Electromagnetic Simulation by Curvilinear Collocated Grid Finite Difference Method

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Abstract

Traditional finite difference method for electromagnetic simulation is based on staggered grid, whilst researches on collocated grid are few. We present a high-order collocated-grid finite-difference method for modelling electromagnetic waves with a topographic ground surface by solving 2D time-domain Maxwell equations in curvilinear coordinates. The proposed method, incorporating curvilinear coordinates, collocated grids and MacCormack finite-difference scheme techniques, can describe the geometry of the irregular interface better and avoid the numerical scattering caused by the staircase approximation in the conventional finite-difference method for electromagnetic wavefield modelling. The first-order 2D Maxwell equations on curvilinear grids are solved by an optimized MacCormack finite-difference scheme, first presented by Hixon (1997). As the collocated grids are implemented, in which the electric and magnetic fields are discretized at the same grids, the interfacial boundary conditions need to be considered. Therefore, a novel effective interface method is presented to handle the conditions. The proposed method is verified by a series of ground penetrating radar application models, such as homogeneous space, multilayered media and buried cavity models, by comparing synthetic waveforms with independent reference solutions, such as the analytical solution, the generalized reflection/transmission method and an open-source program gprMax. Comparisons show that the proposed method does well in handling multiple reflections and curved interfaces.

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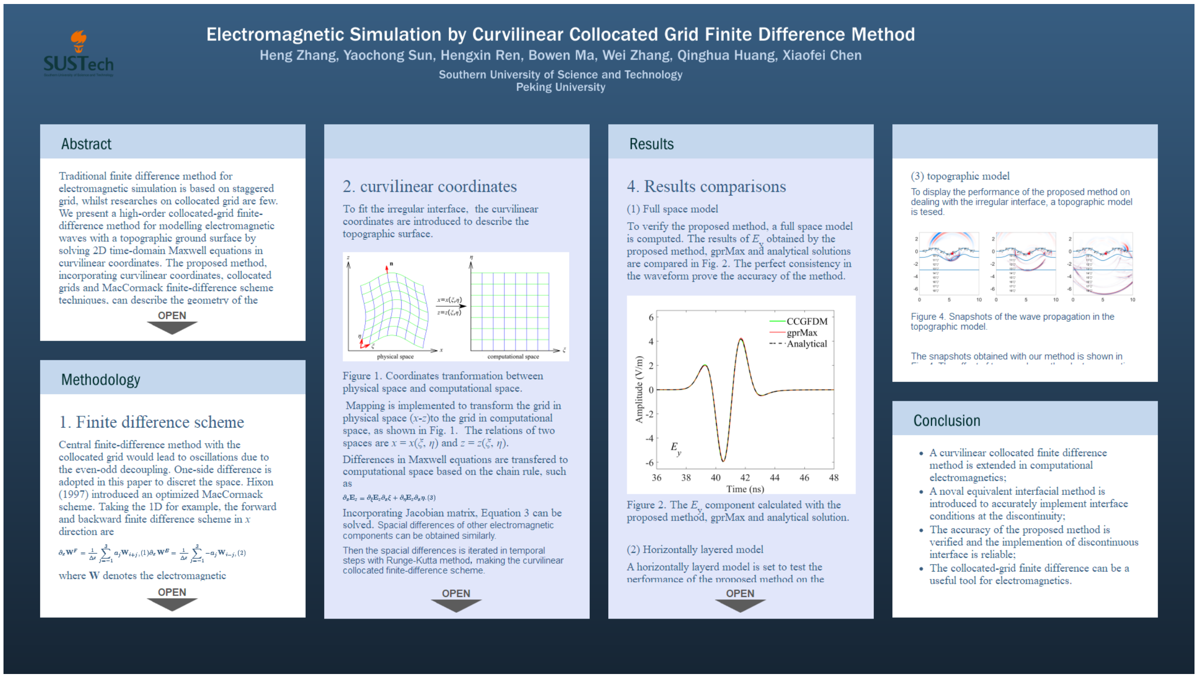

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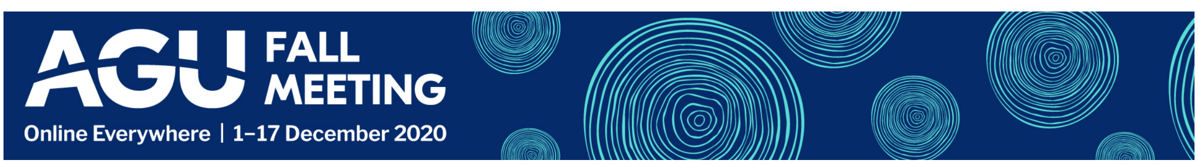


ABSTRACT

Traditional finite difference method for electromagnetic simulation is based on staggered grid, whilst researches on collocated grid are few. We present a high-order collocated-grid finite-difference method for modelling electromagnetic waves with a topographic ground surface by solving 2D time-domain Maxwell equations in curvilinear coordinates. The proposed method, incorporating curvilinear coordinates, collocated grids and MacCormack finite-difference scheme techniques, can describe the geometry of the irregular interface better and avoid the numerical scattering caused by the staircase approximation in the conventional finite-difference method for electromagnetic wavefield modelling. The first-order 2D Maxwell equations on curvilinear grids are solved by an optimized MacCormack finite-difference scheme, first presented by Hixon (1997). As the collocated grids are implemented, in which the electric and magnetic fields are discretized at the same grids, the interfacial boundary conditions need to be considered. Therefore, a novel effective interface method is presented to handle the conditions. The proposed method is verified by a series of ground penetrating radar application models, such as homogeneous space, multilayered media and buried cavity models, by comparing synthetic waveforms with independent reference solutions, such as the analytical solution, the generalized reflection/transmission method and an open-source program gprMax. Comparisons show that the proposed method does well in handling multiple reflections and curved interfaces.

METHODOLOGY

1. Finite difference scheme

Central finite-difference method with the collocated grid would lead to oscillations due to the even-odd decoupling. One-side difference is adopted in this paper to discret the space. Hixon (1997) introduced an optimized MacCormack scheme. Taking the 1D for example, the forward and backward finite difference scheme in *x* direction are

$$\partial_x \mathbf{W}^F = \frac{1}{\Delta x} \sum_{j=-1}^{3} a_j \mathbf{W}_{i+j}, (1)$$

$$\partial_x \mathbf{W}^B = \frac{1}{\Delta x} \sum_{j=-1}^{3} -a_j \mathbf{W}_{i-j}, (2)$$

where **W** denotes the electromagnetic components, *i* is the grid index in the *x* direction, *F* is the forward difference, *B* is the backward difference and para-meters a_j is provided in Hixon (1997). The forth-order Runge-Kutta scheme is combined for time stepping.

2. curvilinear coordinates

To fit the irregular interface, the curvilinear coordinates are introduced to describe the topographic surface.

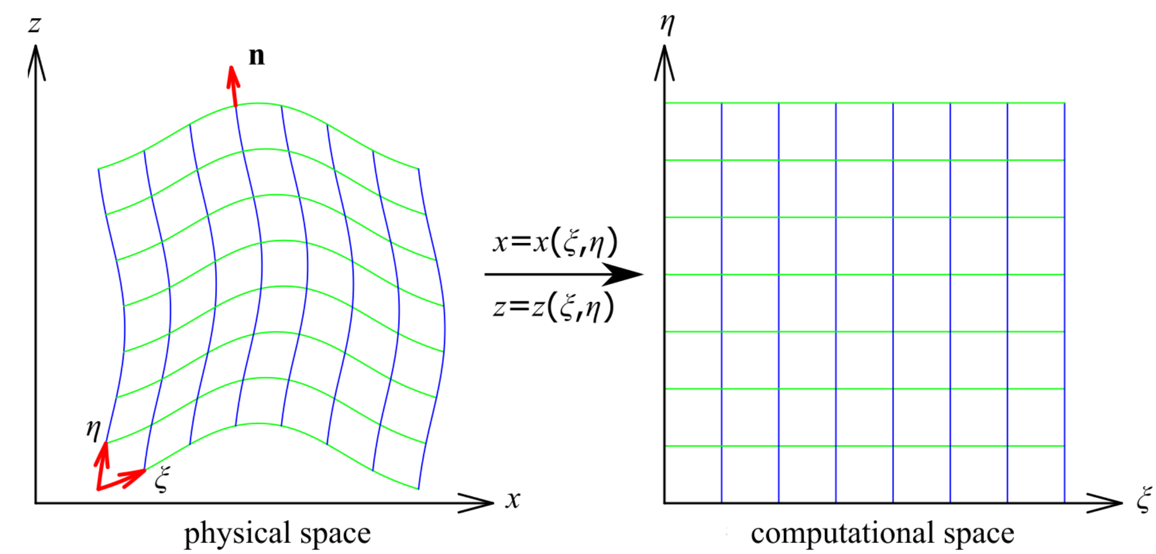


Figure 1. Coordinates tranformation between physical space and computational space.

Mapping is implemented to transform the grid in physical space (*x*-*z*)to the grid in computational space, as shown in Fig. 1. The relations of two spaces are $x = x(\xi, \eta)$ and $z = z(\xi, \eta)$.

Differences in Maxwell equations are transfered to computational space based on the chain rule, such as

$$\partial_x \mathbf{E}_z = \partial_\xi \mathbf{E}_z \partial_x \xi + \partial_\eta \mathbf{E}_z \partial_x \eta. \, (3)$$

Incorporating Jacobian matrix, Equation 3 can be solved. Spacial differences of other electromagnetic components can be obtained similarly.

Then the spacial differences is iterated in temporal steps with Runge-Kutta method, making the curvilinear collocated finite-difference scheme.

3. Interfacial condition

A noval approach of equivalent interface is introduced in our work. This approach is used to implement the discontinuous interfacial problems in computational electromagnetics. Moreover, it is not required to be orthogonal for the coordinates when the interface is irregular, which brings simpler mesh generation.

RESULTS

4. Results comparisons

(1) Full space model

To verify the proposed method, a full space model is computed. The results of E_y obtained by the proposed method, gprMax and analytical solutions are compared in Fig. 2. The perfect consistency in the waveform prove the accuracy of the method.

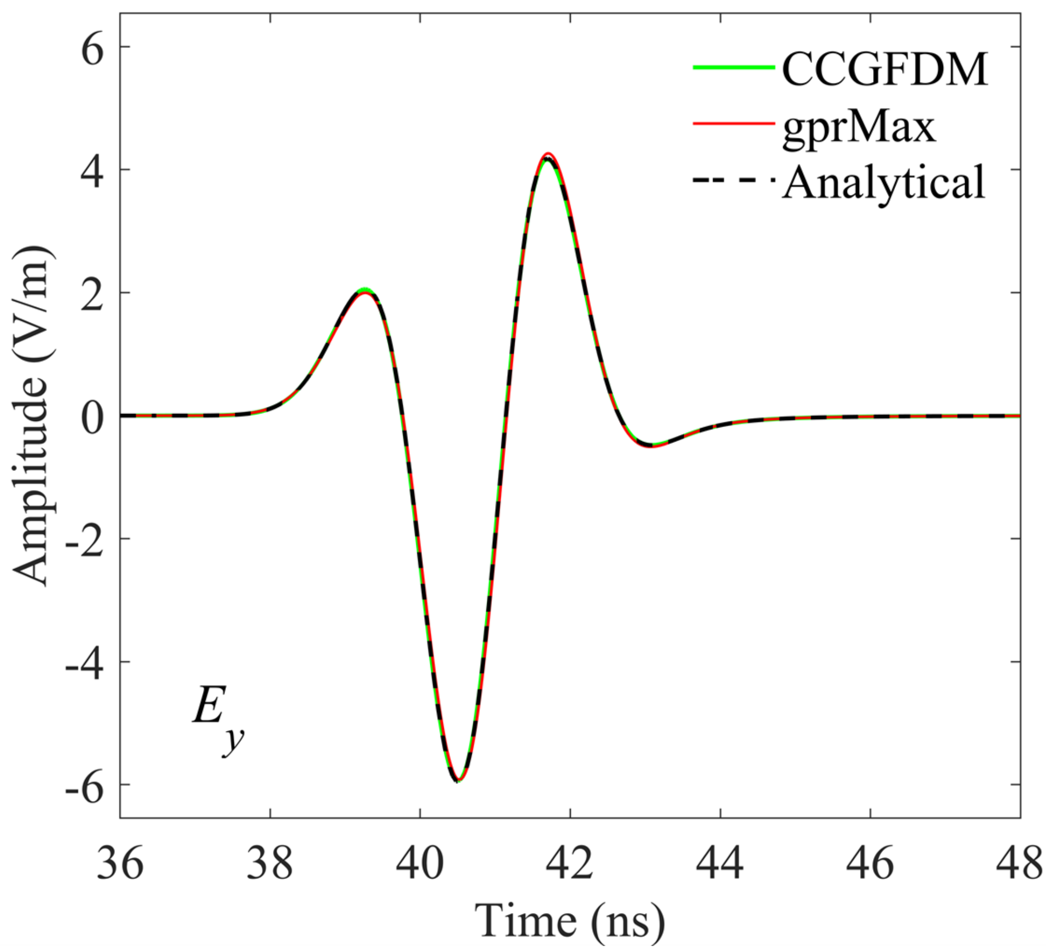
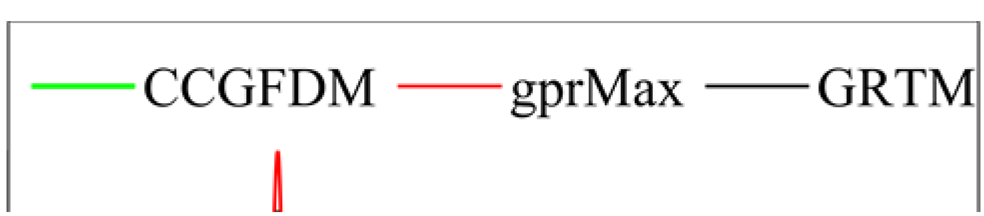


Figure 2. The E_y component calculated with the proposed method, gprMax and analytical solution.

(2) Horizontally layered model

A horizontally layerd model is set to test the performance of the proposed method on the discontinous flat interfaces. The results of discontinuous $\boldsymbol{H}_Z$ obtained by the proposed method, gprMax and analytical GRTM solutions are compared in Fig. 3. The perfect consistency of the proposed method and analytical solution in the waveform prove the accuracy of the method in dealing with the discontinuity. And the difference of gprMax depends on the disadvange of stagger grid at the discontinuous interface.



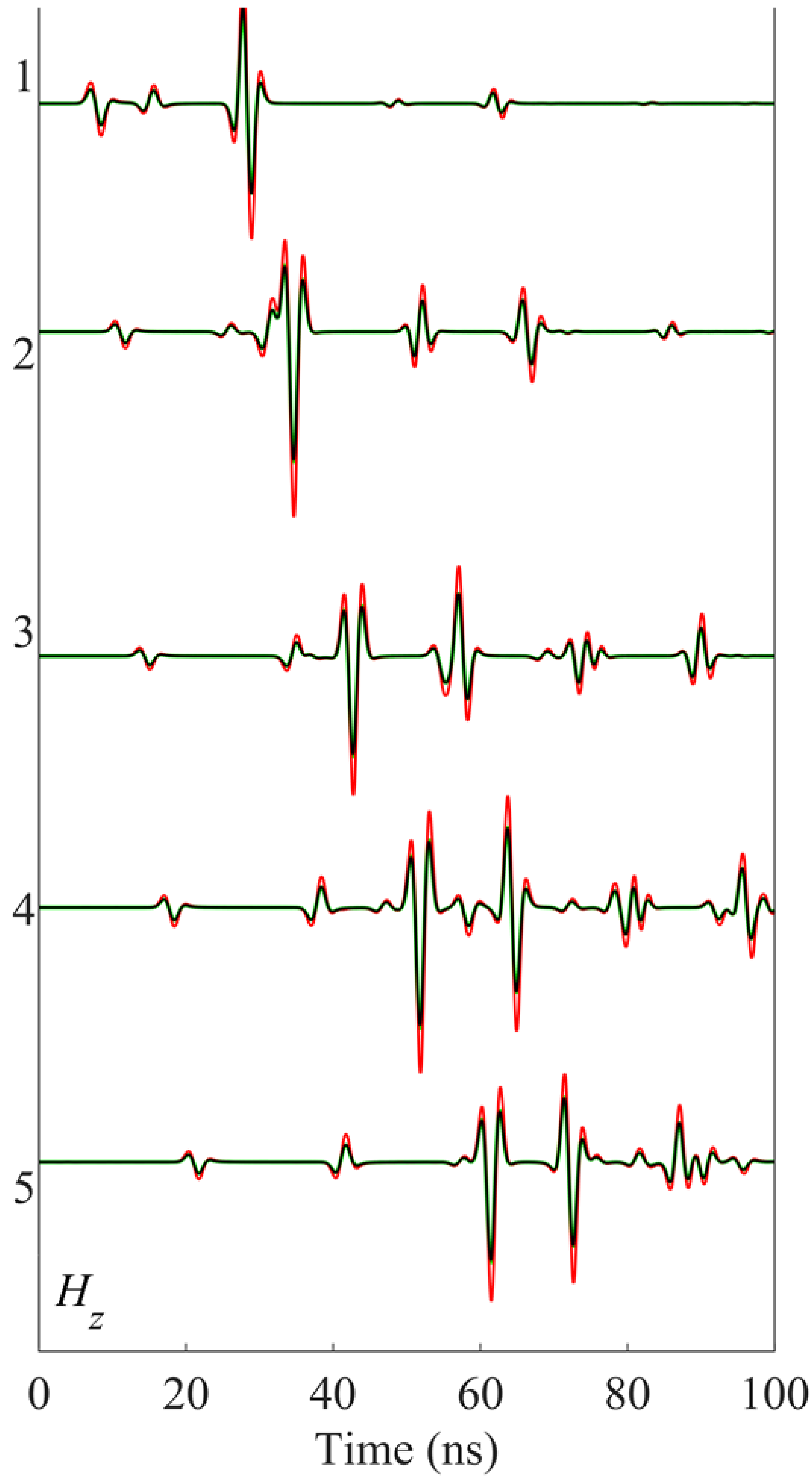

1
2
3
4
5
H_z
0
20
40
60
80
100
Time (ns)

Figure 3. The *Hz* component waveforms of 5 receivers located on the interface, calculated with the proposed method, gprMax and analytical GRTM solution.

(3) Topographic model

To display the performance of the proposed method on dealing with the irregular interface, a topographic model is tesed.

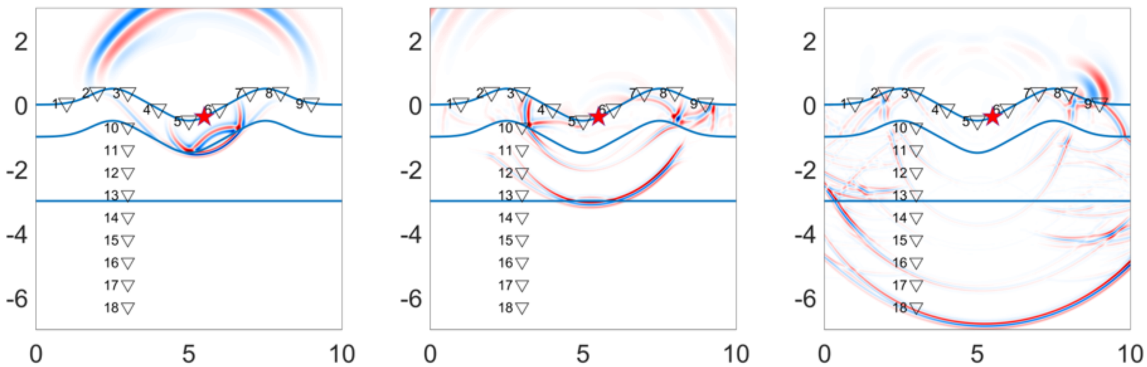


Figure 4. Snapshots of the wave propagation in the topographic model.

The snapshots obtained with our method is shown in Fig. 4. The effect of topography on the electromagnetic wave propagation is fully demonstrated.

CONCLUSION

- A curvilinear collocated finite difference method is extended to the application of computational electromagnetics;
- A noval equivalent interfacial method is introduced to accurately implement interface conditions at the discontinuity;
- The accuracy of the proposed method is verified and the implemention at the discontinuity is reliable;
- The collocated-grid finite difference can be a useful tool for electromagnetics.

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