{loc}^{s_1})$, for all $s_1 \in (0, s)$. As a consequence, as $\varepsilon \to 0$, up to subsequences,

$$(\rho_e^\varepsilon, u_e^\varepsilon) \to (\bar{\rho}_e, \bar{u}_e) \quad \text{strongly in} \quad C([0,T]; H_{loc}^{s_1}). \tag{4.22}$$

As a result, it is sufficient for us to pass the limit in the mass and momentum equations for electrons, as well as the Maxwell equations in (2.14) in the sense of distributions, of which the limiting system is the usual unipolar Euler-Maxwell system for electrons (2.18). Finally, since $(\partial_t E^\varepsilon)_{\varepsilon>0}$ and $(\partial_t B^\varepsilon)_{\varepsilon>0}$ are uniformly bounded in $L^\infty(\mathbb{R}^+; H^{s-1})$. Hence, we have

$$(E^\varepsilon, B^\varepsilon) \to (\bar{E}, \bar{B}) \quad \text{strongly in} \quad C([0,T]; H_{loc}^{s_1}).$$

Combining (2.15) and (4.22) implies (2.19). $\square$

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