Plane bed instability at the granular scale

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November 21, 2022

Abstract

Why is plane-bed topography unstable under certain flow conditions? We investigate the grain-scale mechanisms responsible for topographic instability at the onset of bedform development. Measurements of fluorescent tracer particle motion were used to estimate the ensemble mean particle activity, entrainment rate, hop distance, travel time, and particle velocity characteristic of flow conditions straddling the threshold stress for bedform development. Based on these data, we propose two hypotheses to explain the destabilization of planar topography with rising transport conditions. Hypothesis 1: plane-bed topography is unstable above a theory-predicted entrainment rate threshold that varies primarily as a function of particle diameter. Hypothesis 2: plane-bed topography is unstable above a threshold particle collision frequency that is proportional to bedload flux.The threshold particle collision frequency is predicted analogously to the propensity for congestion shockwaves in vehicular traffic flow theory.

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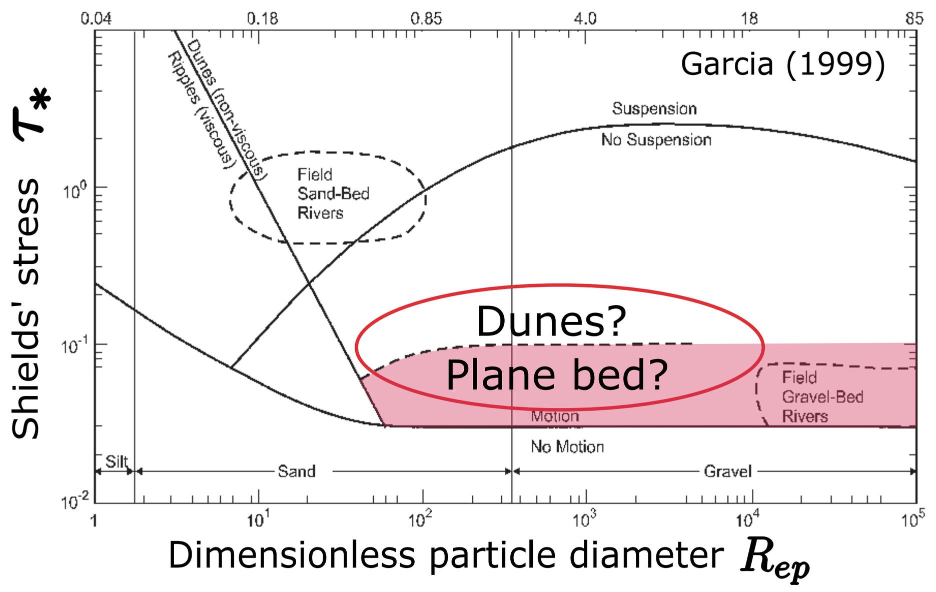
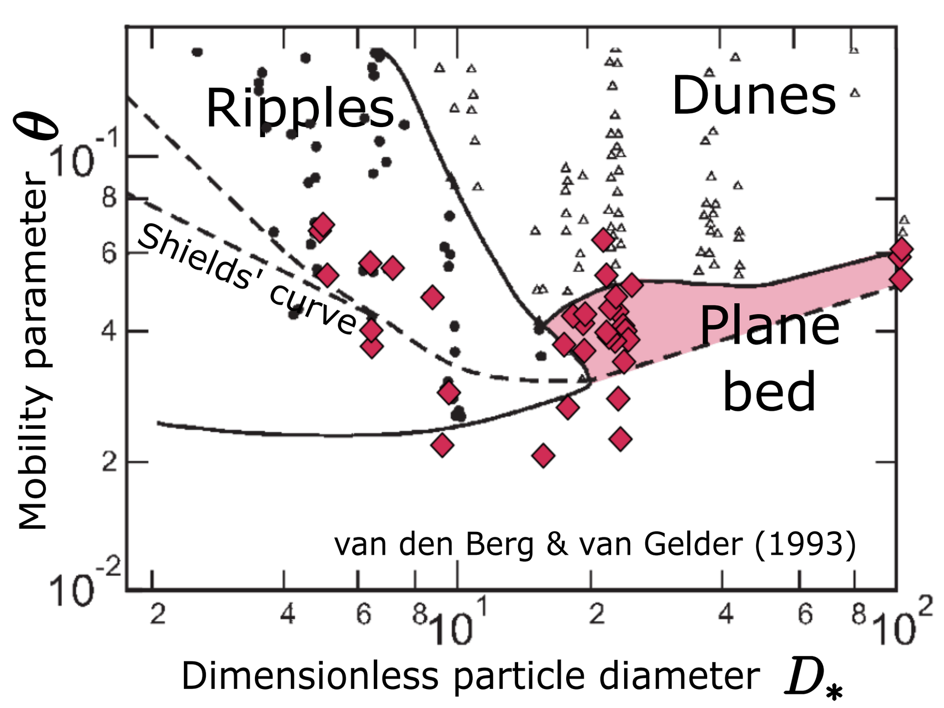
1 MOTIVATION

What process controls the transition from lower-stage plane bed to bedforms?





2 PARTICLE CONGESTION

Hypothesis: Onset of bedform development reflects a critical condition where particle collisions become frequent enough to influence bulk properties of transport.

Model for particle congestion:

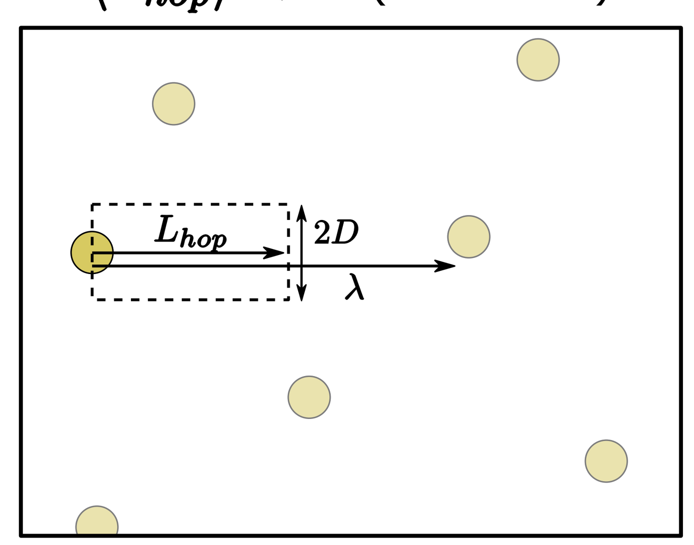
- Mean free path: $\lambda = \frac{1}{2DC_n}$
- Mean particle hop distance: $\langle L_{hop} \rangle$

Critical condition for bedform development:

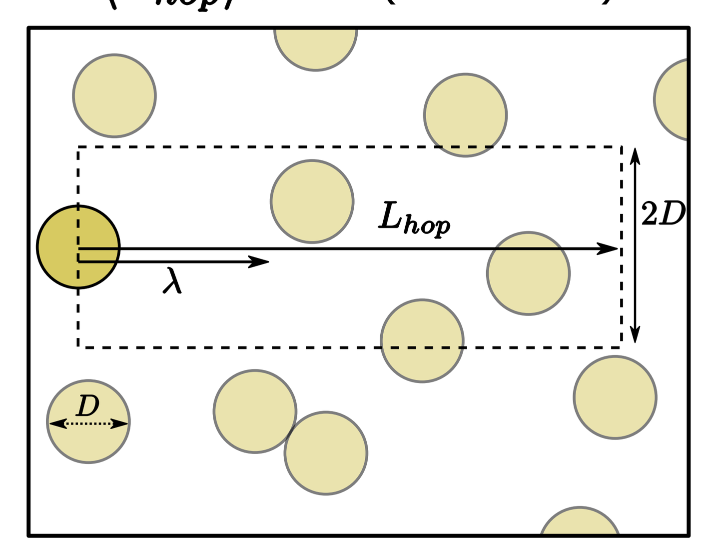
$$\boxed{\frac{\langle L_{hop} \rangle}{\lambda} \approx 1}$$

(average number of collisions per hop is 1, allowing propagation of collision shockwaves)

$\langle L_{hop} \rangle < \lambda$ (Plane bed)



$\langle L_{hop} \rangle > \lambda$ (Bedforms)



Which implies...

$$\frac{\langle L_{hop} \rangle}{\lambda} = \frac{12 q_b \langle T_p \rangle}{\pi D^2} = \frac{127}{\pi} q_b^*$$

Empirical Predictor for $\langle T_p \rangle$
Lajeunesse et al. (2010)

$$\langle T_p \rangle = 10.6 \sqrt{\frac{D}{Rg}}$$

Threshold Criterion:

$$\boxed{q_b^* = 0.025}$$

Variable Definitions:

- C_n Numer of active particles per unit streamed area [L^{-2}]
- D Particle diameter [L]
- T_p Hop duration [T]
- $\langle L_{hop} \rangle$ Mean hop distance [L]

Streamwise hop distance [L]:

$$L_x \approx L_{hop}$$

Particle activity [L]:

$$\gamma = C_n \underbrace{\frac{\pi}{6} D^3}_{\text{Particle volume}}$$

Unit bedload flux [L^{2}T^{-1}]:

$$q_b = \gamma \underbrace{\frac{\langle L_x \rangle}{\langle T_p \rangle}}_{\text{Avg. particle velocity}}$$

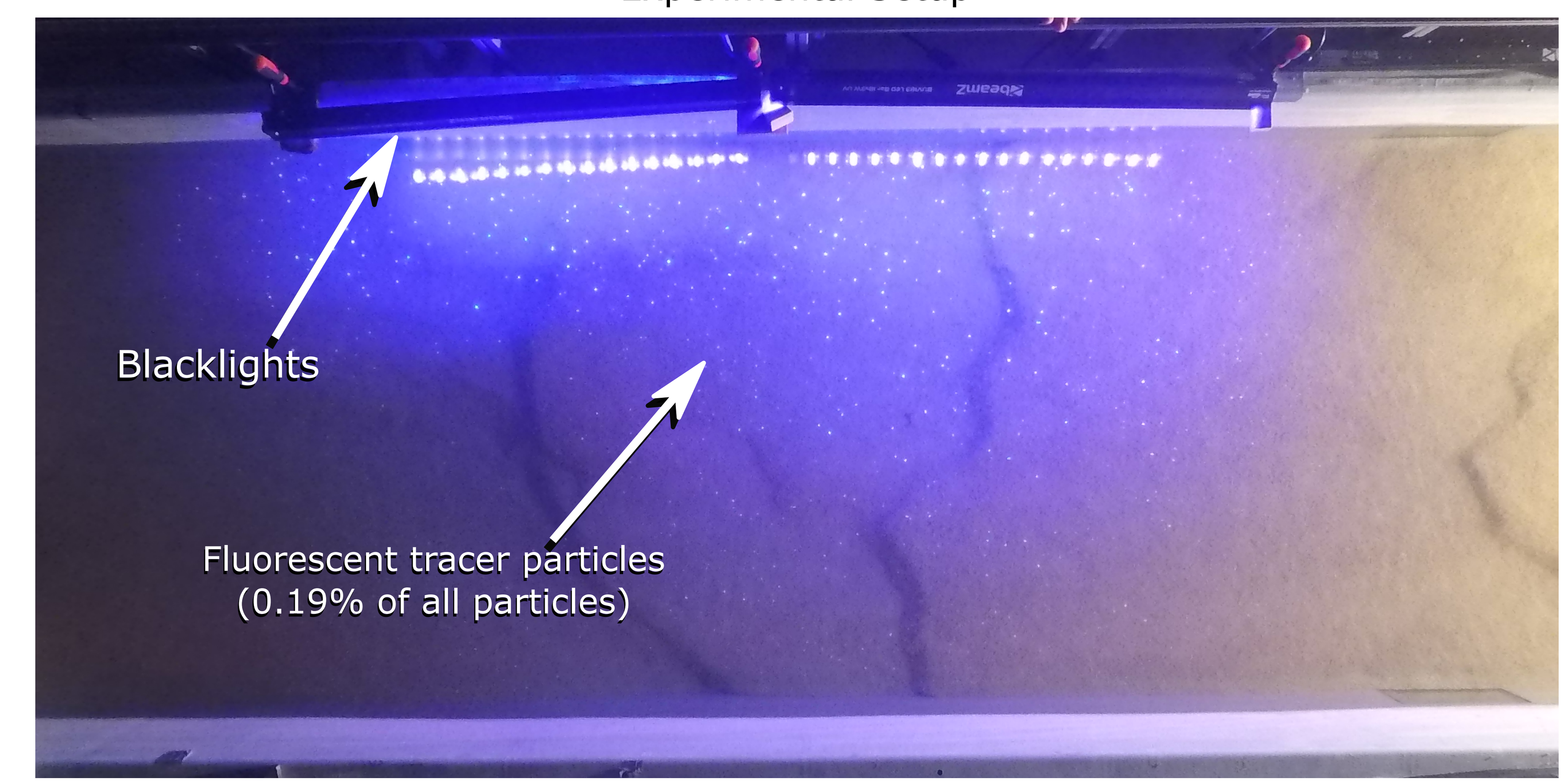
Bedload number:

$$q_b^* = \frac{q_b}{\sqrt{gRD^3}}$$

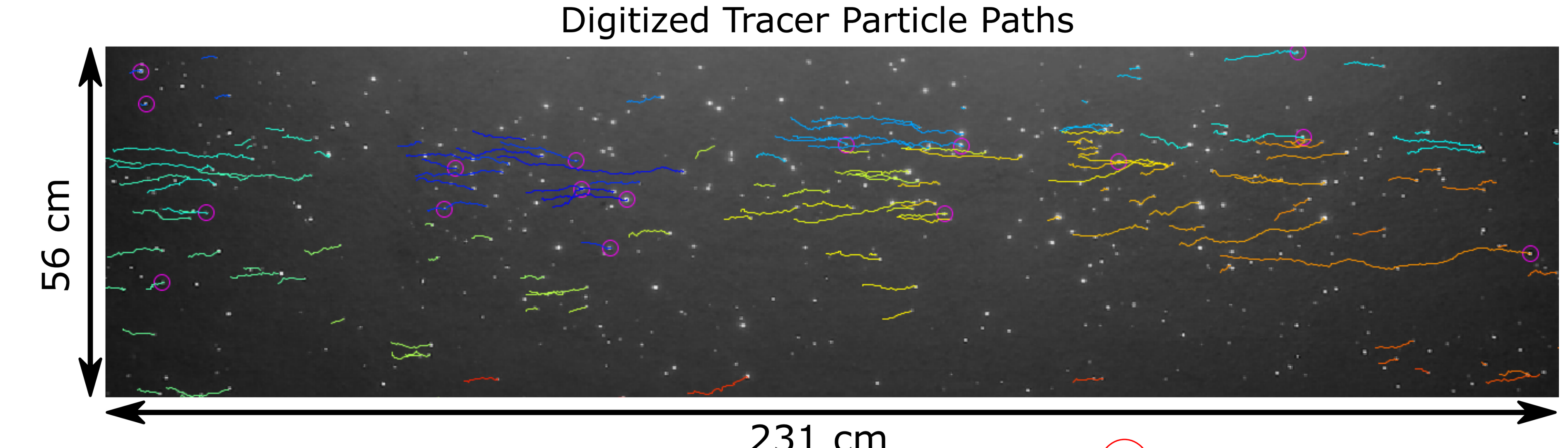
3 EXPERIMENTS

Objective: Measure relevant parameters using tracked tracer particle motions for (a) stable lower-stage plane-bed conditions and (b) unstable plane-bed conditions immediately after an increase in flow strength.

Experimental Setup

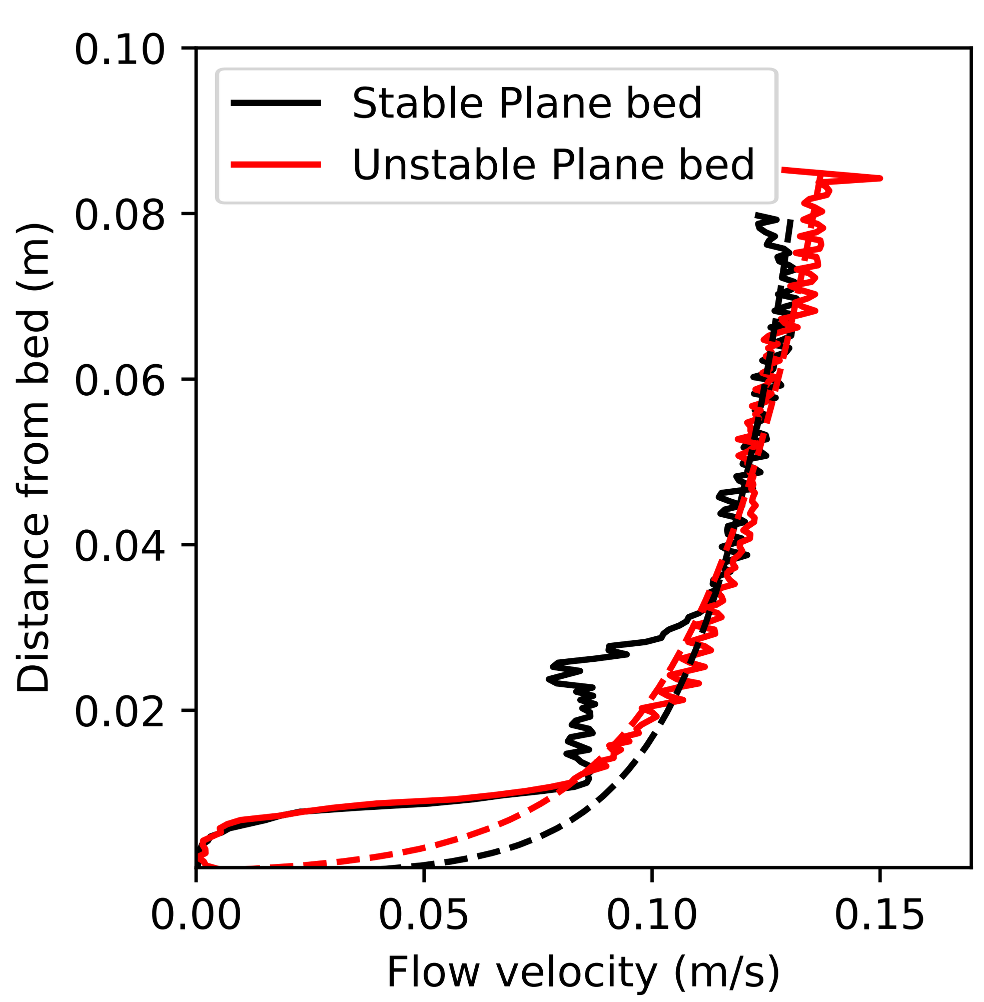


Digitized Tracer Particle Paths



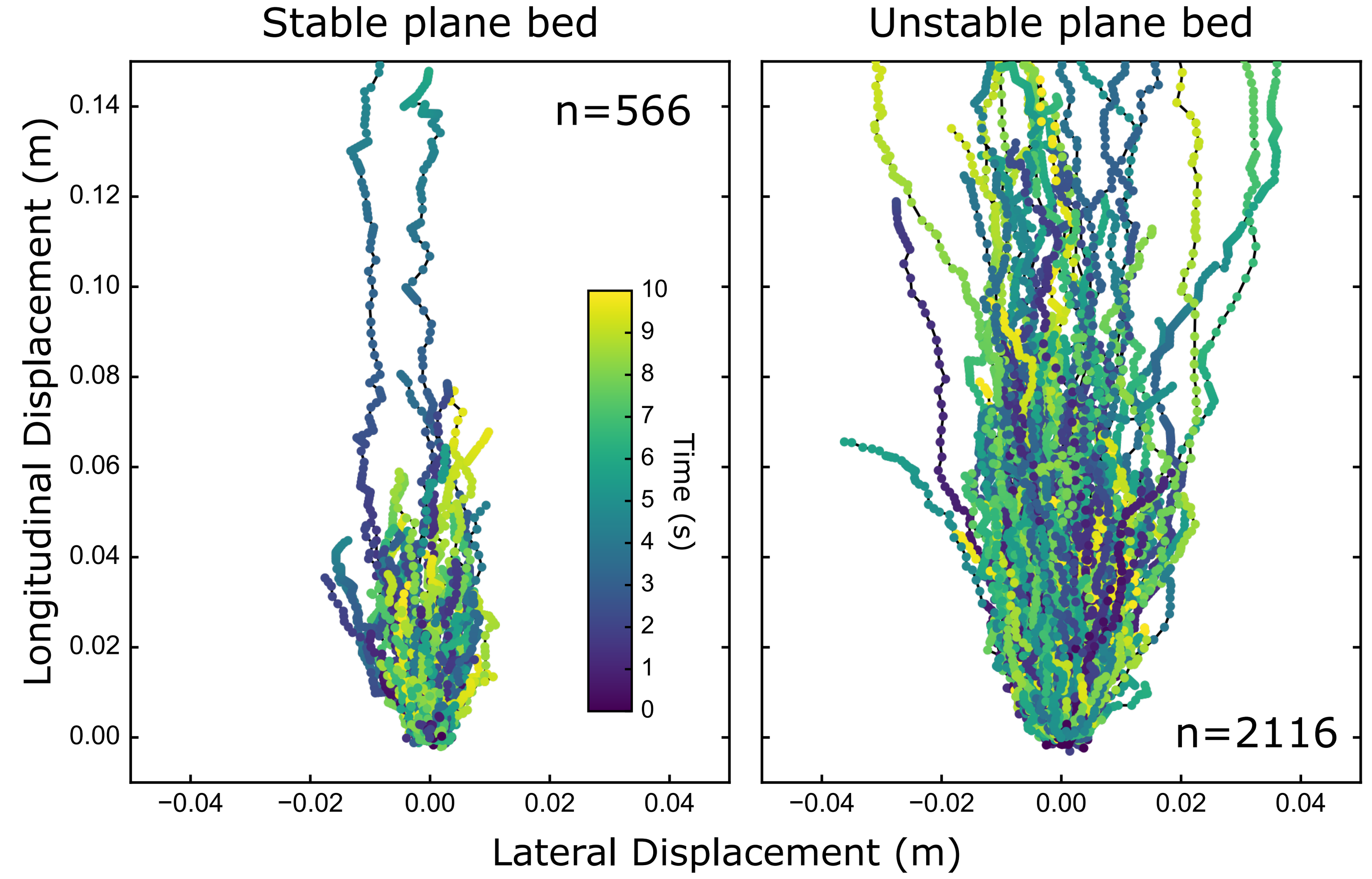
4 RESULTS

Flow conditions



Ensemble average flow velocity profiles recorded using moving ADV.

Particle motions

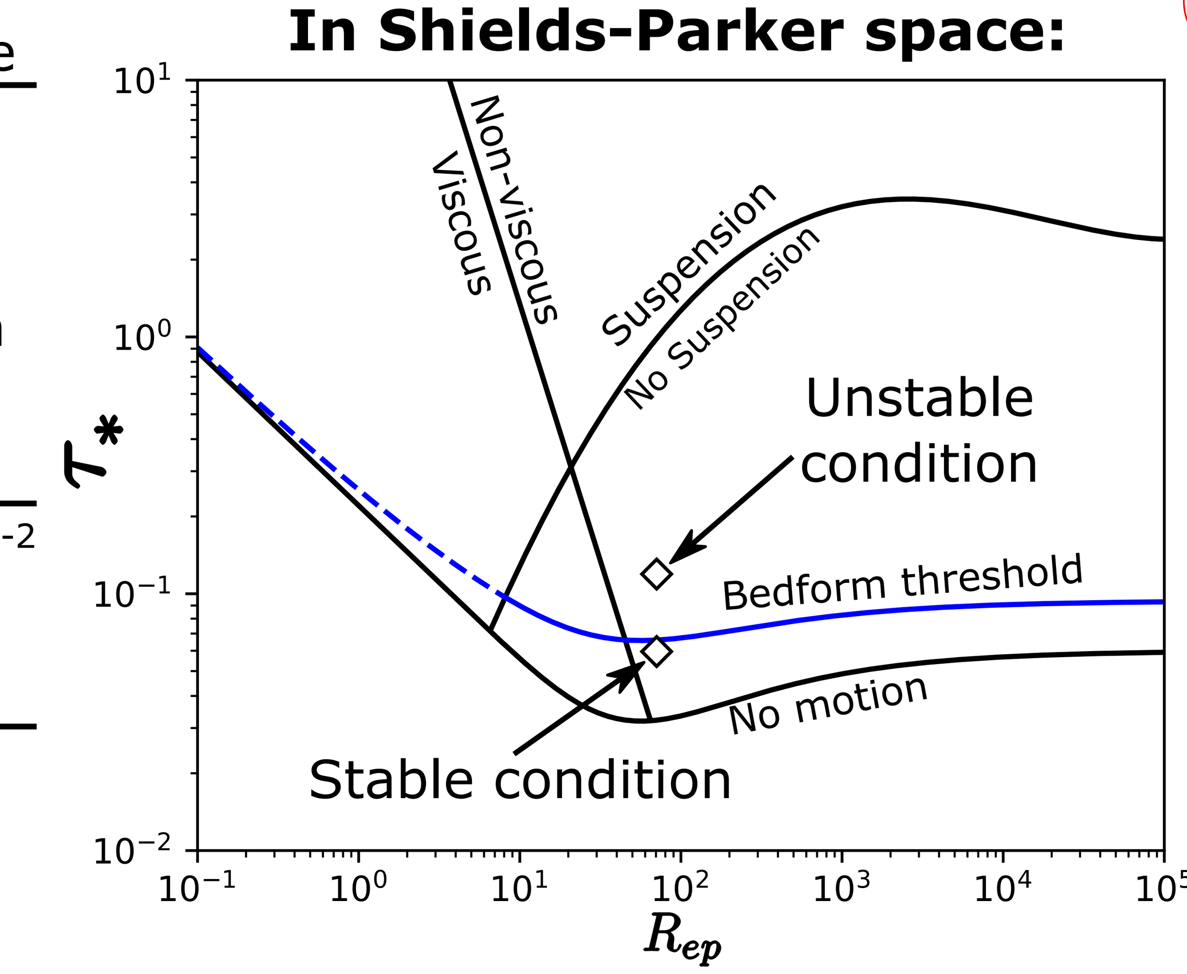


Visualization of tracer particle paths. Each plot shows all paths recorded over two ten second intervals (20 seconds total time represented for each condition).

Summary

	Stable	Unstable
Depth	11 cm	11 cm
Discharge	20 ls^{-1}	25 ls^{-1}
Width	1.2 m	1.2 m
D	2.1 mm	2.1 mm
R	0.055	0.055
R_{ep}	71	71
C_n	2962 m^{-2}	14300 m^{-2}
q_b^*	**0.0051**	**0.029**
$\tau_* - \tau_{*c}$	0.012	0.037
$\langle L_{hop} \rangle$	0.9 cm	1.4 cm
λ	8.0 cm	1.6 cm
$\langle L_{hop} \rangle / \lambda$	**0.11**	**0.85**

In Shields-Parker space:



Bedform threshold computed using Wong & Parker bedload equation as $\tau_* = \tau_{*c} + 0.034$.

5 CONCLUSIONS

- Plane-bed topography is stable when the mean free path significantly exceeds the mean particle hop distance. This suggests that congested transport conditions destabilize plane bed topography leading to bedform development.
- The ratio of particle hop distance to mean free path varies in proportion to the Einstein bedload number near the threshold of sediment motion. The resulting bedform threshold is consistent with observations across a wide range of conditions.