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Dynamic analysis of the different-types elliptic cylindrical inclusions subjected to plane SH-wave scattering

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Abstract

The complex boundary of the elliptical inclusion rendered it difficult to solve the problem of wave scattering. In this study, the steady-state response was analyzed using the wave function expansion method. Subsequently, the Ricker wavelet was employed as the transient disturbance and Fourier transform was used to determine the distribution of transient dynamic stress concentration around the elliptical inclusion. The effects of wave number, elliptical axial ratio and difference in material properties on the distribution of the dynamic stress concentration around the elliptical inclusion were evaluated. The numerical results revealed that the dynamic stress concentration always appeared at both ends of the major axis and minor axis of the elliptical inclusion, and the difference in material properties between the inclusion and medium influenced the variations in the dynamic stress concentration factor with the wave number and elliptical axial ratio.

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Abstract: The complex boundary of the elliptical inclusion rendered it difficult to solve the problem of wave scattering. In this study, the steady-state response was analyzed using the wave function expansion method. Subsequently, the Ricker wavelet was employed as the transient disturbance and Fourier transform was used to determine the distribution of transient dynamic stress concentration around the elliptical inclusion. The effects of wave number, elliptical axial ratio and difference in material properties on the distribution of the dynamic stress concentration around the elliptical inclusion were evaluated. The numerical results revealed that the dynamic stress concentration always appeared at both ends of the major axis and minor axis of the elliptical inclusion, and the difference in material properties between the inclusion and medium influenced the variations in the dynamic stress concentration factor with the wave number and elliptical axial ratio.

Key words: Scattering, Transient response, Elliptical inclusion, Dynamic stress concentration.

1 Introduction

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Due to the discontinuity of structure, inclusions often exist in different objects, such as underground structure in the stratum, impurities in materials and so on. Scattering and dynamic stress concentration will occur around inclusions when subjected to stress wave[1,2], which may cause the structural failure of inclusions. Thus, dynamic response of a medium with inclusions embedded in elastic waves should be considered seriously. Inclusions usually have irregular shapes, which squares and circles are difficult to directly apply to practical research. Ellipse can approach circle and crack with the change in the axis ratio, which exhibits strong flexibility and renders it more suitable for practical engineering. Therefore, the study of the dynamic stress concentration surrounding elliptical inclusions demonstrates an important engineering significance.

In recent decades, the scattering and dynamic stress concentration of stress waves have been extensively studied, the models, methods and wave properties have been developed more maturely.[3-9] Pao and Mow[10] first synthesized and calculated the existing models of the scattering and dynamic stress concentration, obtaining the dynamic stress concentration distribution of a series of models and its influencing factors using the wave function expansion method. Subsequently, investigations have mainly focused on two theoretical types: cavity and inclusion. Liu et al[11]employed the complex variable function to solve the dynamic stress concentration problem surrounding cavity of arbitrary shape in an infinite elastic plane, providing the computational results of the dynamic stress concentration around the cavities of circular, elliptical and horseshoe shape. Tao et al[12] solved the dynamic stress concentration around a circular cavity under the transient P wave disturbance in an infinite homogeneous medium based on the complex variable function and Fourier transform, and the dynamic stress concentration distribution around the circular cavity was observed to affect the Poisson's ratio, wave number and waveform. Li et al[13] explored the application of the complex variable function to determine the scattering of a shallow-buried circular cavity under the transient P wave loads, and analyzed the effects of cavity depth, incident angle and position of wave peak on the dynamic stress concentration factor ($a_{\Delta\Sigma^{*}\Phi}$) distribution. A Butterworth filter was designed to remove the jump points and achieve more reasonable transient response results. Tao et al[14] investigated the utilization of the wave function expansion method based on the Mathieu function to solve the scattering and dynamic stress concentration surrounding the elliptical cavity produced owing to the transient SH wave in the infinite plane, and simulated the plastic deformation of the cavity using LS-DYNA to validate against the numerical result. Ghafarollahi and Shodja[15] implemented the multipole expansion method to present an analytical treatment for the anti-plane scattering of SH-waves by an arbitrarily oriented elliptic cavity/crack is embedded near the interface between the exponentially graded and homogeneous half-spaces.

In addition, the scattering and dynamic stress concentration around inclusions have always been the focus of research too, and the boundary conditions of inclusions are more complicated than cavities. Manoogian and Lee[16] proposed the weighted residual method to the problem of the diffraction and scattering of plane SH-waves by an underground inclusion in half-space, and determined the ground-motion of circular, elliptical and square inclusions. Moreover, the effect of shape and depth of inclusions, frequency and angle of incidence of the incidence wave in ground-motion amplification was analyzed. Yang et al[17] utilized the Green's function to solve the scattering far field solution of SH-wave by a movable rigid cylindrical interface inclusion, indicating that different combinations of medium parameters exhibited a great influence on the far-field solution. Lee and Amornwongpaibun[18] employed the wave function expansion method in the elliptical coordinates and elliptical cosine half-range expansion method to offer an analytical solution to the problem of the scattering around the semi-elliptical hill on half-space, and found that the existence of an elliptical hill causes complicated effects on ground motion. Hei et al[19] presented a universal approach of solving the dynamic stress concentration around a circular inclusion in two-dimensional inhomogeneous medium based on the complex function theory. The inhomogeneity of medium is considered in the calculation process, which expands the research of complex medium. Sheikhhassani and Dravinski[20] derived a non-hypersingular boundary integral equations to compute the stresses and $a_{\Delta\Sigma^{*}\Phi}$ by using a weak form of Helmholtz equation. And using the method to evaluate dynamic stress concentration for the multiple multilayered inclusions embedded in an elastic half-space subjected to SH-waves. Qi et al[21] suggested the way of the complex variable function method, combined with "conformal mapping" method and Green's function method to study the scattering problems of SH-wave by elliptical inclusion with partial debond curve and circular

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cavity in half-space. Results revealed that$a_{\Delta\Sigma^{\gamma}\Phi}$ was influenced by the incident angle, the frequency of incident wave, distance between the defects, depth of the inclusions and partial debond curve angle.

However, the existing research has mainly focused on studying the steady-state stress response and angular dynamic stress concentration around elliptical inclusions while few studies have been devoted to the transient response of stress wave and radial dynamic stress concentration. Thus, it is vital to further extend the investigation of the radial stress concentration distribution around the elliptical inclusions and the dynamic response caused by transient incidence. In this study, theoretical solutions based on the wave function expansion method and Fourier transform were developed for an inclusion in infinite space when subjected to a plane SH-wave. The steady-state and transient responses of the radial and angular stress were determined, and the effects of wave numbers, elliptical axial ratio and material properties on dynamic response were analyzed and discussed.

2 Problem statement and the governing equations

2.1 Problem statement

The deep-buried inclusion has been regarded as a problem of an infinite medium.SH wave as a common stress wave, negatively impacted on the inclusion structure in the propagation process. An elliptic cylindrical inclusion embedded in a full-space is subjected to a plane incident SH-wave. The incident angleϑ is the angle between the incident direction and the positive direction of the x -axis. The geometry of the model is presented in Fig. 1. The major and minor axis of the elliptical inclusion are denoted by l and$h$. Both the medium and inclusion are isotropic elastic materials. Subscript 1 represents the parameters related to the surrounding medium, and subscript 2 refers to the parameters related to the elliptical inclusion. Their material properties include shear modulus μ and wave number k . The shear modulus and wave number of the medium denote μ_1 and k_1, and the shear modulus and wave number of the elliptical inclusion stand forμ_2 and k_2.

2.2 Wave function in elliptical coordinate system

The scattering and dynamic stress concentration around the elliptical inclusion can be solved by the wave function expansion method in the elliptical coordinate system. The elliptical coordinate system is shown in Fig. 2.

The elliptical coordinate system consists of numerous confocal ellipses and hyperbolas with the focal length of $2a$. The transformation from rectangular coordinate system (x, y) to elliptical coordinate system (ξ,η) is defined by[18]:

where ξ and η are the radial and angular coordinates of the elliptical coordinate system, respectively.

The scale factor in the elliptical coordinate system can be expressed as:

Therefore, the major axis, minor axis and axis ratio of the ellipse can be represented as:

The Helmholtz equation is obtained by separating time variables from the wave equation. In the elliptical coordinate system, the Helmholtz equation can be written as[10]:

where φ denotes the potential function of SH wave,c_s denotes the velocity of SH wave, $k$denotes wave number of SH wave and $\kappa=\omega/\varsigma_{\varsigma}$,ω denotes the circular frequency.

By varying separation equation$\varphi(\xi, \eta)=X(\xi)Y(\eta)$, the Mathieu equation can be expressed as:

where$q=(ak)^2/4$, and Eq (5) is referred to as the radial and angular Mathieu equations, respectively. The radial and angular Mathieu functions are obtained by solving the Mathieu equation. In order to obtain the unique single-valued solutions, only those periodic solutions with π or 2π period are of interest to us. This requires that parameters b and$q$ satisfy certain functional relations, which can be expressed as$F(b, q)=0$.

The equation is called as the characteristic equation, and b in the equation represents the characteristic value.

The angular Mathieu function can be expanded by the Fourier series as a series sum of sine and cosine functions as the following symbols[22]:

The radial Mathieu function is expressed by different types of cylinder functions as follows[22]:

where $A_r{}^m$,$B_r{}^m$ denote the expansion coefficient associated with q . In Eq (7),j =1,2,3,4,$Z_m{}^j$ represents the j th type cylinder function of m-order, and the corresponding Mathieu function is termed as the j th type radial Mathieu function. Radial Mathieu function and angular Mathieu function differ in variable by only one imaginary unit.

For simplicity, assuming that the only non-zero displacement component of incident plane SH wave is$u_z{}^{(i)}$, and its maximum displacement is u_0. ϑ is the incident angle and the analytic expression of the incident wave can be expressed as:

Substituting Eq (8) into Eq (1) upon omitting the time-dependent term$\epsilon^{-\imath\omega\tau}$, the expression of incident wave can be simplified as:

The radial Mathieu and angular Mathieu functions have the following integral relation.[23]

Since e^{ikaw} is a periodic function of the variable ϑ , e^{ikaw} can be represented by the expanded form as[10]:

where

In this study, the amplitude of incident wave $u_0$is set as 1. Substituting Eq (12) into Eq (11), the incident wave function expressed by the Mathieu function is derived as[24]:

3 Steady-state response of inclusion

The scattering wave function generated around the elliptical inclusion satisfies the two-dimensional stable wave equation and Sommerfeld's radiation condition. Therefore, according to the asymptotic characteristics of the Mathieu function, the scattering wave φ^S can be expressed as:

where B_m and C_m denote undetermined coefficients for satisfying the boundary conditions, and$q = (ak)^2/4$.

In accordance with the principle of wave superposition, the full wave function in the medium can be expressed as:

In addition, the standing wave generated in the elliptical inclusion, can be represented as[10]:

In the elliptical coordinate system, the stress component is given by the following formula[25]:

The undetermined coefficient can be determined by applying the boundary conditions. The boundary conditions require continuity of the displacement and stress along the inclusion surface, which can be expressed as:

When the incident wave is parallel to the x -axis, $\vartheta=0$, $\sigma\epsilon_\mu(0,\chi)=0$, Eqs (13), (14) and (16) can be simplified as:

Substituting Eqs (19), (20) and (21) into Eq (18) to derive the following two equations:

$\varsigma\epsilon_\mu(\eta,\chi_1)$ and$\varsigma\epsilon_\mu(\eta,\chi_2)$ are not orthogonal to each other, so the undetermined coefficientsB_m and D_m cannot be calculated directly. Thus, the orthogonal condition of angular Mathieu function is used to simplify the calculation. The orthogonal condition can be expressed as:

Multipling both ends of Eq (22) by $\varsigma\epsilon_\nu(\eta,\chi_1)$ and integrating from 0 to 2π to derive the following two simplified equations:

where

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Eliminating B_m from the two equations of Eq (24) to determine an algebraic equation system about D_m .

where

Based on the characteristics of the Mathieu function, it can become a finite series by truncating from N term, and the numerical approximate solution can be obtained. Taking N equations in Eq (26) to compute the coefficients D_0, D_1,D_2, etc. Then bringing back Eq (24) to obtain B_n , which is expressed as follows:

Therefore, the full wave function in the medium can be expressed as:

The dynamic stress concentration factor $\alpha_{\Delta\Sigma\Phi}$around the elliptical inclusion is defined as the ratio of the stress produced by the full wave and the peak stress produced by the incident wave, and it can be expressed as:

Substituting Eq (29) into Eq (30) to obtain the steady-state angular dynamic stress concentration factor$\alpha^{\eta}{}_{\Delta\Sigma\Phi}$ and steady-state radial dynamic stress concentration factor$\alpha^{\xi}{}_{\Delta\Sigma\Phi}$, the results are represented as follows:

Owing to the characteristics of harmonic function, only the real or imaginary part of the result of Eq (31) represents the steady-state-state $\alpha_{\Delta\Sigma\Phi}$. Adding the time-dependent term $\epsilon^{-\iota\omega\tau}$, the real part represents steady-state $\alpha_{\Delta\Sigma\Phi}$ at T=0, and the imaginary part represents steady-state $\alpha_{\Delta\Sigma\Phi}$ at T/4. T denotes the period of the incident wave.

3.1 Case study and verification

In the elliptical coordinate system, the major and minor axis and axis ratio of any ellipse can be calculated by Eq (3). To simplify the problem, the focal length of the ellipse is taken as a =1m. The four cases of radial coordinate ξ = 0.1, 0.2, 0.5, 1.5 are determined to study the influence of different axial ratio on the scattering of incident SH wave around the elliptical inclusion. The smaller ξ , the larger the elliptical axis ratio and the closer the shape is to crack. The larger ξ , the smaller the elliptical axis ratio, and the closer the shape is to circle. The parameter settings of the radial coordinate are illustrated in Table 1.

In this study, the wave velocity c_s was set as 2300 m/s, the incident wave numbers (k) were predetermined to be 0.2, 0.5 and 1, respectively. The range of stress wave numbers generated by earthquake, engineering blasting and mostly impacts was covered here in.[14] The difference in the material properties of the medium and elliptical inclusion additionally affected the scattering of incident SH wave around the elliptical inclusion.[26]This study set up three cases for calculation, as shown in Table 2.

In Table 2,$k^* = k_2/k_1$,$\mu^* = \mu_1/\mu_2$ and three cases correspond to the inclusion being stiffer, softer and much softer than the medium, respectively.

Subsequently, the case was computed when the material properties parameters of the inclusion and medium were the same to verify the correctness of the derivation. When the inclusion and medium possessed the identical material properties parameters, the propagation of SH wave in the inclusion was the same as the propagation in the medium, and dynamic stress concentration was only related to the phase difference in the stress wave. Therefore, incident SH wave only produced the incident wave, and not generated the scattered and standing waves, leading to $\alpha_{\Delta\Sigma\Phi}$ determined only by the incident wave. According to the definition of$\alpha^{\eta}{}_{\Delta\Sigma\Phi}$, the maximum value of$\alpha^{\eta}{}_{\Delta\Sigma\Phi}$ was 1. The numerical results are shown in Fig. 3.

In Fig. 3, as the incident wave number and axial ratio changed,$\alpha^{\eta}{}_{\Delta\Sigma\Phi}$ had a maximum value at the angle perpendicular to the incident direction (both ends of the elliptical minor axis), and a minimum value at the angle of incidence (both ends of the elliptical major axis). The maximum and minimum values of $\alpha^{\eta}{}_{\Delta\Sigma\Phi}$ were 1 and 0.$\alpha^{\eta}{}_{\Delta\Sigma\Phi}$ gradually increased with the angle from 0° to 90°, and distribution of$\alpha^{\eta}{}_{\Delta\Sigma\Phi}$ was symmetrical about the x and y axes. To verify the correctness of the theoretical derivation, the numerical results were compared with those available in literature. As expected, the verification exhibited excellent agreement between the results of the present study and those available in the literature.[10]

3.2 Steady-state response of angular stress

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When computing the steady-state response of angular stress with different axial ratios and different wave numbers in three cases according to the case study, the numerical results were shown in Figs. 4-7. Figures 4-6 presented the distribution of the steady-state$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ around the elliptical inclusion in the three cases. As shown in Figs. 4-6,$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ always had a minimum value at both ends of the elliptical major axis. With variation in the angle, the closer to both ends of the elliptical minor axis, the larger the value of $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$, and the maximum value was obtained at both ends of the elliptical minor axis. Distribution of $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was symmetric about x -axis. In different cases, the minimum value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was 0, but the maximum value of $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was different. The maximum value of $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in Fig. 4 was only 0.781, 0.662, 0.503 and 0.384, respectively. The maximum value of $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in Fig. 5 was 1.105, 1.202, 1.412 and 1.646, respectively. The maximum value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in Fig. 6 was 1.136, 1.284, 2.079, and 1.965, respectively. The value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in case 1 was less than 1, which meant that when the inclusion was stiffer than the medium, the steady-state incidence reduced angular stress dynamic concentration around inclusion. The value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in cases 2 and 3 was larger than 1, which indicated that when inclusion was softer than medium, steady-state incidence aggravated the angular stress concentration around inclusion. The value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in case 3 exceeded that in case 2, which revealed that the softer the inclusion was than the medium, the more significant the angular stress concentration around inclusion was. In addition, the distribution of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ also changed with the axial ratio. As shown in Fig. 4 (d), Fig 5. (d), Figs. 6 (c) and (d), as the shape of inclusion was gradually close to circle and the wave number was high, $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ exhibited multiple extreme values. Because under the condition of high wave number, there were multiple stress wave crests in the inclusion, and the positions of crests were different due to the influence of the phase difference, resulting in multiple extreme values.

Figure 7 showed the changes in the steady-state$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ with the radial coordinates and wave number when η =90°in three cases. As shown in Fig. 7(a), under the condition of constant wave number,$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ of case 1 decreased slowly with an increase in the radial coordinate, and the value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was always less than 1. The results demonstrated that as the inclusion was stiffer than the medium, the steady-state incidence diminished the angular stress concentration, and the more the shape of inclusion was closer to circle, the greater the degree of reduction. In Figs. 7(c) and (e), under the condition of constant wave number,$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ of case 2 and case 3 had a certain volatility as the radial coordinate changes, and had a trend of oscillation. The oscillation at k_{1}=1 was more intense than that at k_{1}=0.5. Moreover, oscillation of $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in case 3 had a higher amplitude, a shorter period, and more intense than that in case 2. This implied that for the inclusion softer than the medium, it had a high sensitivity to radial coordinates, and the greater the difference between the material properties of the inclusion and medium, the higher the sensitivity.

As shown in Fig. 7(b), under the condition of constant elliptical axial ratio, $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ of case 1 decreased with an increase in the wave number, eventually approached 0, and the value of $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was always less than 1. This meant that for the inclusion was stiffer than the medium, the high wave number steady-state incidence decreased the angular stress concentration. In Figs. 7(d) and (f), under the condition of constant elliptical axial ratio, the$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ of cases 2 and 3 experienced obvious volatility with the variation in wave number, along with had a trend of oscillation. The oscillation at ξ=1.5 was more intense than that at ξ=0.5. In addition, oscillation of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in case 3 was higher in amplitude, shorter in period, and more severe than in case 2, which was consistent with the changes of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ with the radial coordinates. The results indicated that for the inclusion was softer than medium, $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ also had a high sensitivity to wave number. The greater the difference between the material properties of inclusion and medium, the higher the sensitivity. The phenomenon was consistent with the existing literature, which further confirmed the validity of the theoretical derivation.[26]

3.3 Steady-state response of radial stress

In the existing studies about the scattering and dynamic stress concentration of elliptical inclusions, a majority of the study content focused on the numerical results for the angular stress concentration and the discussion about the radial failure caused by angular stress concentration. But little study was conducted on the radial stress concentration. The angular failure caused by the radial stress concentration, affected the structural safety of inclusion. Therefore, it is imperative to study the radial stress concentration of elliptical inclusions. The numerical results of radial stress concentration are given below.

3.3.1 Cases of the same material

When the inclusion and medium had the same material properties parameters, the incident wave did not diffract at inclusion. As shown in Fig. 8, $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ had a maximum value at both ends of the elliptical major axis, and a minimum value of 0 at both ends of the elliptical minor axis, which was opposite to the distribution of $\alpha^{\eta}{}_{\Delta\Sigma`\Phi}$. The distribution of $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ was symmetric about the $x$ -axis and the $y$ -axis. Moreover, unlike the distribution of$\alpha^{\eta}{}_{\Delta\Sigma`\Phi}$, the maximum value of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ was not all equal to 1, but approached or equal to 1 at $k_1$=0.2. The distribution of $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ around the elliptical inclusion changed in with the radial coordinate. As observed in Fig. 8, when the radial coordinate was small, the shape of elliptical inclusion was close to crack, the distribution shape of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ around both ends of the elliptical major axis is similar to crack tip, and the value of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ changed drastically. However, when the radial coordinate was large, and the shape of elliptical inclusion was close to circle, the distribution shape of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ around both ends of the elliptical major axis was similar to the circle, and the value of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ changed gently.

3.3.2 Numerical results and analysis

The steady-state response of radial stress with different axial ratios and different wave numbers in the three cases was determined according to the case study. The numerical results were shown in Figs. 9-12. Figures 9-11 presented the distribution of steady-state$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ around the elliptical inclusion in three cases. As shown in Figs. 9-11, when ellipse approached circle and wave number was high, multiple extreme values of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ appeared, and the distribution of $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ on the front wave surface and the back wave surface was different. Although the distribution of $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ was no longer as regular as Fig. (8), the distribution of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ was still symmetrical about the $x$ -axis. In Fig. 9,$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ always had a maximum value at both ends of the elliptical major axis. As the angle was closer to both ends of the elliptical minor axis, the value of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ became smaller, and a minimum value was observed at both ends of the elliptical minor axis. The maximum values of $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ were 3.081, 2.616, 1.985, and 1.432, respectively, which were all larger than 1. This indicated that as the inclusion was stiffer the medium, the steady-state incidence caused significant radial stress concentration. As shown in Figs. 10 and 11, at the small radial coordinate, the distribution of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ was similar to that in Fig. 9, but the maximum value was not larger than 0.28. At the large radial coordinate, the maximum and minimum values of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ no longer occurred at both ends of the major and minor axis of ellipse, and$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ on the front wave surface was more than that on the back wave surface. The maximum value of $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ in Fig. 10 was 1.472, and the maximum value of $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$in Fig. 11 was 1.016.

Figure 12 showed the steady-state$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ changed with radial coordinate and wave number at $\eta$ =0° in three cases. Under the condition of constant wave number,$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ of case 1 decreased slowly with the increase in the radial coordinate. This phenomenon in Fig. 12(a) was consistent with the changes of$\alpha^{\eta}{}_{\Delta\Sigma`\Phi}$ , and the difference was that most of the value of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ was larger than 1. In Figs. 12(c) and (e), under the condition of constant wave number, the variation trends of $\alpha^{\eta}{}_{\Delta\Sigma`\Phi}$ and$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ in the radial coordinate were roughly identical. It implied that when the inclusion was softer than the medium, both $\alpha^{\eta}{}_{\Delta\Sigma`\Phi}$and $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ had high sensitivity to the radial coordinate. The more significant the difference between the material properties of the inclusion and medium, the greater their sensitivity.

As shown in Fig. 12(b), under the condition of constant elliptical axial ratio, the $\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ of case 1 decreased with increasing the wave number. But unlike$\alpha^{\eta}{}_{\Delta\Sigma`\Phi}$ eventually approached 0,$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ eventually approached 0.37. This revealed that for the inclusion stiffer than medium, the high wave number steady-state incidence additionally reduced the radial stress concentration. In Figs. 12(d) and (f), under the condition of constant elliptical axial ratio, the variation trends of$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ and$\alpha^{\eta}{}_{\Delta\Sigma`\Phi}$ in the wave number were roughly identical. This indicated that$\alpha^{\xi}{}_{\Delta\Sigma`\Phi}$ also exhibited a high sensitivity to the wave number, and the larger the difference between the material properties of inclusion and medium, the greater their sensitivity.

Based on the foregoing analysis of the steady-state response, as the inclusion was stiffer than the medium, the dynamic stress concentration around the elliptical inclusion was dominated by the angular stress concentration at both ends of the minor axis.$\alpha_{\Delta\Sigma`\Phi}$ gradually decreased with the increase in radial coordinate and

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wave number, but the final approach values of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ and$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ were different. However, when the inclusion was softer than medium, the dynamic stress concentration around the elliptical inclusion was dominated by the radial stress concentration at both ends of the major axis.$a_{\Delta\Sigma^{\neg}\Phi}$ had high sensitivity to both the radial coordinate and the wave number. The greater the difference between the material properties of inclusion and medium, the larger the amplitude, the shorter the period, and the higher the sensitivity.

4 Transient response of inclusion

The dynamic disturbance in the practical project is a non-periodic transient disturbance, which is different from the simple harmonic wave. The transient wave can be decomposed into the superposition of simple harmonic wave with different frequencies by Fourier transform. The steady-state response has been obtained in the previous section, and the dynamic response of the transient disturbance to the system can be represented as:

where $\chi(\omega)$ denotes steady-state response of the system under the incidence of simple harmonics, namely, the steady-state$a_{\Delta\Sigma^{\neg}\Phi}$ determined in the previous section,$\Phi(\omega)$ denotes the distribution function of the transient disturbance in frequency domain.

In order to calculate the dynamic stress concentration around the elliptical inclusion under practical project disturbance, seismic waves were introduced as the transient disturbance model. In the study of seismic wave forward modeling, seismic wavelet was the basic unit of seismic wave, which can express the basic characteristics of the wave source. Seismic data can be obtained by combining seismic wavelet with convolution model.[27-29] It was generally believed that what a single seismic source excites was a sharp pulse with a short time. When seismic waves propagated in the stratum, due to the attenuation and dispersion effected of the viscoelastic stratum on the high-frequency components, the waveform was elongated and seismic wavelets were formed. Based on the difference of wavelet phase spectrum delay, seismic wavelet can be divided into zero phase wavelet, constant phase wavelet, minimum phase wavelet and mixed phase wavelet. In this study, the Ricker wavelet in the zero-phase seismic wavelet was selected as the transient disturbance model to simulate the disturbance caused by the earthquakes. The distribution of the Ricker wavelet in the time domain and frequency domain was shown in Fig. 13.

The Ricker wavelet is symmetric in the time domain, which can be represented as[30]:

where ω_{π} denotes the dominant frequency of the Ricker wavelet.

Fourier transform requires the distribution of the Ricker wavelet in the frequency domain, $\Phi(\omega)$ can be expressed as:

The distribution of the Ricker wavelet in the time domain and the frequency domain is mainly determined by the ω_{π} . The Ricker wavelet in the time domain has a main lobe and two side lobes, and the duration is short and the convergence is fast. The ratio of the amplitude of the main lobe to the amplitude of the side lobes is $0.5e^{1.5}$, which is approximately equal to 2.241.[31] The Ricker wavelet in the frequency domain always takes the maximum value at the ω_{π} . The larger the ω_{π} , the wider the frequency domain of the Ricker wavelet. In this study, three Ricker wavelets with differentω_{π} were selected for transient incidence.

Substituting Eqs (31) and (34) into Eq (32), and the frequency variablesω was canceled by the integration, then a time-dependent functionu_t was obtained. By selecting different times$t$, the system response at different times in the process of transient wave incident was obtained. Due to the mathematical difficulties in direct integration, trapezoidal approximation was used to determine the integration result. In the practical project, we were more interested in the system response when the incident wave reached the peak, so the case selected the system response at that moment for analysis.

4.1 Transient response of angular stress

The transient response of angular stress with different axial ratios and different ω_{π} in the three cases was analyzed according to the case study. The numerical results were shown in Figs. 14-18. As shown in Figs. 14-16, the spatial distribution of transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was similar to steady-state $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$. Transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$

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always had a maximum value at both ends of the elliptical minor axis, and a minimum value of 0 at both ends of the elliptical major axis. When the shape of inclusion was close to circle and the incident wave was high dominant frequency, the extreme values appeared at other angles. Distribution of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was symmetric about the$x$ -axis and the value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was affected by

ω_{π}.

Fig. 17(a) indicated that when ω_{π} was constant and inclusion was stiffer than the medium, transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ decreased with the reduction in elliptical axial ratio, gradually declined from 1.928 to 0.897. This demonstrated that at the inclusion stiffer than the medium, unlike steady-state $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$always less than 1, the transient incident aggravated the angular stress concentration. Moreover, the angular stress concentration as shape of inclusion approaching crack was more significant than that approaching circle. In Fig. 17(b), for the inclusion softer than the medium, transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ increased with the decrease in the elliptical axial ratio, from 2.712 to 3.705. Numerical results demonstrated that for the inclusion softer than the medium, the angular stress concentration at shape of inclusion approaching circle was more significant than that when shape of inclusion approaching crack. This phenomenon was contrary to Fig. 17(a), indicated that the difference of material properties between the inclusion and medium affected the changes in transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ with elliptical axial ratio.

Fig. 18 illustrated that when elliptical axial ratio andω_{π} were constant, transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ for the inclusion stiffer than the medium was always smaller than that for the inclusion softer than the medium. This phenomenon indicated that the angular stress concentration of the soft inclusion was more significant than that of the stiff inclusion, and the softer the inclusion was, the greater the possibility of failure at both ends of the minor axis. The value of transient $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ in case 2 was smaller than that in case 3, which proved that the greater the difference in the material properties between the medium and inclusion, the more significant the dynamic stress concentration. This phenomenon was consistent with the changes of steady-state$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$.

4.2 Transient response of radial stress

The transient response of radial stress with different axial ratios and different ω_{π} in the three cases was evaluated in the case study. The numerical results were shown in Figs. 19-23. The distribution of transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ was similar to that of steady-state $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$. When the shape of inclusion was close to circle and the incident wave was highω_{π} , transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ exhibited multiple extreme values. Distribution of$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ on the front wave surface was different from that on the back wave surface, but distribution of $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ was symmetric about the x -axis. In Fig. 19,$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ always had the largest value at both ends of the elliptical major axis, with the maximum value reaching 7.869, and the minimum value of 0 at both ends of the elliptical minor axis. The value of transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ in Figs. 20 and 21 was much smaller than the value in Fig. 19, and a majority of the maximum values were around 1. At a small ω_{π} , distribution of transient $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ was similar to Fig.19. At a large ω_{π} , the maximum and minimum values did not always appear at both ends of the major and minor axes of ellipse, while multiple extreme values did.

Fig. 22 indicated that when the elliptical axial ratio andω_{π} were constant, transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ for the inclusion was stiffer than the medium decrease upon reducing the elliptical axial ratio, from 7.214 to 2.710. For the inclusion was softer than the medium, transient $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$increased upon decreasing the elliptical axial ratio, from 0.597 to 1.156. Numerical results demonstrated that as the inclusion was stiffer than the medium, the radial stress concentration for the shape of inclusion close to crack was more significant than that for the shape of inclusion close to circle. But for the inclusion was softer than the medium, the radial stress concentration for shape of inclusion approaching circle was more significant than that as shape of inclusion approaching crack. The difference in material properties between the inclusion and medium also affected the changes in transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ with elliptical axial ratio, which was consistent with transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$. In addition, at the small radial coordinate, the shape of elliptical inclusion was close to crack, the distribution shape of transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ around both ends of the elliptical major axis was similar to crack tip, and the value of transient $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ changed drastically. However, at the lager radial coordinate, the shape of elliptical inclusion was close to circle, the distribution shape of transient $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ around both ends of the elliptical major axis was approximate to the circle, and the value of transient $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$varied slightly. This behavior was consistent with the distribution of stead-state $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$.

Fig. 23 demonstrated that at the constant elliptical axial ratio andω_{π} , the transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ with the inclusion stiffer than the medium was much larger than the transient$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ for the inclusion softer than the medium. The numerical findings indicated that the radial stress concentration of the stiff inclusion was more significant than that of soft inclusion, and the stiffer the inclusion, the greater the possibility of failure at both ends of the major axis. The value of transient $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ in case 2 was higher than that in case 3, which confirmed that the greater the difference in material properties between the medium and inclusion, the more significant the dynamic stress concentration. This behavior was in accordance with the changes in transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$.

5 Discussion

In this study, theoretical solutions based on the wave function expansion method and Fourier transform were obtained for an inclusion in infinite space when subjected to a plane SH-wave. First, the steady-state response was analyzed using the wave function expansion method. Then, the Ricker wavelet was introduced as the transient disturbance. Finally, the Fourier transform was used to determine the distribution of transient dynamic stress concentration around the elliptical inclusion. The numerical results indicated that the dynamic stress concentration generated by the steady-state and transient incident waves was different. However, the dynamic stress concentration distribution in the two states was dependent on the elliptical axis ratio, incident wave number, difference in material properties between the medium and inclusion.

As reported in literature, the main content is analyzing angular stress concentration, while the radial stress concentration is often ignored. In this study, the angular and radial stress expressions under the steady-state incidence and transient incidence were obtained theoretically, and two types of distribution of dynamic stress concentration around the elliptical inclusion was calculated. It is found that regardless of the steady-state or transient incidence, for the inclusion stiffer than the medium, significant radial stress concentration appeared at both ends of the elliptical major axis. At the elliptical axis ratio of 10, the maximum steady-state$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ attained 3.081, and the maximum transient $a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$reached 7.869. The angular stress concentration was observed at both ends of the elliptical minor axis, but the value of$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was slightly small. The maximum value of steady-state$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was 0.764, and the maximum value of transient$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ was 1.969. The closer the inclusion shape to crack, the larger the value of$a_{\Delta\Sigma^{\neg}\Phi}$ and the more significant the dynamic stress concentration. Therefore, the structural failure was likely to occur at both ends of the major axis and the minor axis of elliptical inclusion, and the radial stress concentration at both ends of the major axis of inclusion was more likely to cause structural failure. At this moment, the closer the inclusion shape to crack, the higher the possibility of the structural failure. However, for the inclusion softer than the medium, significant angular stress concentration was observed at both ends of the elliptical minor axis. The steady-state$a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ achieved the maximum value of 2.079 at the elliptical axis ratio was 2.16, and the transient state $a^{\eta}{}_{\Delta\Sigma^{\neg}\Phi}$ attained the maximum value of 4.588 when elliptical axis ratio of 1.1. The radial stress concentration was noted at both ends of the elliptical major axis. When the axial ratio was 1.1, the maximum value of steady-state$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ attained 1.472, and the maximum value of transient state$a^{\xi}{}_{\Delta\Sigma^{\neg}\Phi}$ reached 2.476. The more approximate the inclusion shape to circle, the larger the value of$a_{\Delta\Sigma^{\neg}\Phi}$ and the more significant the dynamic stress concentration. Therefore, the structural failure of inclusion was likely to occur at both ends of the major axis and the minor axis of elliptical inclusion, and the angular stress concentration at both ends of the minor axis of inclusion was more likely to result in the structural failure. At this time, the closer the shape of inclusion to circle, the greater the possibility of structural failure.

Compared with the wave function expansion method based on the Fourier-Bessel expansion and conformal transformation[32], the Mathieu function was more convenient to deal with elliptical boundary problems, because it can avoid complicated boundary mapping, and the mathematical form was more concise. Moreover, the correctness of the theoretical derivation was verified by calculating distribution of dynamic stress concentration with the same material properties parameters between inclusion and medium. The results indicated that the spatial distribution of angular stress concentration was only related to the phase difference of the

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incident wave, and the maximum value of 1 was obtained in the vertical direction of the incident angle, which was consistent with the previous literature.[10] The steady-state$a^{\eta}{}_{\Delta\Sigma^{\circ}\Phi}$ gradually decreased with increasing the wave number for stiffer inclusion than medium. But when the inclusion was softer than the medium, the steady-state$a^{\eta}{}_{\Delta\Sigma^{\circ}\Phi}$ oscillated with the change in the wave number, the greater the difference of material properties between the inclusion and medium, the more intense the oscillation. This phenomenon indicated that$a^{\eta}{}_{\Delta\Sigma^{\circ}\Phi}$ had a high sensitivity to wave number, and the greater the difference in material properties between inclusion and medium, the higher the sensitivity.[26]

However, during the process of realizing the algorithm by mathematical software, the calculation of the Mathieu function was complex in some aspects. For example, under the limitation of boundary conditions, only the case of 0° incidence can be calculated. The wave number k of the incident wave was related to the q variable in the Mathieu function, the q value was related to the truncation series of the Mathieu function. In order to ensure accuracy, the truncation series must be changed when the k value was changed. In addition, the Mathieu function with no primitive function or the primitive function was difficult to express. As a result, only numerical integration methods can be used in Fourier integration, which greatly increased the amount of calculation. In order to ensure the accuracy of the results, this study compared the error between the sum of N and N+1 term. The Mathieu functions were generally truncated to the 8th or 9th to ensure accuracy. The Mathieu function was determined N=8-16, and the error between N=12 and N=16 was found to be less than 0.01%. Therefore, N=12 was selected to reduce unnecessary calculations while ensuring accuracy.

6. Conclusion

In this study, theoretical solutions based on the wave function expansion method and Fourier transform were obtained for an inclusion in infinite space when subjected to a plane SH-wave. The effects of wave number, elliptical axial ratio and difference of material properties on the distribution of dynamic stress concentration around the elliptical inclusion were analyzed. The numerical findings revealed that the dynamic stress concentration was noted always to appear at both ends of the major axis and minor axis of elliptical inclusion. For the inclusion stiffer than the medium, the radial stress concentration at both ends of the elliptical major axis was more significant than the angular stress concentration at both ends of the elliptical minor axis. When the inclusion was softer than the medium, the phenomenon was opposite. The difference of material properties between inclusion and medium affected the changes of $a_{\Delta\Sigma^{\circ}\Phi}$ with wave number and elliptical axial ratio, the greater the difference of the material properties between the medium and inclusion, the more significant the dynamic stress concentration was. Besides, the distribution of$a^{\xi}{}_{\Delta\Sigma^{\circ}\Phi}$ at both ends of the elliptical major axis was similar to the shape of the inclusion.

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Conflicts of interests

The authors declare no potential conflict of interests.

References

1. Jiang GXX, Yang ZL, Sun C, Sun BT, Yang Y. Dynamic analysis of anisotropic half space containing an elliptical inclusion under SH waves. *Math Methods Appl Sci.* 2020;43(11):6888-6902.*https://doi.org/10.1002/mma.6431.*

2. Qi H, Chu FQ, Guo J, Sun RC. Dynamic analysis for a vertical interface crack and the nearby circular cavity located at the piezoelectric bi-material half-space under SH-waves. *Acta Mech.*2021;232(3):1113-1129.*https://doi.org/10.1007/s00707-020-02812-6.*

3. Tao M, Ma A, Zhao R, Hashemi SS. Spallation damage mechanism of prefabricated elliptical holes by different transient incident waves in sandstones. *Int J Impact Eng.* 2020;146.*https://doi.org/10.1016/j.ijimpeng.2020.103716.*

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4. Zhao R, Tao M, Zhao HT, Cao WZ, Li XB, Wang SF. Dynamics fracture characteristics of cylindrically-bored granodiorite rocks under different hole size and initial stress state. *Theor Appl Fract Mech.* 2020;109.*https://doi.org/10.1016/j.tafmec.2020.102702*.

5. Tao M, Zhao HT, Li ZW, Zhu JB. Analytical and numerical study of a circular cavity subjected to plane and cylindrical P-wave scattering. *Tunn Undergr Space Technol.* 2020;95.*https://doi.org/10.1016/j.tust.2019.103143*.

6. Liu QJ, Zhao MJ, Liu ZX. Wave function expansion method for the scattering of SH waves by two symmetrical circular cavities in two bonded exponentially graded half spaces. *Eng Anal Bound Elem.*2019;106:389-396.*https://doi.org/10.1016/j.enganabound.2019.05.015*.

7. Gu HL, Tao M, Li XB, Li QY, Cao WZ, Wang F. Dynamic response and failure mechanism of fractured coal under different soaking times. *Theor Appl Fract Mech.* 2018;98:112-122.*https://doi.org/10.1016/j.tafmec.2018.09.001*.

8. Tao M, Zhao HT, Li XB, Li X, Du K. Failure characteristics and stress distribution of pre-stressed rock specimen with circular cavity subjected to dynamic loading. *Tunn Undergr Space Technol.*2018;81:1-15.*https://doi.org/10.1016/j.tust.2018.06.028*.

9. Aklouche O, Pelat A, Maugeais S, Gautier F. Scattering of flexural waves by a pit of quadratic profile inserted in an infinite thin plate. *J Sound Vib.* 2016;375:38-52.*https://doi.org/10.1016/j.jsv.2016.04.034*.

10. Pao YH, Mow CC. *Diffraction of elastic waves and dynamic stress concentrations.* New York: Crane and Russak; 1973.

11. Liu DK, Gai BZ, Tao GY. On Dynamic stress concentration in the neighborhood of a cavity. *Earthq Eng Eng Vib.* 1980(00):97-110.*https://doi.org/10.13197/j.eeev.1980.00.009*.

12. Tao M, Li ZW, Cao WZ, Li XB, Wu CQ. Stress redistribution of dynamic loading incident with arbitrary waveform through a circular cavity.*Int J Numer Anal Methods Geomech.* 2019;43(6):1279-1299.*https://doi.org/10.1002/nag.2897*.

13. Li ZW, Tao M, Du K, Cao WZ, Wu CQ. Dynamic stress state around shallow-buried cavity under transient P wave loads in different conditions. *Tunn Undergr Space Technol.* 2020;97.*https://doi.org/10.1016/j.tust.2019.103228*.

14. Tao M, Zhao R, Du K, Cao WZ, Li ZW. Dynamic stress concentration and failure characteristics around elliptical cavity subjected to impact loading. *Int J Solids Struct.* 2020;191:401-417.*https://doi.org/10.1016/j.ijsolstr.2020.01.009*.

15. Ghafarollahi A, Shodja HM. Scattering of SH-waves by an elliptic cavity/crack beneath the interface between functionally graded and homogeneous half-spaces via multipole expansion method. *J Sound Vib.* 2018;435:372-389.*https://doi.org/10.1016/j.jsv.2018.08.022*.

16. Manoogian ME, Lee VW. Diffraction of SH-Waves by Subsurface Inclusions of Arbitrary Shape. *J Eng Mech.* 1996;122(2):123-129.*https://doi.org/10.1061/(asce)0733-9399(1996)122:2(123)*.

17. Yang ZL, Liu DK, Shi WP. Scattering far field solution of SH-wave by movable rigid cylindrical interface inclusion. *Acta Mech Solida Sin.* 2002;15(3):214-226.

18. Lee VW, Amornwongpaibun A. Scattering of anti-plane (SH) waves by a semi-elliptical hill: I-Shallow hill. *Soil Dyn Earthq Eng.*2013;52:116-125.*https://doi.org/10.1016/j.soildyn.2012.08.005*.

19. Hei BP, Yang ZL, Sun BT, Wang Y. Modelling and analysis of the dynamic behavior of inhomogeneous continuum containing a circular inclusion. *Appl Math Model.* 2015;39(23-24):7364-7374.*https://doi.org/10.1016/j.apm.2015.03.015*.

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20. Sheikhhassani R, Dravinski M. Dynamic stress concentration for multiple multilayered inclusions embedded in an elastic half-space subjected to SH-waves. *Wave Motion.* 2016;62:20-40.*https://doi.org/10.1016/j.wavemoti.2015.11.002*.

21. Qi H, Chen HY, Zhang XM, Zhao YB, Xiang M. Scattering of SH-wave by elliptical inclusion with partial debond curve and circular cavity in half space. *Explos Shock Waves.* 2018;38(06):1344-1352.*https://doi.org/10.11883/bzycj-2017-0142*.

22. Abramowitz M, Stegun IA, Miller D. Handbook of Mathematical Functions With Formulas, Graphs and Mathematical Tables (National Bureau of Standards Applied Mathematics Series No. 55). *J Appl Mech.*1965;1(1):239-239.*https://doi.org/10.1115/1.3625776*.

23. Mclachlan NW. Theory and Application of Mathieu Functions.*Math Gaz.* 1968;52(379):94-95.*https://doi.org/10.2307/3614519*.

24. Wong HL, Trifunac MD. Scattering of plane SH waves by a semi-elliptical canyon. *Earthq Eng Struct Dyn.* 1974;3(2):157-169.*https://doi.org/10.1002/eqe.4290030205*.

25. Liang JW, Jia F. Surface motion of a semi-elliptical hill for incident plane SH waves. *Earthq Sci.* 2011;24(5):447-462.*https://doi.org/10.1007/s11589-011-0807-1*.

26. Jiang GXX. *Dynamic stress concentration around the inclusion in different inhomogeneous media under SH wave.* School of Aerospace and Civil Engineering, Harbin Engineering University; 2020.

27. Keiiti A, Anders C, Eystein S. H. Determination of the three-dimensional seismic structure of the lithosphere. *J Geophys Res.* 1977;82(2):277-296.*https://doi.org/10.1029/jb082i002p00277*.

28. Berenger JP. A perfectly matched layer for the absorption of electromagnetic waves. *J Comput Phys.* 1994;114(2):185-200.*https://doi.org/10.1006/jcph.1994.1159*.

29. Robert C, Bkorn E. Absorbing boundary conditions for acoustic and elastic wave equations. *Bull Seismol Soc Am.*1977;67(6):1529-1540.*https://doi.org/10.1785/BSSA0670061529*.

30. Zhang YY, Wang YZ, Shi Y, Ke X. Frequencies of the Ricker wavelet.*Prog Geophys.* 2017;32(005):2162-2167.*https://doi.org/10.6038/pg20170542*.

31. Ricker N. The form and nature of seismic waves and the structure of seismograms. *Geophysics.* 1940;5(4):348-366.*https://doi.org/10.1190/1.1441816*.

32. Liu DK, Gai BZ, Tao GY. Applications of the method of complex functions to dynamic stress concentrations. *Wave Motion.*1982;4(3):293–304.*https://doi.org/10.1016/0165-2125(82)90025-7*.

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