Cross-diffusion effects on the double-diffusive convection in a rotating vertical porous cylinder with vertical throughflow

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June 2, 2023

Abstract

The impacts of vertical throughflow, rotation, cross-diffusion, and vertical heterogeneous permeability on the double-diffusive convection in a finite rotating vertical porous cylinder have been studied. The fluid in the cylinder is warmed and salted from beneath, and its top and lower walls are taken to be isothermal, isosolutal and permeable. In the model formulation, the Brinkman model was adopted, coupled with the Boussinesq approximation. The normal mode technique is used to perform linear stability analysis and single term Galerkin technique is employed to solve the eigenvalue problem. Further, the influence of vertical heterogeneity, vertical throughflow, thermal and solute Rayleigh, Taylor, and the Soret and Dufour numbers on the fluid system instability has been investigated. We found, among other results, that vertical heterogeneity may either stabilize or destabilize the fluid system. The Dufour number delays both the stationary and oscillatory convection onsets. The positive Soret number is found to have a stabilizing effect on the stationary convection case, with a destabilizing effect on the oscillatory convection case.

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Keywords: Throughflow, Double-diffusive convection, Porous media, Rotating Vertical cylinder, Soret and Dufour effects.

Nomenclature

Latin Symbols

a	wave number
h	height of the cylinder
r_0	radius of the cylinder
R	aspect ratio $(= \frac{r_0}{h})$
T	temperature
C	solute concentration
t	time
P	pressure
$\vec{V}$	fluid velocity
V_0	basic flow velocity
$\vec{g}$	gravitational acceleration
e_z	z-direction unit vector
K	permeability

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Ra_T	thermal Rayleigh number
Ra_S	salinity Rayleigh number
Le	Lewis number$(=\frac{k_T}{k_S})$
D_u	Dufour number $(=\frac{D_{TS}\Delta C}{k_T\Delta T})$
S_r	Soret number $(=\frac{D_{ST}\Delta T}{k_S\Delta C})$
Pr	Prandtl number
Q	Peclet number $(=\frac{HV_0}{k_t})$
Ta	Taylor number$(=\frac{4\Omega^2H^4}{\mu^2})$

Greek symbols

β_T	thermal expansion coefficient
β_S	concentration expansion coefficient
K_T	thermal diffusivity
K_S	solute diffusivity
ϕ	porosity
μ	fluid dynamic viscosity
$\overline{\mu}$	optimum viscosity for the Brinkman term
ν	kinematic viscosity $(=\frac{\mu}{\rho_0})$
Λ	effective Darcy number $(=\frac{\overline{\mu}k_T}{\mu H^2})$
σ	volumetric heat capacity

Subscripts

b	basic state
c	critical value

Superscripts

$'$	perturbed quantities
$*$	non-dimensional quantities

1. Introduction

The study of double-diffusive convection in saturated porous media in different settings has received significant coverage recently due to its vast range of engineering applications. These applications range from heating and cooling processes, grain storage, fibrous insulation, geothermal systems, petroleum reservoirs recovery, among others. In general, double diffusive convection is a type of fluid flow that occurs when two components with different diffusivities are subject to a vertical temperature gradient. This phenomenon is also known as thermohaline convection or salt-finger convection, depending on the nature of the two components involved. Double diffusive convection can also occur in porous media, such as in geological formations or in industrial processes where porous media are used for heat and mass transfer. In porous media, fluid flow is affected by the pore structure, which can create additional transport mechanisms, such as advection and dispersion, that interact with thermal and diffusive buoyancy. In the case of double diffusive convection in porous media, two components with different diffusivities, such as temperature and salt concentration, can interact to create convective flow patterns. These patterns can result in the formation of salt fingers or convective rolls, depending on the properties of the system. Double diffusive convection in porous media has important implications for various fields, including geology, hydrology, and engineering. For example, it can affect the transport of heat and mass in underground aquifers, and can play a role in the formation of mineral deposits. In industrial applications, double diffusive convection in porous media can be used to optimize heat and mass transfer in various processes, such as in fuel cells and heat exchangers. The onset of convection in a horizontal porous layer with heat supplied from below studied by [1, 2] where they employed linear stability theory approach. A comprehensive review of double-diffusive convection in porous media is given in [3–6].

The rotation effect on double-diffusive convection in a sparsely filled porous layer was investigated by Rudraiah et al. [7]. They found that rotation prolongs the convection onset. The nonlinear double-diffusive convection in a rotating porous medium was studied by Lombardo and Mulone [8] with the aid of the Lyapunov direct method. Malashetty et al. [9] studied the problem in a rotating horizontal saturated porous layer with fluid and solid phases that were not in local thermodynamic equilibrium. They found among other results that rotation is to improves the stability of the fluid system. Wang and Tan [10] studied the onset of Darcy–Brinkman thermosolutal convection using the linear stability approach. They showed that the Darcy number destabilizes the fluid system while increasing the normalized porosity has the opposite effect.

Griffiths, and Murray and Chen [11, 12] studied double-diffusive convection in s porous medium experimentally. It has been shown that when heat and mass movement occur concurrently in a fluid flow, the fluxes and generating potentials relationship gets complex. These studies proved that energy flow may be caused by gradients in composition as well as temperature. The flux of energy due to the compositional gradient is referred to as the diffusion-thermo or Dufour effect where as the flux of mass due to the thermal gradients is known as the thermal-diffusion or Soret effect. Cross-diffusion in porous media which is saturated is complicated by the coupling of the fluid with the porous composite, and accurate estimates of cross-diffusion parameters are difficult to obtain. Hence it is difficult to perform an experimental study on the influence of cross-diffusion. Studies on cross diffusion effects on the diffusive convection in a porous medium are found in [4, 13–16]. Bouachir et al. [17] investigated the Soret and Dufour effects on convection in a vertically oriented porous enclosure. They found, among other results, that the Soret number has both stabilizing and destabilizing effects depending on the numerical value, but the Dufour number always has a destabilizing influence on the fluid system.

In some situations, such as geothermal flows, the influence of lateral boundaries on the convective process must be considered. Bau and Torrance [18] reported a practical and an analytical analysis of low Rayleigh number case for convection in a vertically oriented circular cylinder filled with saturated porous materials. Other notable analytical predictions of the critical Rayleigh number for a porous layer enclosed within a vertical concentric cylinder are found in [18–25]. These studies confirmed the stabilizing and convective shape controlling effects of the lateral walls. Wooding [19] studied asymmetric saline convection flows in an endless vertical tube. Zebib [20] observed that asymmetric flows are frequently preferable in a cylinder with impermeable upper and lower boundaries. Further, Haugen and Tyvand [21] noted that, for conducting cylindrical calls, the axisymmetric mode is favourable for all aspects ratios.

The heterogeneity permeability and thermal conductivity effects on marginal stability of porous media fluid was first considered by [4, 26]. Further, other characteristics of conductivity heterogeneity in general have been investigated by Braester and Vadasz [27]. Nield [28] investigated heterogeneity influence on the onset of convection in a porous media. Futhere, the effects of strong heterogeneity has been investigated by Nield and Simmons [29], Nield et al. [30]. They found that when the attributes fluctuate in a nonlinear or linear way, the influence of substantial heterogeneity on the critical Rayleigh number is of second order. Nield and Kuznetsov [31] studied vertical throughflow effects in a heterogeneous vertically oriented cylinder. They concluded that, under the weak throughflow and weak heterogeneity assumptions, throughflow has a stabilizing impact regardless of its direction. However, with regard to the orientation of quadratic variation, the influence of heterogeneity can either stabilize or destabilize the fluid system. There are numerous studies on the vertical throughflow effects on diffusive convection onset in a soaked porous medium including [4, 32–35]. Shivakumara et al. [36] investigated the throughflow effects on double-diffusive convection in a porous layer. They concluded that, when the lower and upper boundaries are of similar, the effect of throughflow is to destabilize the fluid system, and may either stabilize or destabilize when the boundaries are unique. To the best of our knowledge, heterogeneity and throughflow interaction in a rotating vertical cylinder with cross-diffusion effect has not been studied. Therefore, in this study, among others, we investigate vertical throughflow, Soret, Dufour and heterogeneity effects. We assume vertical heterogeneity with second-order as discussed by Nield and Kuznetsov [31].

2. Mathematical formulation

A single-phase flow in a vertically oriented cylinder of radius r_0 and height h filled with a saturated porous medium and constantly rotating with angular velocity Ω about the vertical axis is considered. The aspect ratio r_0/h is represented by R. We assume a vertical permeability $K(z^*)$ within the cylinder. We assume that the basic flow is uniform and has velocity V_0 in the z-direction, see, Figure 1.

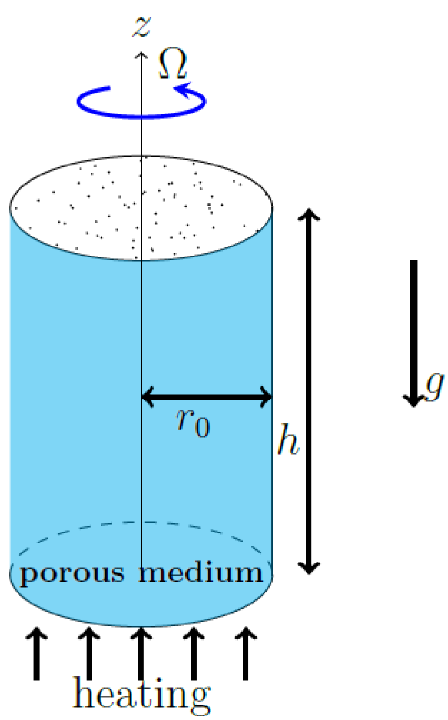


Figure 1: Geometry of the problem.

We consider constant temperatures T_0, T_1 and uniform concentrations C_0, C_1 at the top and lower boundaries ($T_0 < T_1$ and $C_0 < C_1$), respectively. The temperature, solute concentration and velocity are represented by T^*, C^*, V^* respectively. We assumes Boussinesq approximation, the walls of the cylinder are impermeable, isothermal and isosolutal, and the extended Brinkman Darcy model [4, 10]. Introducing the Soret and Dufour effects terms, the governing equations are given as

$$\nabla^* \cdot V^* = 0, \tag{1}$$

$$\overline{\mu}\nabla^{*2}V^* - \frac{\mu}{K(z^*)}V^* + 2\Omega_z \times V^* = \nabla^* P^* - \rho g e_z, \tag{2}$$

$$\sigma\frac{\partial T^*}{\partial t^*} + V^* \cdot \nabla^* T^* = k_T \nabla^{*2} T^* + D_{TS}\nabla^{*2}C^*, \tag{3}$$

$$\phi^*\frac{\partial C^*}{\partial t^*} + V^* \cdot \nabla^* C^* = k_S \nabla^{*2} C^* + D_{ST}\nabla^{*2}T^*, \tag{4}$$

$$\rho = \rho_0[1 - \beta_T(T^* - T_0) - \beta_S(C^* - C_0)]. \tag{5}$$

Where P^* denotes excess pressure considered to be above the hydrostatic reference value, ρ_0 is the reference density of the fluid at reference temperature T_0, σ is the saturated porous medium to fluid heat capacity ratio ($\sigma = (\rho c)_m/(\rho c)_f$), k_T is the thermal diffusivity ($k_T = k_m/(\rho c)_f$) where k_m is the porous medium effective thermal conductivity, k_S is the solutal diffusivity, and ϕ^* is the porous medium porosity.
Introducing the following non-dimensional quantities

$$\mathbf{x} = \frac{\mathbf{x}^*}{h},\ V = \frac{h}{k_T}V^*,\ t = \frac{k_T}{\sigma h^2}t^*,\ T = \frac{T^* - T_0}{\Delta T},\ C = \frac{1}{Le}\frac{C^* - C_0}{\Delta C},\ P = \frac{K_H}{\rho_0 P_r k_T^2}[P^* - \rho_0 g z^*], \tag{6}$$

where $Le = \dfrac{k_T}{k_S}$ is the Lewis number, $\nu = \dfrac{\mu}{\rho_0}$ denotes kinematic viscosity, $P_r = \dfrac{\nu}{k_T}$ is the Prandtl number, and K_H is the $K(z^*)$ mean harmonic value. Using the nondimensional quantities Eq. (6) in equations (1)–(5) yields

$$\nabla \cdot V = 0, \tag{7}$$

$$\nabla P = \Lambda \nabla^2 V - \frac{1}{K(z)}V + \sqrt{Ta}e_z \times V + [Ra_T T + Ra_S C]e_z, \tag{8}$$

$$\frac{\partial T}{\partial t} + V \cdot \nabla T = \nabla^2 T + D_u \nabla^2 C, \tag{9}$$

$$\phi\frac{\partial C}{\partial t} + V \cdot \nabla C = Le^{-1}[\nabla^2 C + S_r \nabla^2 T], \tag{10}$$

where $Da = \dfrac{K_H}{h^2}$ is the Darcy number, $\Lambda = \dfrac{\overline{\mu}Da}{\mu}$ is the effective Darcy number, $Ta = \dfrac{4\Omega^2 K_H^4}{\mu^2}$ is the Taylor number, $Ra_T = \dfrac{\beta_T g h K_H \Delta T}{\nu k_T}$ is the thermal Rayleigh number, $Ra_S = \dfrac{\beta_S g h K_H \Delta C}{\nu k_S}$ is the solutal Rayleigh number, $D_u = \dfrac{D_{TS}\Delta C}{k_T \Delta T}$ is the Dufour number, $S_r = \dfrac{D_{ST}\Delta T}{k_S \Delta C}$ is the Soret number, $\phi = \dfrac{\phi^*}{\sigma}$ is the normalized porosity. Following [31] we express the permeability as $K(z) = \dfrac{K(z^*)}{K_H}$.
Equations (7)–(10) have basic state solution of the form

$$V_b = (0, 0, Q),\ T_b(z),\ C_b(z),\ p_b(z),$$

with $Q = \dfrac{V_0 H}{k_t}$ being the throughflow depend on Péclet number, where

$$
\begin{aligned}
&V_b = Q e_z, && (11)\\
&\frac{dp}{dz} = \frac{-1}{K(z)} Q + Ra_T T_b + Ra_S C_b, && (12)\\
&Q \frac{dT_b}{dz} = \frac{d^2 T_b}{dz^2} + D_u \frac{d^2 C_b}{dz^2}, && (13)\\
&LeQ \frac{dC_b}{dz} = \frac{d^2 C_b}{dz^2} + S_r \frac{d^2 T_b}{dz^2}. && (14)
\end{aligned}
$$

Equations (13)–(14) are solved subject to boundary conditions to give

$$
T_b = 1, \quad C_b = \frac{1}{Le}, \quad at \quad z = 0, \quad T_b = 0, \quad C_b = 0, \quad at \quad z = 1.
$$

The gives the basic temperature solution

$$
T_b = b_1 + b_2 e^{\lambda_2 z} + b_3 e^{\lambda_4 z}, \tag{15}
$$

and the basic solute solution

$$
C_b = \frac{1}{Le}(b_4 + b_5 e^{\lambda_2 z} + b_6 e^{\lambda_4 z}), \tag{16}
$$

where

$$
\begin{aligned}
\lambda_1 &= \frac{Q(1 + Le + \sqrt{(Le-1)^2 + 4D_u S_r Le})}{2(D_u S_r - 1)},\\
\lambda_2 &= \frac{Q(-1 - Le + \sqrt{(Le-1)^2 + 4D_u S_r Le})}{2(D_u S_r - 1)},\\
\lambda_3 &= \frac{Q\sqrt{(Le-1)^2 + 4D_u S_r Le}}{D_u S_r - 1},\\
\lambda_4 &= -\lambda_1 = \frac{Q(1 + Le + \sqrt{(Le-1)^2 + 4D_u S_r Le})}{2(1 - D_u S_r)},\\
b_1 &= \frac{(1 - 2e^{\lambda_2} + e^{\lambda_3})\sqrt{(Le-1)^2 + 4D_u S_r Le} + (2D_u + Le - 1)(e^{\lambda_3} - 1)}{2(e^{\lambda_1} - 1)(e^{\lambda_2} - 1)\sqrt{(le-1)^2 + 4D_u S_r le}},
\end{aligned}
$$

$$
\begin{aligned}
b_2 &= \frac{(2D_u + Le - 1) + \sqrt{(Le-1)^2 + 4D_uS_rLe}}{2(1-e^{\lambda_2})\sqrt{(Le-1)^2 + 4D_uS_rLe}},\\
b_3 &= \frac{(e^{\lambda_1} - e^{\lambda_3})(2D_u + Le - 1 - \sqrt{(Le-1)^2 + 4D_uS_rLe})}{2(e^{\lambda_1}-1)(e^{\lambda_2}-1)\sqrt{(Le-1)^2 + 4D_uS_rLe}},\\
b_4 &= \frac{\sqrt{(Le-1)^2 + 4D_uS_rLe}(1 + e^{\lambda_3} - 2e^{\lambda_2}) + (2S_rLe - Le + 1)(e^{\lambda_3} - 1)}{2(e^{\lambda_1}-1)(e^{\lambda_2}-1)\sqrt{(Le-1)^2 + 4D_uS_rLe}},\\
b_5 &= \frac{\sqrt{(Le-1)^2 + 4D_uS_rLe} + (2S_rLe - Le + 1)}{2(1-e^{\lambda_2})\sqrt{(Le-1)^2 + 4D_uS_rLe}},\\
b_6 &= \frac{(e^{\lambda_1} - e^{\lambda_3})(2S_rLe - Le + 1 - \sqrt{(Le-1)^2 + 4D_uS_rLe})}{2(e^{\lambda_1}-1)(e^{\lambda_2}-1)\sqrt{(Le-1)^2 + 4D_uS_rLe}},
\end{aligned}
$$

are nonzero constants. To avoid singular solutions, we assume that $D_uS_r \neq 1$.

3. Linear stability analysis

To investigate the linear stability, we impose perturbations on the basic state solution, the fluid quantities are then written as

$$V = V_b + V', P = P_b + P', T = T_b + T', C = C_b + C'. \tag{17}$$

Using equation (17) into equations (7)–(10) and linearizing we have

$$
\begin{aligned}
&\nabla \cdot V' = 0, &(18)\\
&\nabla P' = \Lambda \nabla^2 V' - \frac{1}{K(z)} V' + \sqrt{Ta} e_z \times V' + [Ra_T T' + Ra_S C'] e_z, &(19)\\
&\frac{\partial T'}{\partial t} + w' \frac{dT_b}{dz} + Q \frac{\partial T'}{\partial z} = \nabla^2 T' + D_u \nabla^2 C', &(20)\\
&Le\phi \frac{\partial C'}{\partial t} + Lew' \frac{dC_b}{dz} + LeQ \frac{\partial C'}{\partial z} = \nabla^2 C' + S_r \nabla^2 T'. &(21)
\end{aligned}
$$

Taking the curl of equation (19) twice in e_z direction, and following Nield and Kuznetsov [31] assuming weak heterogeneity, we obtain

$$
\begin{aligned}
&\Lambda \nabla^2 \xi = \frac{1}{K(z)} \xi + \sqrt{Ta} \frac{\partial w}{\partial z}, &(22)\\
&\Lambda \nabla^4 w' - \frac{1}{K(z)} \nabla^2 w' = -\sqrt{Ta} \frac{\partial \xi}{\partial z} - Ra_T \nabla_H^2 T' - Ra_S \nabla_H^2 C', &(23)
\end{aligned}
$$

where ∇_H^2 is the horizontal laplacian operator and ξ denotes the vorticity vector. Substituting Eqs. (15) and (16) into Eqs. (20) and (21) we obtain

$$
\begin{aligned}
&\frac{\partial T'}{\partial t} + Mw' + Q \frac{\partial T'}{\partial z} = \nabla^2 T' + D_u \nabla^2 C', &(24)\\
&Le\phi \frac{\partial C'}{\partial t} + Nw' + LeQ \frac{\partial C'}{\partial z} = \nabla^2 C' + S_r \nabla^2 T', &(25)
\end{aligned}
$$

where

$$M = \frac{dT_b}{dz} = b_2\lambda_2 e^{\lambda_2 z} + b_3\lambda_4 e^{\lambda_4 z},$$

and

$$N = Le\frac{dC_b}{dz} = b_5\lambda_2 e^{\lambda_2 z} + b_6\lambda_4 e^{\lambda_4 z}.$$

Assuming that the impermeable upper, lower and lateral boundaries to be isothermal and isosolutal, the boundary conditions are then given as

$$w' = \frac{\partial \xi}{\partial z} = T' = C' = 0 \quad at \quad z = 0 \quad \text{and} \quad z = 1, \tag{26}$$

$$u' = \xi = T' = C' = 0 \quad \text{at} \quad r = R. \tag{27}$$

The normal mode technique is employed to solve equations (22)–(25) subject to homogeneous boundary conditions (26) and (27), to obtain

$$[w', T', C'] = [W, \Theta, \Gamma] J_n(ar)\sin(m\pi z)\cos(n\phi)\exp(st), \tag{28}$$

$$\xi = Z J_n(ar)\cos(m\pi z)\cos(n\phi)\exp(st), \tag{29}$$

where W, Θ, Γ, Z are constants, s is the growth rate, m and n are integers, we consider only the minimum mode in the vertical direction $m = 1$, which reflects the highest unstable mode, J_n is a first kind Bessel function of order n, $a = j_n/R$, and j_n is the smallest zero of $J_n(ar)$. Substituting equations (28) and (29) into equations (22)–(25), we get

$$\left\{\left(\Lambda\alpha^2 + \frac{1}{K(z)}\right)Z + \pi\sqrt{Ta}W\right\}\cos(\pi z) = 0, \tag{30}$$

$$\left\{\left(\Lambda\alpha^4 + \frac{1}{K(z)}\alpha^2\right)W - a^2(Ra_T\Theta + Ra_s\Gamma) - \pi\sqrt{Ta}Z\right\}\sin(\pi z) = 0, \tag{31}$$

$$\left\{\left(\alpha^2 + s\right)\Theta + D_u\alpha^2\Gamma + MW\right\}\sin(\pi z) + m\pi Q\Theta\cos(\pi z) = 0, \tag{32}$$

$$\left\{\left(\alpha^2 + Le\phi s\right)\Gamma + S_r\alpha^2\Theta + NW\right\}\sin(m\pi z) + \pi LeQ\Gamma\cos(\pi z) = 0, \tag{33}$$

where $\alpha^2 = \pi^2 + a^2$. Equations (30)–(33) with respect to the boundary conditions (27) denotes an eigenvalue problem with Ra_T considered as the eigenvalue. The single-term Galerkin technique is used to solve the closed form eigenvalue problem. After applying the orthogonality of trial functions we obtain

$$\left(\Lambda\alpha^2 + L_1\right)Z + \pi\sqrt{Ta}W = 0, \tag{34}$$

$$\left(\Lambda\alpha^4 + \alpha^2 L_2\right)W - a^2 Ra_T\Theta - a^2 Ra_s\Gamma - \pi\sqrt{Ta}Z = 0, \tag{35}$$

$$\left(\alpha^2 + s\right)\Theta + \alpha^2 D_u\Gamma + U_1 W = 0, \tag{36}$$

$$\left(\alpha^2 + Le\phi s\right)\Gamma + S_r\alpha^2\Theta + U_2 W = 0, \tag{37}$$

where

$$L_1 = 2\int_0^1 \frac{1}{K(z)}\cos^2 \pi z \quad dz, \tag{38}$$

$$L_2 = 2\int_0^1 \frac{1}{K(z)}\sin^2 \pi z \quad dz, \tag{39}$$

$$U_1 = 2\int_0^1 M \sin^2 \pi z \quad dz = 4\pi^2 \left[b_2 \frac{e^{\lambda_2} - 1}{4\pi^2 + {\lambda_2}^2} + b_3 \frac{e^{\lambda_4} - 1}{4\pi^2 + \lambda_4^2} \right], \tag{40}$$

$$U_2 = 2\int_0^1 N \sin^2 \pi z \quad dz = 4m^2\pi^2 \left[b_5 \frac{e^{\lambda_2} - 1}{4m^2\pi^2 + \lambda_2^2} + b_6 \frac{e^{\lambda_4} - 1}{4\pi^2 + \lambda_4^2} \right]. \tag{41}$$

The system of equations (34)–(37) only admits a non-trivial solution if the determinant

$$\begin{vmatrix} \pi\sqrt{Ta} & \Lambda\alpha^2 + L_1 & 0 & 0 \\ \alpha^2(\Lambda\alpha^2 + L_2) & -\pi\sqrt{Ta} & -a^2 Ra_T & -a^2 Ra_s \\ U_1 & 0 & (\alpha^2 + s) & D_u\alpha^2 \\ U_2 & 0 & S_r\alpha^2 & (\alpha^2 + Le\phi s) \end{vmatrix} = 0.$$

Expanding the determinant and solving for Ra_T, we obtain

$$Ra_T = \frac{\left\{\alpha^2(\Lambda\alpha^2 + L_1)(\Lambda\alpha^2 + L_2) + \pi^2 Ta\right\}\left\{D_u S_r \alpha^4 - (\alpha^2 + Le\phi s)(\alpha^2 + s)\right\}}{a^2\left\{\Lambda\alpha^2 + L_1\right\}\left\{(U_1 - D_u U_2)\alpha^2 + Les\phi U_1\right\}} - Ra_s\left\{\frac{(U_2 - S_r U_1)\alpha^2 + sU_2}{(U_1 - D_u U_2)\alpha^2 + Les\phi U_1}\right\}, \tag{42}$$

where s is a complex quantity. For marginal curves to exist, the real part of s should be equal to zero, thus, $s = i\omega$, where ω represents frequency in real dimensions, then equation (42) gives

$$Ra_T = \Delta_1 + i\omega\Delta_2, \tag{43}$$

where:

$$\Delta_1 = \frac{\alpha^2\left\{(\Lambda\alpha^2 + L_1)(\Lambda\alpha^2 + L_2)\alpha^2 + \pi^2 Ta\right\}\left\{(U_1 - D_u U_2)(D_u S_r - 1)\alpha^4 - \omega^2 Le\phi(Le\phi U_1 + D_u U_2)\right\}}{a^2(\Lambda\alpha^2 + L_1)\left\{(U_1 - D_u U_2)^2\alpha^4 + (\omega Le\phi U_1)^2\right\}} - Ra_s\left\{\frac{(U_1 - D_u U_2)(U_2 - S_r U_1)\alpha^4 + Le\phi U_1 U_2 \omega^2}{(U_1 - D_u U_2)^2\alpha^4 + (\omega Le\phi U_1)^2}\right\}, \tag{44}$$

$$\Delta_2 = \frac{\left\{\pi^2 Ta + (\Lambda\alpha^2 + L_1)(\Lambda\alpha^2 + L_2)\alpha^2\right\}\left\{Le\phi U_1(\alpha^4(1 - D_u S_r) - \omega^2 Le\phi) - \alpha^4(U_1 - D_u U_2)(1 + Le\phi)\right\}}{a^2(\Lambda\alpha^2 + L_1)\left\{(U_1 - D_u U_2)^2\alpha^4 + (\omega Le\phi U_1)^2\right\}}$$
$$+ \alpha^2 Ra_s\left\{\frac{Le\phi U_1(U_2 - S_r U_1) - U_2(U_1 - D_u U_2)}{(U_1 - D_u U_2)^2\alpha^4 + (\omega Le\phi U_1)^2}\right\}. \tag{45}$$

Here Ra_T must be a real physical quantity, therefore, from Eq. (43), it follows that either $\omega = 0$ or $\Delta_2 = 0$.

3.1. Stationary convection

For the stationary Rayleigh number, we substitute $\omega = 0$ into equation (43), and note that direct bifurcation occurs when $Ra_T = Ra_T^{st}$. Then, the stationary Rayleigh number Ra_T^{st} is given as

$$Ra_T^{st} = \frac{\alpha^2\left\{\pi^2 Ta + \alpha^2(\Lambda\alpha^2 + L_1)(\Lambda\alpha^2 + L_2)\right\}\left\{D_u S_r - 1\right\}}{a^2(\Lambda\alpha^2 + L_1)(U_1 - D_u U_2)} - Ra_s\left\{\frac{U_2 - S_r U_1}{U_1 - D_u U_2}\right\}. \tag{46}$$

It is noticeable that the expression of the stationary Rayleigh number that in Eq. (46) is independent of the normalized porosity. Thus the normalized porosity does not have an effect on the stationary convection.

The critical stationary Rayleigh number given by

$$Ra_{T\,c}^{st} = \frac{\alpha_c^2\left\{\pi^2 Ta + \alpha_c^2(\Lambda\alpha_c^2 + L_1)(\Lambda\alpha_c^2 + L_2)\right\}\left\{D_u S_r - 1\right\}}{a_c^2(\Lambda\alpha_c^2 + L_1)(U_1 - D_u U_2)} - Ra_s\left\{\frac{U_2 - S_r U_1}{U_1 - D_u U_2}\right\}, \tag{47}$$

where a_c is the minimal a in the set j_n/R. In Table 1 we find the values of a for the three minimum modes in the azimuthal direction ϕ. The axisymmetry mode $n = 0$ is always prefect [21]. The critical wave number is $a_c = \dfrac{2.405}{R}$. We define the stream function $\psi(r, z)$ as

$$\frac{\partial\Psi}{\partial r} = rw,$$

and by integration we get

$$\psi = \frac{r}{a_c} J_1(a_c r)\sin\pi z. \tag{48}$$

The axisymmetry mode radial velocity is given by:

$$u = -\frac{1}{r}\frac{\partial\Psi}{\partial z} = \frac{-\pi r}{a_c} J_1(a_c r)\cos\pi z. \tag{49}$$

We note from Eq. (46) in the case of the homogeneous case ($L_1 = L_2 = 1$) without throughflow

Table 1: The wave number a at the double-diffusive convection onset in a rotating heterogeneous porous cylinder for the azimuthal modes $n = 0$ (axisymmetry), $n = 1$ and $n = 2$, for different aspect ratio R.

R	$a = j_n/R$		
	n = 0	n = 1	n = 2
0.01	240.48	552.01	865.37
0.02	120.24	276.01	432.69
0.05	48.10	110.40	173.07
0.1	24.05	55.20	86.54
0.2	12.02	27.60	43.27
0.5	4.81	11.04	17.31
1	2.40	5.52	8.65
2	1.20	2.76	4.33
5	0.48	1.1	1.73
10	0.24	0.55	0.87
20	0.12	0.28	0.43

($Q \to 0$), without the Brinkman term and Taylor number, and $D_u = S_r = 0$, then $U_1 = U_2 = -1$, and the stationary Rayleigh number gives to

$$Ra_T{}^{st} + Ra_s = \frac{(a^2 + \pi^2)^2}{a^2},$$

with critical value $Ra_{T_c}^{st} = 4\pi^2$, for $a_c = \pi$. This is in agreement with the known result of Niled and Kuznetsov [31].
Now for the homogeneous case with throughflow without the Brinkman term and rotation effects, Eq. (47) reduces to

$$\frac{(U_1 - D_u U_2)}{D_u S_r - 1} Ra_T + \frac{(U_2 - S_r U_1)}{D_u S_r - 1} Ra_s = \frac{(\pi^2 + a_c^2)^2}{a_c^2}.$$

When $D_u = S_r = 0$, with weak throughflow these results reduce to those of Nield and Kuznetsov [31]. Following [31], we allow for the variation of the vertical permeability $K(z)$ in the form:

$$\frac{1}{K(z)} = \frac{1 + \gamma z + \frac{\beta}{2} z^2}{1 + \frac{\gamma}{2} + \frac{\beta}{6}}, \tag{50}$$

where γ, and β are variables denoting the degree of heterogeneity, and have magnitudes less than unity. Substituting equation (50) in equations (38)–(39) we obtain

$$L_1 = 1 + \frac{3\beta}{2\pi^2(6 + 3\gamma + \beta)} \approx 1 + \frac{\beta}{4\pi^2}, \tag{51}$$

$$L_2 = 1 - \frac{3\beta}{2\pi^2(6 + 3\gamma + \beta)} \approx 1 - \frac{\beta}{4\pi^2}. \tag{52}$$

For the non-homogeneous case, substituting equations (51) and (52) in equation (47), we obtain

$$Ra^{st}_{T\,c} = \frac{\alpha_c^2\Big\{\pi^2 Ta + \alpha_c^2(\Lambda\alpha_c^2 + 1 + \dfrac{\beta}{4\pi^2})(\Lambda\alpha_c^2 + 1 - \dfrac{\beta}{4\pi^2})\Big\}\Big\{D_u S_r - 1\Big\}}{a_c^2(\Lambda\alpha_c^2 + 1 - \dfrac{\beta}{4\pi^2})(U_1 - D_u U_2)} - Ra_s\Big\{\frac{U_2 - S_r U_1}{U_1 - D_u U_2}\Big\}. \tag{53}$$

When Soret and Dufour effects are non-existent, Eq. (53) reduce to

$$\frac{4\pi^2 a_c^2}{\alpha_c^2(4\pi^2 + Q^2)} Ra^{st}_{T\,c} + \frac{4\pi^2 a_c^2}{\alpha_c^2(4\pi^2 + Le^2 Q^2)} Ra_s = \alpha_c^2\left(\Lambda\alpha_c^2 + 1 - \frac{\beta}{4\pi^2}\right) + \frac{\pi^2 Ta}{\left(\Lambda\alpha_c^2 + 1 + \dfrac{\beta}{4\pi^2}\right)}. \tag{54}$$

Thus, if β is positive there is a reduction in the critical Rayleigh number and in this case the heterogeneity has a destabilizing effect at the second-order Taylor expansion for small value of β. For $Ta = \Lambda = 0$ and weak throughflow, equation (54) becomes

$$\left(1 - \frac{Q^2}{4\pi^2}\right) Ra_c{}^{st} + \left(1 - \frac{Le^2 Q^2}{4\pi^2}\right) Ra_s = \frac{(\pi^2 + a_c^2)^2}{a_c^2}\left(1 - \frac{\beta}{4\pi^2}\right),$$

which agrees with the result of Nield and Kuznetsov [31].

3.2. Oscillatory convection

With rotation, solutal gradient and temperature gradient present, oscillatory motions are possible ($\omega \neq 0$). In this case $\Delta_2 = 0$, so the equation (45) becomes

$$\begin{aligned}\omega^2 = &\frac{\Big\{\Lambda\alpha^2 + L_1\Big\}\Big\{Le\phi U_1(U_2 - S_r U_1) - (U_1 - D_u U_2)U_2\Big\}a^2\alpha^2 Ra_s}{Le^2\phi^2 U_1\Big\{\alpha^2(\Lambda\alpha^2 + L_1)(\Lambda\alpha^2 + L_2) + \pi^2 Ta\Big\}} \\ &+ \frac{\alpha^4\Big\{Le\phi U_1(1 - D_u S_r) + (1 + Le\phi)(D_u U_2 - U_1)\Big\}}{Le^2\phi^2 U_1}.\end{aligned} \tag{55}$$

Eq. (55) gives the oscillatory mode frequency. If positive ω^2 does not exist, then an oscillatory instability cannot be observed. If positive values of ω^2 exist, then the oscillatory Rayleigh number is found by inputting the positive values of ω^2 in Eq. (43). Now the thermal oscillatory Rayleigh number Ra_T^{osc} is given by

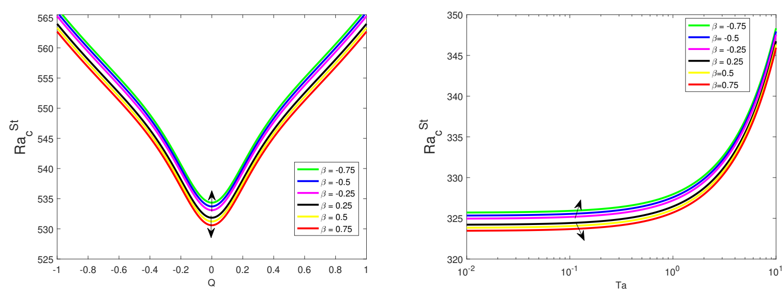
$$\begin{aligned}Ra_T^{osc} \\ = &\frac{\alpha^2\Big\{(\Lambda\alpha^2 + L_1)(\Lambda\alpha^2 + L_2)\alpha^2 + \pi^2 Ta\Big\}\Big\{(U_1 - D_u U_2)(D_u S_r - 1)\alpha^4 - \omega^2 Le\phi(Le\phi U_1 + D_u U_2)\Big\}}{a^2(\Lambda\alpha^2 + L_1)\Big\{(U_1 - D_u U_2)^2\alpha^4 + (\omega Le\phi U_1)^2\Big\}} \\ &- Ra_s\Big\{\frac{(U_1 - D_u U_2)(U_2 - S_r U_1)\alpha^4 + Le\phi U_1 U_2\omega^2}{(U_1 - D_u U_2)^2\alpha^4 + (\omega Le\phi U_1)^2}\Big\},\end{aligned} \tag{56}$$

It is evident that the oscillatory convection is depend on the parameters $S_r, D_u, Le, Ta, Ra_S, Le, \phi$ and Λ. The oscillatory critical Rayleigh number ${Ra_T^{osc}}_c$ is calculated from Eq. (56) for different parameter values.

4. Results and discussion

The linear stability of double-diffusive convection in a rotating vertical cylinder packed with a heterogeneous porous media was studied. This investigation aimed to investigate the influence of the heterogeneity, rotation, throughflow, Soret, and Dufour numbers on the onset of instability in a fluid layer. The Galerkin approximation method was used to solve the resulting eigenvalue problem. The stationary Rayleigh number is given by Eq. (46), and the oscillatory thermal Rayleigh number is given by Eq. (56). The parameter values are taken from the literature [4, 17, 37]. The influence of the parameters on the onset of convection is presented in Figures 2–6. Figure 2 shows the effect of various degrees of heterogeneity on the stationary Rayleigh number in Eq. (54) with changes in Péclet and Taylor numbers.

2a.eps 2b.eps



(a) (b)

Figure 2: Changes in critical thermal Rayleigh number ${Ra_c}^{St}$ with (a) Péclet–Darcy number Q, (b) Taylor number Ta. For various degrees of heterogeneity parameter β. When $S_r = 0,\ D_u = 0,\ Ta = 100,\ \Lambda = 0.1,\ Q = 0.5,\ Ra_s = 20, Le = 20, R = 0.4$.

The effect of heterogeneity is moderately destabilizing when β is positive and slightly stabilizing for negative β. Therefore, the effect of heterogeneity is to stabilize or destabilize the fluid system, depending on the direction of the quadratic variation. Figure 2(a) depicts that the Péclet number has a stabilization effect, as the critical stationary Rayleigh number increases with Péclet–Darcy number for both the upward and downward throughflow. Figure 2(b) shows that ${Ra_c}^{St}$ increases as Ta increases, indicating that rotation has a stabilizing effect on the system.

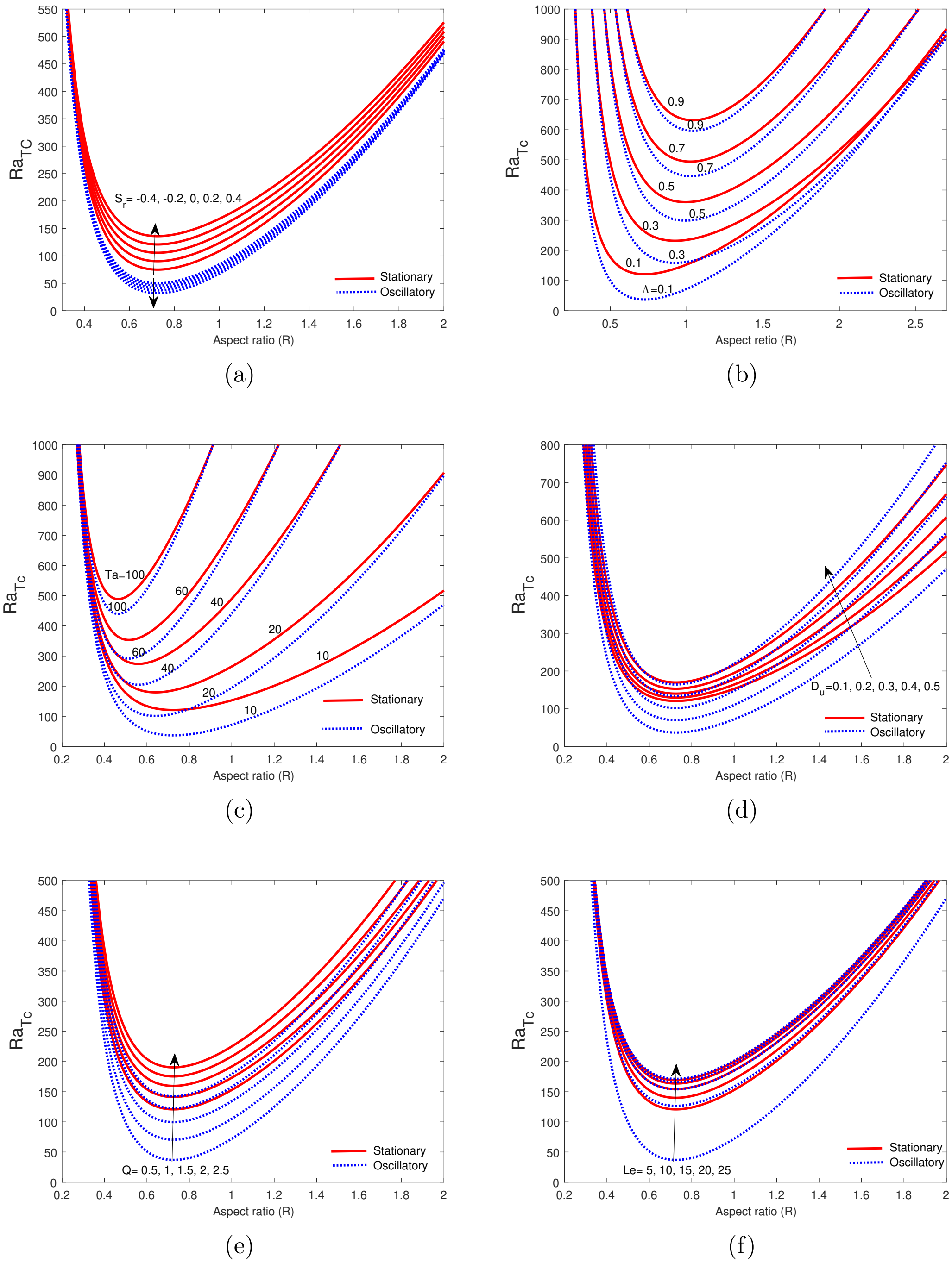


Figure 3: Neutral stability curves with aspect ratio R for various values of (a) Soret number S_r, (b)effective Darcy number Λ, (c) Taylor number Ta, (d) Dufour number D_u, (e) Péclet number Q, and (f) Lewis number Le. When $\beta = 0.25, D_u = 0.1, S_r = 0.2, Ta = 10, Le = 2, Q = 1, \Lambda = 0.1, Ra_s = 10, \phi = 0.1$.

Figure 3 depicts stability neutral curves for critical stationary and oscillatory Rayleigh numbers against the aspect ratio R of the vertical slender cylinder for various parameter values with other fixed parameters values as $\beta = 0.25, D_u = 0.1, S_r = 0.2, Ta = 10, Le = 5, Q = 0.5, \Lambda = 0.1, Ra_s =$

$100, \phi = 0.1$.
Figure 3(a) depicts the effect of the Soret number. It shows that the minimum stationary Rayleigh number increases by increasing the positive Soret number and decreases by increasing the negative magnitude of the Soret number, showing that a positive Soret number stabilizes the stationary convection and a negative Soret number destabilizes it. The critical oscillatory Rayleigh number decreases by increasing the positive value of S_r, and increases by increasing negative values of S_r, showing that the positive Soret number destabilizes the oscillatory convection and negative Soret number stabilizes the oscillatory convection. Figures 3(b–f) depict the effect of Darcy, Taylor, Dufour, Péclet, and Lewis numbers respectively. As the values of these variables are increased, the critical stationary and oscillatory Rayleigh number increases, showing that the factors postpone the double-diffusive convection onset in the fluid system.

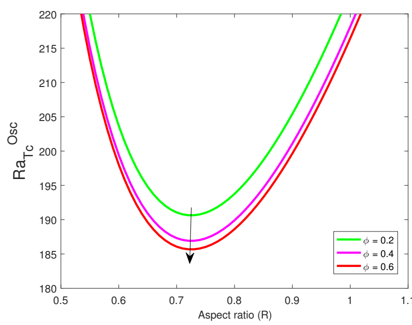


Figure 4: Normalized porosity ϕ effect on the oscillatory critical Rayleigh number ${Ra_T}_c^{osc}$ when $\beta = 0.25, Ta = 10, D_u = 0.1, S_r = 0.2, Q = 0.5, \Lambda = 0.1, Le = 5, Ra_s = 10$.

Figure 4 displays the effect of normalized porosity on the oscillatory neutral curves. We find that when normalized porosity increases, the minimal critical oscillatory Rayleigh number decreases, showing that normalized porosity accelerates the oscillatory convection onset.

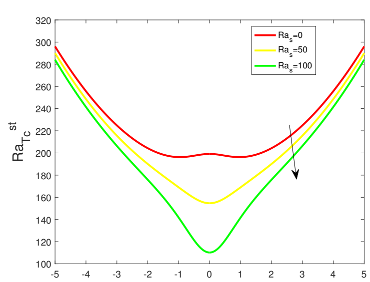


Figure 5: Solute Rayleigh number effect on stationary critical Rayleigh number ${Ra_T}_c^{st}$ with Péclet–Darcy number Q. When $\beta = 0.25, Ta = 10, D_u = 0.1, S_r = 0.2, \Lambda = 0.1, Le = 5$.

Figure 5 illustrates the critical Rayleigh number against Péclet number for various values of Ra_s. The figure shows that, for $Ra_s = 0$ with increasing Q, the critical Rayleigh number decreases initially and then increases. Thus, the throughflow first destabilizes the system and then stabilizes it. For $Ra_s > 0$, the throughflow effect always stabilizes the system. These findings are in agreement with those of Shivakumara [36]. Also, Figure 5 shows that Ra_s has a destabilizing effect.

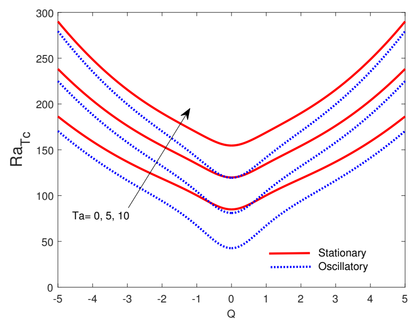


Figure 6: Critical Rayleigh number $Ra_{T\,c}$ variation with with Péclet–Darcy number Q, for diffrent values of Taylor number Ta, when $\beta = 0.25, D_u = 0.1, S_r = 0.2\Lambda = 0.1, Le = 5, \phi = 0.1$.

Figure 6 shows the variation of critical Rayleigh number against the Péclet number for different values of Taylor number. The figure shows that increasing the rotation delays the onset of stationary and oscillatory instabilities.

5. Conclusion

A linear stability analysis is carried out to investigate double-diffusive convection in a rotating vertical cylinder filled with heterogeneous porous media and vertical throughflow in the presence of Soret and Dufour influences. The Brinkman model was employed in the system of governing equations. The effect of the normalized porosity of the porous medium, heterogeneity, Dufour, Soret, Lewis, Darcy, Taylor, Péclet, and solute Rayleigh numbers on the stationary and oscillatory convection have been presented. In summary, we observe the following

- As heterogeneity is increased, both destabilizing and stabilizing effects are experienced.
- In the absence of the diffusing component, throughflow destabilizes the system before stabilizing it. When Ra_s is greater than zero, the throughflow always stabilizes the system.
- The stationary and oscillatory convection onsets are delayed by increasing the Dufour, Taylor, Lewis, and Darcy numbers.
- The Soret parameter stabilizes the fluid system in the stationary mode and destabilizes it in the oscillatory mode.
- Through delaying the start of convection instabilities, the effective Darcy number has a stabilizing impact on stationary and oscillatory convection.
- Increasing the normalized porosity reduces the critical oscillatory Rayleigh number, hence it has a destabilization effect.

The authors are grateful to the University of KwaZulu-Natal for its support.

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